ARITHMETIC IN PRIMARY SCHOOL IN BRAZIL: END OF THE NINETEENTH CENTURY

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The arithmetic is part of mathematical knowledge based on the idea of the number. The teaching of intuitive calculation in Brazil in primary education level at the end of the nineteenth century and early twentieth century seems to be influenced directly by the "Cartas de Parker". These arithmetic charts based on the ideas of Pestalozzi, Froebel and Herbart were diffused in arithmetic textbooks and educational journals, testimonies of their strong influence in Brazil. This article is based on methodological presuppositions of the Cultural History, of the History of School Disciplines and the studies on the School Culture.

Keys-words: Arithmetic, Intuitive Calculation, Cartas de Parker, Grube's Method, Elementary level.

INTRODUCTION

This article presents a partial result of the literature research related to a doctorate thesis, still under development. It aims to investigate the historical route Mathematical Education in Brazilian primary education teaching. It seeks to analyze the part "to count" of "the school of reading, writing and counting"; and includes understanding the process of its teaching by seeking answers to questions like, for example: which textbooks were adopted for the teaching of arithmetic at school? What was the role of Psychology in the evolution of the textbooks of arithmetic for primary education teaching? How were the contents of arithmetic school in the textbooks modified? What kind of modifications have the arithmetic's textbooks been under to?

Considering the contributions of the Cultural History, the History of the School Disciplines and the studies on the School Culture, this research focuses the documentary sources such as textbook, school files, legislative texts relating to teaching as well as old daily materials (teachers' personal files, pupils' books, tests, periodic school magazines and exams questions)[1] .

According to Enfert (2003), unlike what occurred to the research of the French's history of the primary education teaching, the history of the teaching of mathematics at this level did not receive the attention which it deserves. Except some cases of specialized studies, research, in a general way, mostly treated mathematics teaching at the secondary or higher level. The history of this discipline has not been treated as a whole (Arithmetic, Geometry, Geometrical Drawing, Algebra, Accountancy, etc), nor over its long duration.

In the History of the School Disciplines, Chervel (1998) defines a particular phenomenon called "vulgata". At each time, the teaching given by teachers is, *grosso modo*, identical, similar for the same discipline and at the same level. All textbooks, or nearly all, say the same thing then, or almost. The concepts, the adopted terminology, the succession of the headings and the chapters, the organization of the corpus of the knowledge, even the examples or types of exercises performed are identical, except for some small variations. These variations justify the publication of new textbooks although they present only tiny variations.

The description and the analysis of the "vulgatas" are fundamental tasks for the School Discipline's historian. If it is not possible to examine into the entire editorial production carefully, they must determine a sufficiently representative corpus of their various aspects. This is the only way that the historian can arrive at concrete and conclusive results.

The research in the teaching of mathematics in Brazil in primary education level at the end of the nineteenth century, particularly among textbooks of representative authors' of their community, revealed a reference particular called "Cartas de Parker". Their contents appears as a model and reference adopted by several textbooks published at the beginning of the twentieth century, and it seems to be like a "vulgata" and influences the teaching of the rudiments of calculus on this level of education.

INTUITIVE CALCULATION

According to Buisson (1880), intuitive calculation is a term which means a way of teaching the first elements of calculation. This methodology was borrowed from Germany and diffused in Russia, in the Netherlands, in Sweden and found a strong adhesion in the United States. This way of teaching was called Grube's method.

In 1842, Grube published in Berlin the first edition of his *Leitfaden für das Rechnen in der Elementarschule nach den Grundsätzen einer heuristischen Methode* (Guide for calculation in the elementary classes, following the principles of a heuristic method). This "*Essai d'instruction éducative*", as he called it, after causing warm discussions, was approved by membership of the class of teacher. His book was successfully in agreement with the new system of weight and measurements and got to its 5th edition in 1873. Many textbooks, in all the languages, were reproduced, imitated or applied the Grube's method.

The Grube's method consists in making the pupils to do themselves, by intuition, the fundamental operations of elementary calculation. Such method aims to make them known the numbers: to understand an object, which it is not only to know its name, but to apprehend it in all its forms, in all its states and in its various relations with other objects; to be able to compare it with others, to follow the transformations, to write it and measure it, compose it and break up of them, at their will.

By treating the numbers as unspecified objects that are familiar to the pupils, Grube opposes to the old long-established method in arithmetic which is calculated to teach the first four processes of addition, subtraction, multiplication, division, in the order in which they are named, finishing addition with small and large numbers, before subtraction is begun, and so on. An improvement on this method consisted in excluding the larger numbers altogether at the beginning and dividing the numbers on which the first four processes were taught, into classes, or so-called *circles*. The pupil learns each of the four processes with the small numbers of the first circle (i.e., from 1 to 10) before larger numbers are considered; then the same processes are taught with the numbers of the second circle, from 10 to 100, then of the third, from 100 to 1000, and so on.

Grube, however went beyond this principle of classification. He discarded the use of large numbers, hundreds and thousands, at the beginning of the course, as others had done before him; but instead of dividing the primary work in arithmetic into three or four circles or parts only, i.e., from 1 to 10, 10 to 100, etc., he considered each number as a circle or part by itself. He recommended that the pupil should learn each of the smaller numbers in succession, and all the operations within the range of each number, before proceeding to the next higher one, addition, subtraction, multiplication, and division, before proceeding to the consideration of the next higher number.

Treating, for instance, the number 2, Grube leads the child to perform all the operations that are possible within the limits of this number, i.e., all those that do not presuppose the knowledge of any higher number, no matter whether in the usual classification these operations are called addition, subtraction, multiplication, or division. The child has to see and to keep in mind that

$$1 + 1 = 2, \quad 2 \times 1 = 2, \quad 2 - 1 = 1, \quad 2 \div 1 = 2, \text{ etc.}$$

The whole circle of operations up to 2 is exhausted before the pupil proceeds to the consideration of the number 3, which is to be treated in the same way.

The four processes are the direct result of comparing, or “measuring”, as Grube calls it, two numbers with each other. Only when the child can perform all these operations, for instance, within the limits of 2, can it be supposed really to have a perfect knowledge of this number. So Grube takes up one number after the other, and compares it with the preceding ones, in all imaginable ways, by means of addition, subtraction, multiplication and division. This comparing or “measuring” takes place always on external, visible objects, so that the pupil can see the objects, the numbers of which he has to compare with each other.

This methodology does not only prepare the pupil to study the arithmetic, but it offers an advantage over the other methods about the necessary conditions to the promotion of mental calculation. The pupils subjected to this method do not become slaves of the numbers and pencils and their “armed operations”.

Soldan (1878) exposes the six most important points about the Grube's method of teaching:

a) *Language* - the language is the only way that the teacher will have access to what the pupil is thinking, because it is not requested any records of the calculations made by them. A complete answer must be required from the pupil, because it is only by doing it that the teacher will be able to evaluate what the pupil learned or not.

b) *Questions* - teachers should avoid asking too many questions. Such questions, moreover, as, by containing half the answer, prompt the pupils, should be omitted. The pupils must speak themselves as much as possible.

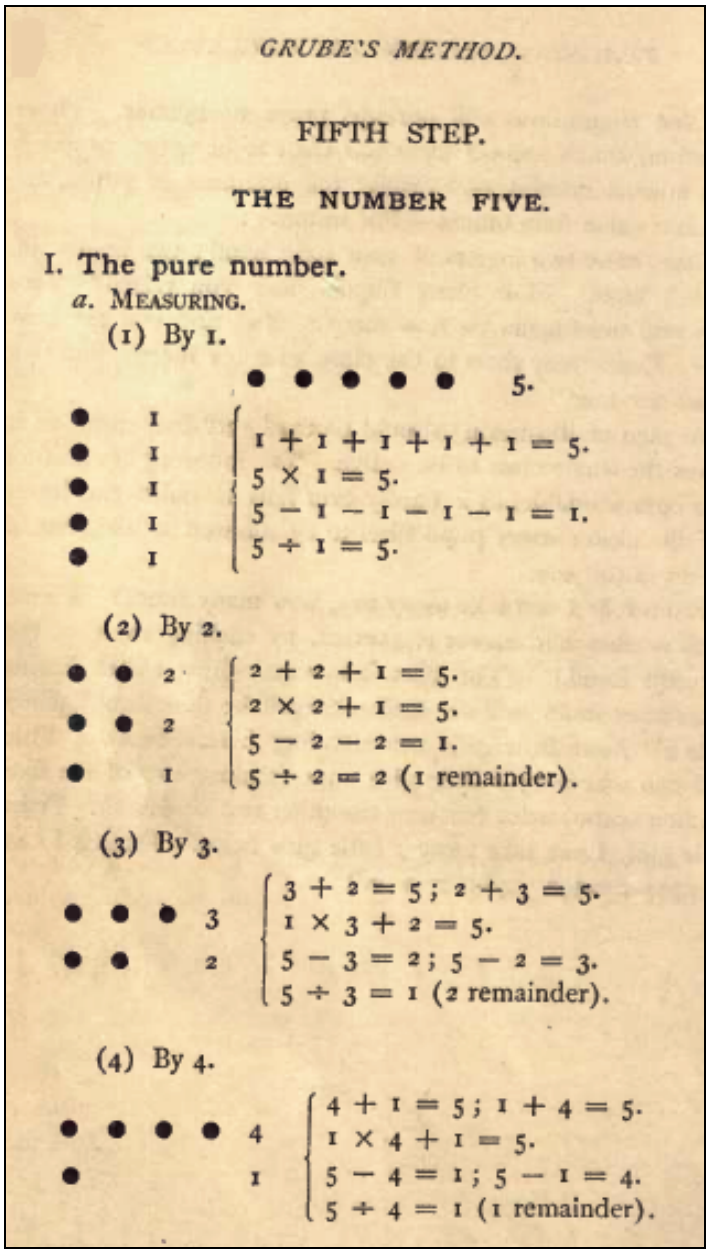
c) *Individual recitation* and *jointly with the class* - In order to animate the lesson, answers should be given alternately by the pupils individually, and by the class in concert. The typical *numerical diagram* [2] are especially fit to be recited in concert.

d) *Illustration* – Every process and each example should be illustrated by means of objects. Fingers, lines, or any other objects can be used to answer the purpose, but some kinds of objects must always be presented to the class.

e) *Comparison and measurement* – the operation of each new stage consist in comparing or measuring each new number with the preceding ones. Since this measuring can take place either in relation to difference (arithmetical ratio), or in relations to quotient (geometrical ratio), it will be found to comprise the first four rules. A comparison of two numbers can only take place by means of one of the four processes. This comparison of the two numbers, illustrated by objects, should be followed by exercises of fast-solving problems and a view of the numerical relations of the numbers just treated, in more difficult combinations. The latter offer a good test as to whether the results of the examination of the arithmetical relations of the number treated have been converted into ideas by a process of mental assimilation. In connection with this, a sufficient number of examples in *applied numbers* are given to show that applied numbers hold the same relation to each other that *pure numbers* [3] do.

f) *Writing of figures* – on neatness in writing the figures, the requisite time must be spent. Since an invariable diagram for each number will re-appear in all stages of this course of instruction, the pupil will soon become able to prepare the work for each coming number by writing its *numerical diagrams* on their slates.

The study of the Grube's methodology turns possible to hypothesize the influences of Grube's methodology into the publications of Mr. Parker.



GRUBE'S METHOD.

FIFTH STEP.

THE NUMBER FIVE.

I. The pure number.

a. MEASURING.

(1) By 1.

● ● ● ● ● 5.

● 1
● 1
● 1
● 1
● 1

1 + 1 + 1 + 1 + 1 = 5.
5 × 1 = 5.
5 − 1 − 1 − 1 − 1 = 1.
5 ÷ 1 = 5.

(2) By 2.

● ● 2
● ● 2
● 1

2 + 2 + 1 = 5.
2 × 2 + 1 = 5.
5 − 2 − 2 = 1.
5 ÷ 2 = 2 (1 remainder).

(3) By 3.

● ● ● 3
● ● 2

3 + 2 = 5; 2 + 3 = 5.
1 × 3 + 2 = 5.
5 − 3 = 2; 5 − 2 = 3.
5 ÷ 3 = 1 (2 remainder).

(4) By 4.

● ● ● ● 4
● 1

4 + 1 = 5; 1 + 4 = 5.
1 × 4 + 1 = 5.
5 − 4 = 1; 5 − 1 = 4.
5 ÷ 4 = 1 (1 remainder).

Fig. 1 – The Grube's Method.

INTUITIVE'S METHOD AND THE "CARTAS DE PARKER" (NUMERICAL DIAGRAMS)

Research on the teaching of mathematics in Brazil in primary education level at the end of the nineteenth century through the sources, revealed a particular reference to Mr. Parker, this eminent American teacher, author of "Cartas de Parker".

According to Montagutelli (2000), Francis Wayland Parker (1837-1902) developed an educational system which was recognized by John Dewey as the "father of progressive education", also inspiring a few years later Granville Stanley Hall. Coming from a family of educators, Parker became a teacher when he was sixteen years, and later also served in the army at the time of the Succession War in the United States. At the end of the hostilities, he took the direction of a school in Ohio.

In 1872, he did a study trip in Europe: in Germany, he became familiarized with Herbart's pedagogy. It is possible that he took note of the Grube's method by this time. In 1875, he got back to the United States, where he became the supervisor of the schools of the town of Quincy, in Massachusetts. By this time, Parker develops the so-called "Quincy System". In an atmosphere without the rigid discipline imposed in the majority of the schools of this time, the pupils read newspapers or texts composed by their teachers; on the basis of knowledge, they approached the new concepts concretely followed by working groups besides also the practice of drawing and music.

Parker published five books on education: *Talks on Teaching* [4] (New York, 1883); *The Practical Teacher* (1884); *Course in Arithmetic* (1884); *Talks on Pedagogies* (1894) and *How to Teach Geography* (1885).

An important educational journal of the beginning of the twentieth century, "Revista de Ensino", created in 1902 by the Association of Public's Teacher of São Paulo (Brazil), devoted in several editions, in its section called Teaching Practice, several articles about the way of using the "*Cartas de Parker*".

According Pierre Ognier (1984), the educational journal, is one of vast documentary corpus, because it is a living witness evidence of teaching methods from an era and the conceptions of moral ideology, social and politics of a professional group. This makes it an excellent observatory, a picture of the ideology that governs.

Accordingly, it is a practical guide to everyday educational and school, allowing the researcher to study the pedagogical thought of one determined sector or a social group from the analysis of reported speech and resonance of the issues discussed within and outside the universe school.

This educational publication, "Revista de Ensino", over a number of editions, published about fifty charts, diffusing them in Brazil. These charts concretize the appropriation by Parker of the *numerical diagrams* stated in the Grube's method. They represent the way of treating the teaching of Arithmetic in an intuitive way. Moreover, they are presented like references for the development of textbooks of mathematics for the first levels.

By a heuristic process, i.e., a procedure which consists in discovering by the pupil what exactly wants to teach to him, the teacher questioned the pupil in front of the chart. Example extracted the fourth chart (see Figure 2): in the items ***h***, ***i*** and ***l*** are representative drawings of the number *ten*. And by the observation, the pupil should give his answers or make remarks about this number formation. Thus, in the letter ***h***, it is needed two *five* to have a *ten*; in letter ***l*** we find *three* + *three* + *four* to have a *ten*; in letter ***i***, it is needed five times of *two* to have a *ten*. This way the pupil learned how to compose and break up the number into equal or unequal parts. The idea of the addition, subtraction, multiplication as of division is concomitantly subjacent with this process.

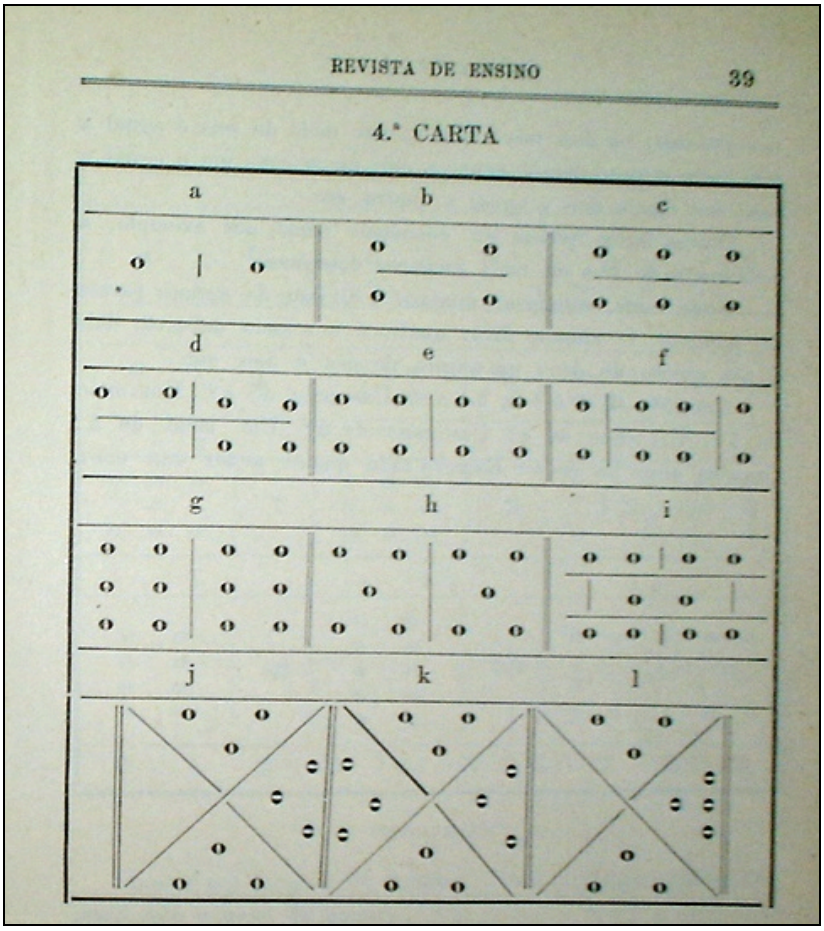


Fig. 2 – 4th Carta de Parker

In Brazil, in addition to the quotations and the articles of "Revista de Ensino" on "Cartas de Parker", an important textbook of the beginning of the twentieth century, written by Arnaldo de Oliveira Barreto, *Série Graduada de Matemática Elementar*, published by the Salesians, in São Paulo, in 1912, quotes the name of Parker and the "Cartas de Parker" in the foreword signed by Oscar Thompson, director of the Normal School (Teacher School). There are also quotations in the presentation of the book and the final comments relating to the conferences pronounced by Parker.

The effective methodology of teaching during this time treated intuitive method which had been adopted in second half of the nineteenth century in the European, American and Brazilian schools; it was based on the ideas of Pestalozzi and Fröbel.

For Valdemarin (1998), the intuitive method was influenced directly by the current empiric of philosophy, carried by Francis Bacon and John Locke (seventeenth century) by determining the procedures of teaching based on the *observation*.

This method was presented in the form of a response to the abstract character and little utility of the instruction up to that point of use, by developing new didactic materials and a diversification of the teaching activities. It also brought others

innovations that were spread on successive Universal Expositions which were organized for the diffusion of teaching practices, like the ones held in London (1862), in Paris (1867), Vienna (1873) and Philadelphia (1876).

The presence of the intuitive method in teaching of arithmetic reveals a new teaching method which is opposed to the preceding way of teaching where the memorizing of the knowledge was privileged. The "Cartas de Parker" are the elements that made possible to associate the influence of this intuitive movement of the teaching of arithmetic in Brazil at this time. Evidences of dissemination of this methodology are present in articles in major educational journals such as the "*Revista do Ensino*" and of the textbooks like "*Aritmética Escolar*" of Ramon Roca Dordal [5] or "*Contador Infantil*" of Heitor Lacerda [6], among others.

CONCLUSION

According to Chervel (1998), the first task of the School Disciplines's historian is to study the explicit contents of disciplinary teaching. The study of a "vulgata", configured as "Cartas de Parker" enables us to connect the form and the contents of the teaching of mathematics in the primary education level at the end of the nineteenth century - beginning of the twentieth century in Brazil, becoming an important element of the writing of the History of Mathematical Education in Brazil.

Moreover, this study allows hypothesizing that the relation is given at educational backgrounds of the ideas that circulated in the late nineteenth century in Europe and materialize in Brazil on publications of textbooks and articles in educational journals. This seems to point towards the influence of intuitive teaching, conceived by their European authors as a pedagogical tool capable of reversing not only the inefficiency of school, but also reduce the existing economic development gap, since the emergent industrial labour demanded literate and think quickly and creatively individuals.

According to Valdemarin (1998) this inefficiency of school teaching was characterized by the formation of pupils with insufficient reading and the writing notions and also without satisfactory concepts of calculation, mainly because of the learning based exclusively on memory, giving priority to the abstraction, enhancing the value of repetition to the detriment understanding and impose contents without examination and discussion.

The explicit proposal of the "Cartas de Parker" appears to be consistent with the aspirations of a time that rejects the methods primarily based on the memory and develops the observation as a way of effective training of calculation.

It is through historical studies that we have access the way that great teaching thinkers thought about the teaching of mathematics and the way it echoes in Brazil.

NOTES

1. This research is subordinated to one of the thematic projects which are developed by the GHEMAT – Grupo de Pesquisa de História da Educação Matemática do Brasil (Group of Search for History of the Mathematical Education of Brazil): "A EDUCAÇÃO MATEMÁTICA NA ESCOLA DE PRIMEIRAS LETRAS, 1850-1950" coordinated by Prof. Dr. Wagner Rodrigues Valente and financed by the FAPESP. Through a financial support obtained from CNPq – Conselho Nacional de Desenvolvimento Científico e Tecnológico (National Council of Technological and Scientific Development), I have been developed my research of doctorate at INRP/SHE (Institut National Recherche Pédagogique, Service d'Histoire de l'Education – Paris – France) under supervision of Prof. Dr. Alain Chopin (05/2008 to 04/2009).

2. The *numerical diagram* of the Grube's method will be presented later on in this article as "Cartas de Parker".

3. A *pure number* also called an *abstract number*, which is that makes mention only quantity. Four, thirty, twelve are examples of *pure numbers*. Applied to an object, it will be called a *applied number* or *concrete number*. Thirty apples, four trees, three meters, are examples of *applied numbers* or *concrete numbers*.

4. This book was translated into Portuguese by Arnaldo de Oliveira Barreto in 1909 and edited by Livraria Francisco Alves: "*As Conferências de Parker*".

5. See article Costa, D.A., Valente, W.R. (2007). Análise da Arithmética Escolar de Ramon Roca Dordal. In: Simpósio Internacional do Livro Didático, 2007, São Paulo. Livro Didático - Educação e memória. São Paulo: Centro de Memória da Educação – FEUSP, v.1.

6. See *Revista do Ensino*, 1902, p.146.

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