HOW CAN GAMES CONTRIBUTE TO EARLY MATHEMATICS EDUCATION? – A VIDEO-BASED STUDY

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In recent years early mathematics education has become an area of increased interest and research activity. Consequently, a growing number of educational programs and especially developed materials are published and used in kindergarten. Games, however, are an often underestimated yet promising approach for the early years. We asked if, how, and under what conditions early mathematics education (3- to 6-year-olds) can be organized with everyday materials, for example games. In a two-phase design, we first developed criteria based on didactical considerations to assess materials. In the following empirical study we videotaped children using selected materials. The research resulted in first descriptions of the conditions under which potentially suitable materials can develop mathematical potential in young children.

Keywords: number concept, arithmetic skills, early childhood education, kindergarten, learning materials, video study, grounded theory, games

1 THE CONSTRUCTION OF NUMBER CONCEPT

Since the late 1990s a growing research activity can be observed in the field of *early mathematics education*. Within this research there is a consensus about the *contents* that should be part of a *preschool curriculum*. The answers differ in detail but many authors focus on fundamental ideas or important aspects of mathematical thinking like number and quantitative thinking, geometry and spatial thinking, algebraic reasoning (patterns, relationships) or data and probability sense (cf. Ramani & Siegler, 2008; Peter-Koop & Grüßing, 2007; Clements & Sarama, 2007a/b; Baroody et al, 2006; Lorenz, 2005; Balfanz et al, 2003; Krajewski, 2003; Arnold et al, 2002;). Some authors also mention process ideas like mathematization and communication or argumentation (cf. Perry et al, 2007; Clements & Sarama, 2007b, 463).

Our research relates to the *construction of number concept and quantitative thinking*, because "for early childhood, number and operations is arguably the most important area of mathematics learning. In addition, learning of this area may be one of the best developed domains in mathematics research" (Clements & Sarama, 2007b, 466). Consequently, there are not only a lot of games and materials for kindergarten which address this area, but there also exists a well-developed theory on the construction of number concept our research can be based on. Although our research concentrates on this area we know that early childhood education needs a broader approach and a widespread fostering of abilities.

In the past fifty years, the research on children's development of quantitative thinking and construction of number concept has seen a change from Piaget's *logical-foundation-model* to the current *skills-integration-model* (cf. Baroody et al, 2006; Clements, 1984; Peter-Koop & Grüßing, 2007).

Piaget's developmental theory emphasizes that the construction of number concept depends on the development and synthesis of logical thinking abilities, especially of classifying and ordering (cf. Piaget, 1964, 50ff). According to this view counting does hardly benefit the construction of number concept but might rather be an obstacle. The logical thinking abilities are not available until concrete operational stage, that is at the age of seven (cf. Piaget, 1952, 74). Therefore the construction of number concept is not possible until primary school and activities to foster this goal do not make any sense in kindergarten. In the pedagogical practice Piaget's theory led to set theory that postponed teaching number and arithmetic concepts until preschool and primary school (cf. for example Neunzig, 1972).

Particularly since the late 1970s Piaget's theory has given rise to a lot of criticism. In contrast to Piaget, Gelman and Gallistel (1978) underline the meaning of counting for the construction of number concept. In their opinion counting principles are innate and therefore available in kindergarten. Starkey and Cooper (1980) demonstrated that even infants are capable of distinguishing sets of small numbers and Wynn (1998) even speaks of infants' sensitivity to numbers. Thus nowadays there is a wide consensus that *preschoolers* show *considerable informal arithmetic knowledge* in spite of the existence of large inter-individual differences (cf. Baroody et al, 2006; Schipper, 1998). A well-developed number concept is not naturally given but requires nurturing: Learning number words for example may help to construct an understanding of number. There is also agreement on the *skills-integration-model*. The following skills seem to be central for the years before school attendance (cf. Resnick, 1989; Gerster & Schultz, 2000; Krajewski, 2003; Lorenz, 2005):

- *Perceptual and conceptual subitizing*: Perceptual subitizing is the spontaneous recognition of recurrent configurations up to sets of four that are associated with number words; whereas conceptual subitizing allows the instant recognition of sets bigger than four. Conceptual subitizing requires visual structuring processes (numbers as units of units) (cf. Clements 1999).
- *Verbal and object counting*: Verbal counting extends from simply reciting the number line (string level) to skills like counting forwards, backwards, counting on, counting in steps (bidirectional chain level) (cf. Fuson, 1988, 34–60); object counting contains counting sets and naming the number word (cardinality rule); and counting out objects to a given number word.
- *Comparing and ordering sets*: Comparison and ordering of sets is possible on a perceptual level (more, less, even) and on a numerical level (5 is more than 3). For small sets it is possible by perceptual subitizing.

- *Part-whole-connections, composing and decomposing sets*: These skills are closely connected to conceptual subitizing and the numerical comparison of sets. Understanding that a number is composed of other numbers is seen as the central skill for the construction of number concept (cf. Resnick 1989).
- *Beginning addition and subtraction* with material and in concrete contexts: Children can use either counting procedures and/or visual structuring processes to solve first arithmetical problems.

In a longitudinal study Krajewski (2003) proved that some of these skills are of great importance for later school achievement and success. They even allow the statistical prediction of marks in primary school mathematics.

2 RESEARCH QUESTIONS

In recent years different approaches to early mathematics education have been developed. One can distinguish at least two types:

- *Course-like educational programs* in kindergarten, focussing on the purposeful construction of specific mathematical skills, sometimes even following a relatively strict curriculum (e.g. in Germany Preiß, 2004/05; Krajewski et al, 2007; in the USA Clements & Sarama, 2007a; Ramani & Siegler, 2008).
- *Implementation of games, educational materials and informal learning opportunities* in the daily kindergarten practice, subsequent to joint activities, realized in a playful way, aiming at a wide spread fostering of children's abilities (e.g. in Germany Hoenisch & Niggemeyer, 2004; Müller & Wittmann, 2002/04; e.g. in the USA Balfanz et al, 2003).

Our study refers to the latter approach which seems promising but often underestimated. Examples for materials can be

- well-known *commercially available games* like common board games, card games and dice games,
- special *educational games and materials* to foster arithmetic skills which can be either purchased or developed by the educational staff (and the children) themselves.

The goal of our study is to analyze the role of these materials in early mathematics education. In detail we ask the following research questions:

1. What (theoretical) potential for children's construction of number concept do these materials have in principle?
2. Under what conditions can potentially suitable games and materials can develop their mathematical potential?
3. In which way can games contribute to early mathematics education? Is it possible to organize early mathematics education, at least partially, with games?

3 RESEARCH METHODS AND METHODOLOGY

Our research follows a qualitative design. According to the research questions it is a two-phase design (cf. figure 1) that will lead to a (grounded) theory about the conditions for a substantial and rich mathematical learning environment (cf. Strauss & Corbin, 1996):

- The first phase is a *theoretical analysis of games and educational materials*. We established theory-driven criteria on the basis of didactical considerations (cf. section 1) to assess the suitability of materials for the construction of number concept (cf. Schuler, 2008).
- The second phase is an *empirical evaluation of selected, theoretically proved games and educational materials*. A theoretical study can never capture all aspects of a learning environment. Thus we started a video-based study in cooperation with the staff of a selected kindergarten to test the criteria's workability, to develop further and more detailed criteria and to develop learning environments with materials that meet the criteria's requests. In a first step of data inquiry we videotaped educators while playing with children during an open offer at several occasions with selected materials. In a second step the researcher took the role of an educator and offered games during free play at several occasions.

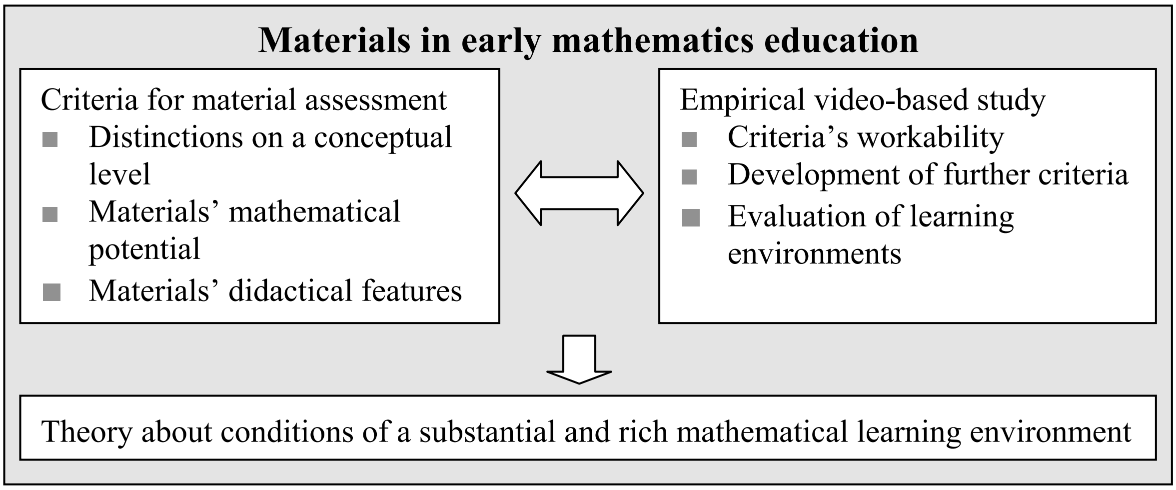

Figure 1: Two-phase research design

According to the *methodology of Grounded Theory* (Strauss & Corbin, 1996), which requires the ongoing change and interplay between action (data inquiry) and reflection (data analysis and theory construction) (cf. Mey & Mruck 2007, 13), the video-based study is still in progress. Basis of the data analysis are transcripts of video sequences. These transcripts do not include only verbal data but also the paraphrase of actions, gesture, facial expressions, as well as screenshots and a storyboard. The data analysis provided first answers to some of the earlier questions and led to further research activities following *theoretical sampling* (cf. Strauss &

Corbin, 1996, 148ff). Using the three most important tools in Grounded Theory methodology – theoretical coding, theoretical sampling, and permanent comparison – there was reason to believe that, aside from the material chosen, the educator's role is crucial to the development of mathematical potential. It has become obvious that the initial criteria need supplementing because *the development of the mathematical potential is linked to conditions*.

4 RESEARCH RESULTS

4.1 Criteria for material assessment

During the past decade many suggestions for early mathematics education were published. Thus it seems necessary to develop criteria to assess these materials and to choose carefully (cf. Schuler, 2008).

1. In accordance with previous remarks, we first distinguished the materials from one another *on a conceptual level*.
   - Does mathematics appear as a part of kindergarten everyday life or is there the idea of a special class?
   - Does the material aim at support of at-risk children or of all children?
   - Does the material support one content idea (e.g. number) or different content ideas?
2. Following the skills-integration-model about the construction of number concept we asked what *mathematical content and potential* is inherent in the material. For the content idea "number and quantitative thinking" the skills mentioned in section 1 guide the analysis:
   - Does the material make it possible to compare sets on a perceptual and a numerical level?
   - Does the material support the construction of mental images of numbers (for example following the patterns of dice images)?
   - Does the material prompt counting activities (forward, backward, counting in steps, precursor/successor)?
   - Are composing, decomposing and first arithmetic activities possible?
3. Following the idea of an early mathematics education implementing mathematics in every day practices and fostering all children of different ages, we asked in addition the following questions:
   - Does the material meet different levels of previous knowledge?
   - Does the material allow access and challenge at different levels?

Mathematical content and potential	
Comparing and ordering sets	+
Constructing ideas of dice images (up to 6)	++
Constructing ideas of other images (up to 6)	++
Counting objects	++
Assigning sets to numerical symbols	+
Assigning numerical symbols to sets	+
Counting verbally	
Finding precursor/successor	
Composing and decomposing set images/numbers	+
Beginning addition and subtraction	+

+: possible ++: appropriate, highly supported

Table 1: Implementation of the criteria for the chips game

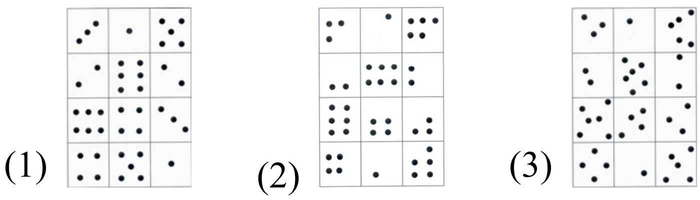

Figure 2: Boards for the chips-game

Games are one possible material to meet the conceptual needs. We want to illustrate the implementation of the criteria by an *example* (see table 1). The *chips-game* is played by two persons. Each person gets a board (three or more alternative versions, see figure 2) and chips of one colour. Throwing alternately one puts chips on the matching square. The person who covers all squares first wins. Variations take into account different levels of previous knowledge, access and challenge:

- playing and covering alone with or without a dice,
- boards with different images,
- two persons playing on one board with chips of different colours,
- covering the squares with number cards.

General mathematical skills like describing, giving reasons, arguing, forming hypotheses or making predictions are not material inherent. But data analysis showed that they can be stimulated by the educator's questions (see section 4.2). Thus *process ideas* can be described as *mathematical potential that develops in interaction*. One goal of the video data analysis is to generate more knowledge about how mathematical potential develops.

4.2 The video-based study

As mentioned above data inquiry, data analysis, and theory construction are still in progress. Therefore the following section reflects the contemporary status of the research process and the results we have got so far. In a first step coding and comparing sequences of the kindergartens educators on the one hand and the researcher taking the role of an educator on the other hand, led to *three preconditions on the part of the educator* to *develop a game's mathematical potential*:

- *Mathematical and didactical competence* contains the analysis, assessment, choice and presentation of materials and results in sensitivity for possibilities and variations in the games course.
- *Individual presence* emphasizes that the educator's actions and support depend on the individual child's needs and competences. The educator's presence can support affordance and lasting involvement with the material by creating game situations, explaining rules and goals, helping to follow the rules, to solve conflicts and to facilitate feelings of competence.
- *Conversational competence* means to develop the mathematical potential through comments on the game's course, questions that stimulate objective explanations, reflections on actions and thoughts, interchange between children, assumptions and hypotheses.

Concerning these three preconditions we observed difficulties on the part of the educators. Except for counting activities they were mostly not aware of the game's mathematical potential. They consequently could not stimulate other mathematical opportunities. Supporting presence during free play was often an organisational problem and aggravated the perception and realisation of individual needs. The educators questioning repertoire was mainly reduced to narrow questions like: How many are there? How many chips do you need? Where are five? Examples for questions to understand and stimulate the child's thinking are open and reasoning questions: How have you seen these are precise five? How do you know here are more/less than /just as many as there?

In a second step we started to investigate the mathematical opportunities during the game sequences. According to the differences in mathematical potential we distinguished different game sequences:

- introduction of a new game or material (1),
- game situation with fostering elements (2),
- game among children of similar age (3),
- game among children of different age (4).

Detailed analysis and open coding of transcripts of type (2), mostly a one-to-one-situation of educator and child, revealed so far the following characteristics:

- *Individual affordance* (cf. Lewin following Heckhausen 2006, 31, 105ff) *by optical or haptic features*: An example for optical affordance is a child's confusion and curiosity about differing set images in the chips game (see board 3 in figure 2). Haptic affordance can manifest in covering the set images with chips without using a dice.
- *Demonstration of skills and abilities*: In a game situation with fostering elements, children want to show what they already can. One can distinguish explicit ways of demonstration like "I can those." or "This is easy for me." from implicit ways that manifest in the child's increased gestural and verbal engagement.
- *Gestural and verbal explanation*: The chips game can be played on different levels of articulation – actions (having a throw, covering), gestural and verbal comments on actions (naming and showing dice and board images), gestural and verbal explanations (showing and explaining the differences and similarities between images of board 2/3 and dice images). The latter level requires the educator's purposeful questions and stimuli.

6 DISCUSSION

As we expounded in section 1 there is a wide consensus about contents in early mathematics education and about the importance of the construction of number concept and quantitative thinking. The theoretical analysis of selected games could show that games have a mathematical potential concerning the number concept. To identify this potential, central skills were reformulated for the analysis of kindergarten materials (see table 1).

Aside from contents, the question of *methods in early mathematics education* is an interesting and still little investigated issue: „little is known about preschool teachers' role in promoting math skills" (Arnold et al 2002, 762). One can distinguish different statements about this subject:

- General statements about how children can learn mathematics emphasize the area of conflict between construction and instruction: "Early childhood educators face a balancing act – that is, an approach that is neither too direct nor too hands off" (Baroody et al, 2006, 203).
- A further discussion focuses on the role of playing and learning: "Play is not enough. […] children need adult guidance to reach their full potential" (Balfanz et al, 2003).
- In addition, some authors stress the differences in content and method between kindergarten and primary school. "Early childhood mathematics should not involve a push-down curriculum" (Balfanz et al, 2003, 266) and kindergarten aims

at “preparing children for school but not by school methods” (Woodill et al, 1992, 77).

Our data analysis indicates so far that potentially suitable games need a competent educator with regard to didactical and conversational aspects. For one type of sequences – game situation with fostering elements – we phrased characteristics. These characteristics imply and allow more specific statements about an educator’s didactical and conversational competence. The educator has to discern the child’s individual approach to the material and has to consider the mathematically productive aspects. He has to make possible the demonstration of abilities and has to facilitate and challenge gestural and verbal explanations through suitable game materials, stimuli and questions.

For other types of sequences this work still is to come. We expect new findings from sequences where children play with other children of the same or of a different age and from sequences which have both elements – children playing together with selective educator’s interventions. Whereas we could find some answers to the still little investigated educator’s role in early mathematics education we do not know much about what children at this age can actually learn with and from each other. We also have to do further research on suitable ways of interventions to make a game mathematically productive without reducing the game’s idea and affordance.

Games can be described as one possibility to organize early mathematics education in correspondence with the daily kindergarten practice. But as we have seen this is not without requirements. These requirements simultaneously show the limitations of this approach.

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