EXPLORING THE RELATIONSHIP BETWEEN JUSTIFICATION AND MONITORING AMONG KINDERGARTEN CHILDREN

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This paper investigates the types of justifications given by kindergarten children as well as the monitoring behavior exhibited by these children as they work on number and geometry tasks. Results showed that kindergarten children are capable of using valid mathematical procedures as well as the critical attributes of geometric figures in their justifications. Children also exhibited monitoring behaviors on both tasks. The study suggests a possible reciprocal relationship between giving justifications and monitoring behaviors in young children.

INTRODUCTION

According to the Principles and Standards for School Mathematics (NCTM, 2000), "Instructional programs from prekindergarten through grade 12 should enable all students to recognize reasoning and proof as fundamental aspects of mathematics" (p. 122). It is important to note that reasoning and proof are not relegated solely to the upper elementary and high school grades. These aspects of mathematics may be nurtured and should be nurtured from a young age. Two fundamental components of children's reasoning processes are justifications and metacognition (Tang & Ginsburg, 1999). In this article, justification refers to the act of defending or explaining a statement. Metacognition includes monitoring one's work. In this article, monitoring refers to those managerial skills which guide the problem solving process. This study is an initial investigation into kindergarten children's reasoning process and the possible relationship between giving justifications and monitoring.

THEORETICAL FRAMEWORK

In analyzing the long-term cognitive development of different types of reasoning, Tall and Mejia-Ramos (2006) described three mental worlds of mathematics: the conceptual-embodied, the proceptual-symbolic, and the axiomatic-formal. The thought-processes of early childhood are said to be embedded in the first two worlds and may be used to describe the types of reasoning displayed by young children as they develop geometrical orientation and number concepts. The first world focuses on objects and begins with perceptions based on the physical world. Through the use of language, children refine their mental perceptions by focusing on the object's properties, leading to the use of definitions which in turn are used to make inferences. This world is particularly apt for describing the development of geometric reasoning as described by the van Hiele levels (van Hiele & van Hiele, 1958). The proceptual-symbolic world builds on actions or procedures. These are encapsulated into symbols that function both as "processes to do and concepts to think about" (Tall, 2004, p. 285). For example, the act of counting leads to the concept of number.

Different types of justifications are an outgrowth of the different cognitive worlds. "Initially, something is true in the embodied world because it is 'seen' to be true" (Tall, 2004, p. 287). Later on, justifications are based on definitions such as used in Euclidean geometry. In the proceptual world, something is true because some procedure shows it to be true. As reasoning in this world develops, justifications are given using symbolic manipulations. Yet, knowing how to use some procedure or knowing the definitions of some concepts are not always enough. Mason and Spence (1999) differentiated between knowing-about the subject and knowing-to act in the moment. They claimed that students do not always appear to know-to use what they have learned and that it is essential to raise students' awareness of their behaviors.

Awareness and expression of one's thinking and behaviors, as well as recognition of mistakes and adaptability contribute to students' success in problem solving (Pappas, Ginsburg, & Jiang, 2003). Schoenfeld (1992), building on Poya's (1945) work of problem solving, pointed to several important aspects of monitoring: the ability to plan, assess progress "on line," act in response to this assessment, and look back. Research has shown that secondary school students, as well as undergraduate students, exhibit few monitoring behaviors during the problem solving process (Jurdak & Shahin, 2001; Lerch, 2004). At the elementary level, Nelissen (1987) reported significant differences in monitoring behaviors between high-achieving and low-achieving students. Preschool children were shown to have little awareness of mistakes and little ability to select appropriate strategies without adult assistance (Pappas, Ginsburg, & Jiang, 2003). All in all, students of different age levels were found to encounter difficulties with monitoring. Yet, since these processes are important, they should be an integral part of mathematics instruction (NCTM, 2000). Being that mathematics is part of the kindergarten curriculum, we should also look for ways to foster monitoring among young children. It has been suggested that, for school-age students, the act of explaining and justifying one's responses may facilitate monitoring (Pape & Smith, 2002). Is this true also for young children? And is this relationship reciprocal? May the act of monitoring provide an impetus for children to justify their responses?

This paper focuses on justification and monitoring among kindergarten children. Specifically we investigate (1) the types of justifications given by young children, (2) the existence of monitoring among young children, and (3) the possible relationship between justification and monitoring among young children.

METHOD

Fourteen preschool classes in low-socioeconomic neighborhoods participated in this study. Each class consisted of approximately 30 pre-kindergarten and kindergarten children between the ages of four and six years old. In this paper we focus on different types of monitoring and justifying responses given by the kindergarten children (between the ages of five and six years old), to two tasks. These children were expected to enter first grade in the upcoming school year.

Two main focal points of the kindergarten curriculum are number concepts (counting objects, identifying number symbols, and comparing the number of items in different sets) and geometry (identifying different two-dimensional and three-dimensional geometrical shapes). In this paper we describe the children's responses to two tasks. Each child sat with the researcher in a quiet corner of the class. Verbal responses as well as gestures were recorded by the researcher.

Task one: Which has more? Two bunches of nine and 12 bottle caps, respectively, were placed on a table before the child. All the bottle caps were of the same shape and size. Each bunch was placed by the fistful on the table, keeping the caps bunched together, without any set order of placement. The child was asked two questions: (1) Which bunch has more bottle caps? (2) Can you check? The questions which accompanied this task were designed to assess children's ability to estimate amounts as well as their ability to check their estimation. The request for monitoring (Can you check?) came from the researcher. Our aim was to investigate if this request would lead the child to justify his answer and if so, what type of justification would the child give.

Task two: Is this a pentagon? For this task, children were shown six cards, two cards, each with a drawing of a pentagon, and four cards, each with a drawing of a non-pentagon shape. Children were asked two questions: (1) Is this a pentagon? (2) Why? The questions which accompanied this task were designed to assess children's ability to identify a pentagon as well as their ability to use the critical attributes of a pentagon in their justifications. Reasoning based on critical attributes indicates a more mature level of reasoning than merely visualizing the whole shape (van Hiele & van Hiele, 1958). In this activity, the researcher asked for a justification. Our aim was to investigate if the request for a justification would then lead the child to monitor his answer.

Analyzing the results. Students' responses were assessed on two levels. First, the type of justifications given were analyzed according to Tall's (2006) theory of the three mental worlds of mathematics described previously. Second, the types of monitoring behaviors exhibited by the children were analyzed with a focus on the following behaviors: (1) expression of one's thinking, (2) planning, (3) assessing progress "on line", (4) awareness of mistakes, and (5) looking back.

RESULTS

In this section we offer a sample of the justifications and monitoring exhibited by kindergarten children in the tasks described above. Samples were chosen in order to illustrate typical responses as well as to demonstrate the range of justifications and monitoring exhibited by these children.

Task one: Which has more?

We begin by presenting children who offered correct estimations, with valid and invalid justifications. We then present a child who offered an incorrect estimation.

Correct estimations and valid justifications. One of the strategies used to check which bunch had more bottle caps was counting. Counting the number of bottle caps in each separate bunch was considered a valid justification.

> C1: (The child counts the bottle caps in each bunch separately.) I told you that I know there are more bottle caps here (pointing to the bunch of 12 caps).
>
> C2: We can count. (The child proceeds to count the bottle caps in each bunch separately and smiles in recognition of her correct estimation.) I was right!
>
> C3: We can count. (The child proceeds to count the bottle caps in each bunch separately.) Here (pointing to the bunch of 12 bottle caps) there are more. Twelve is bigger than nine.

The reasoning exhibited by all three children was embedded in the proceptual-symbolic world. All of the above children took action upon being requested to monitor their estimation and each had a valid procedure used to justify their estimations. C1 and C2 both followed their actions with an assessment of their initial estimations. In other words, an external request for monitoring was followed by a justification, which in turn was followed by monitoring (looking back). Yet the quality of their monitoring had a subtle difference. C1's response, "I told you", hints at the child's response being directed outward, toward the interviewer. C2's smile, along with his response "I was right" was directed inward and hints at the possibility that the outside request for monitoring led to a more introspective form of monitoring. C3 had a method for monitoring his estimation (counting) which was followed by a justification (12 is bigger than nine). This justification indicates that the child has possibly abstracted the bottle caps to numbers and can now compare the number concepts without reference to the physical objects at hand. Both C2 and C3 expressed their thoughts ("We can count") before plunging into actions. Yet, C3 does not look back.

One child was unsure of how to apply the counting procedure:

> C4: (The child counts the smaller bunch first, stops, and looks at the second bunch.) Should I continue from here? (C4 considers if he should continue the counting sequence by counting the second bunch starting from 10.) Or should I start from the beginning? (C4 does not wait for an answer but proceeds to count the second bunch of 12 caps correctly, starting from 1 and concluding with 12.) Here (pointing to the bunch of 12) there are more.

C4 is developing his reasoning ability within the proceptual-symbolic world. He knows he ought to use a counting procedure. He monitors his procedure "on line" by stopping mid-way and thinking of how to proceed. C4 is struggling to connect the procedure with the concept. By monitoring his actions he switches from *doing* mathematics to *thinking* about mathematics.

Not all children responded immediately to the question of which bunch had more bottle caps. Instead, when asked which bunch had more, one child responded, "I need to count." Only after she was told to answer first without counting did she choose the bunch with 12 bottle caps as having more than the other. In other words, this child had a plan which she wished to implement before answering the question.

Other than the counting procedure, children relied on the principle of one-to-one correspondence to compare the amount of bottle caps in each bunch:

> C5: (The child lines up each bunch in two separate rows, making sure that each cap touches the next. He then compares the length of each row.) This one is longer.

C5 compared the lengths of the two rows of bottle caps. As the caps were all of the same size and each cap touched the following one, this was a valid method. For C5, the procedure of lining up the bottle caps led to a reflection on the concept of length.

Correct estimations but invalid justifications. Some children estimated correctly which bunch had more but replied with invalid justifications stemming from improper use of the counting procedure.

> C6: (The child counts the smaller bunch first, 1…9, and proceeds to count the second bunch, 10…21.) There are 21 bottle caps in this bunch (pointing to bunch of 12 caps).

Unlike C4, who had thought about counting both bunches together but did not, the counting activity of C6 may be considered a rote procedure divorced from conceptual meaning.

An invalid justification sometimes left the child unable to assess the correctness of his estimation:

> C7: (The child counts the bunch of 12 bottle caps but does not count the bunch of nine bottle caps.)
>
> Researcher: And how do you know that there are more in this bunch than in the other bunch?
>
> C7: I don't know.

Other children, although correctly estimating which bunch had more, did not respond with justifications based on mathematical procedures or concepts:

> Researcher: How do you know which bunch has more?
>
> C8: Because we see.
>
> Researcher: Can you check?
>
> C8: Yes.

Researcher: How?

C8: With the eyes.

This child seems to be reasoning within the conceptual-embodied world instead of choosing an action or procedure. His correct estimation was based solely on his visual perception. The outside call for monitoring did not trigger a switch to an appropriate mathematical procedure.

Incorrect estimation but correct conclusion. The opportunity to monitors one's thinking was noticeable when a wrong estimation was given. For example, one child incorrectly estimated that the bunch of nine bottle caps had more caps than the bunch of 12 bottle caps. When asked to check, he responded:

C9: (The child counts each bunch separately and smiles.) Oh! This bunch (pointing to the 12 bottle caps) has more.

For C9, the external request for monitoring was followed by a valid action and justification, which in turn was followed by the awareness ("oh!") that a mistake was made.

Task two: Is this a pentagon?

Children were shown six different shapes and asked to identify the shapes as pentagons or non-pentagons and to justify their identification. At times, their initial identifications remained unchanged and at times children's final identifications differed from that of their initial identifications. In this section we review typical responses to one pentagon shape and to one non-pentagon shape (see Figure 1).

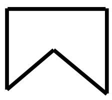
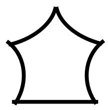
 Pentagon	 Non-pentagon

Figure 1: Two shapes presented to children for the pentagon task

Correct initial and final identifications with critical attribute reasoning. Regarding the pentagon, children who identified this shape correctly often justified their identification by referring to critical attributes of the pentagon.

C10: It has five vertices, it's a closed shape, and it has five straight lines.

Regarding the non-pentagon, some children who correctly identified this shape as a non-pentagon referred in their justifications to "crooked" or "rounded" lines. One child justified his correct identification by saying, "It's not (a pentagon) because it has two rounded sides… actually is has four rounded sides… it doesn't matter." This child assessed his justification "on line". At first he noticed two rounded sides. Then he took a closer look and noticed four rounded lines. However, he realized immediately, that in fact it does not matter how many rounded sides the shape has, because even one is sufficient to nullify the shape as a pentagon. This child exhibited monitoring, not of his solution (which was correct) but of his justification. As he was justifying his conjecture, he monitored the correctness and perhaps quality of his justification.

Regarding both shapes, some children first counted the vertices or sides and only then responded to the question of identification. Such children thought about how to go about identifying the shape, acted on their plan, identified the shape and then justified their identification.

Incorrect initial identification but correct final identification with critical attribute reasoning. Children who corrected their initial incorrect identifications, typically referred to the critical attributes of a pentagon in their justifications. Regarding the pentagon:

> C11: It's not a pentagon. Let's check. (The child counts the vertices.) It is a pentagon because it has five sides and five vertices and it's closed.

> C12: It's not a pentagon. The line here points to here (referring to the concaveness of the pentagon). (The child counts the vertices.) It is a pentagon.

C11 immediately went to check his conjecture, even before the researcher had a chance to ask him why he claimed the shape was not a pentagon. In other words, he initiated the monitoring (when he declared "let's check" and counted the vertices) which in turn led to a correct identification based on a correct justification. C12 initially used a justification based on a non-critical attribute (the direction of the line). This justification was followed by monitoring (counting the vertices) which in turn led to a correct identification. Both C11 and C12 exhibit reasoning which integrates both the conceptual-embodied world with the proceptual-symbolic world. They begin by using perceptual reasoning. This reasoning is monitored by using the counting procedure and number concepts of the proceputal world, which ultimately leads back to reasoning based on properties and critical attributes.

Regarding the non-pentagon, one child claimed at first that this shape was a pentagon. When asked why he thought it was a pentagon, he proceeded to count the points and said, "Yes… uh… no. It has five vertices but it's not straight." In this case, justifying the conjecture led to self-initiated monitoring.

Incorrect initial and final identification with critical attribute reasoning. At times, children gave incorrect identifications along with critical attribute reasoning. For example, regarding the pentagon:

> C13: It's not a pentagon. It doesn't have five sides. (There was no indication that the child had counted the sides.)

It seems that C13 gave a verbal justification without carrying out any action. Although he gave a justification befitting his (incorrect) identification, the request for justification did not lead this child to monitor his response. He did not look back and was not aware of his mistake.

Unchanging identifications (correct and incorrect) with visual reasoning. Not all children justified their identifications using the critical attributes of a pentagon. Regarding the pentagon:

> C14: It's a pentagon because it looks like a pentagon.
>
> C15: It's not a pentagon because it looks like a tooth.
>
> C16: It's not a pentagon because it doesn't have the shape of a pentagon.

Regarding the non-pentagon:

> C17: It's not a pentagon because it looks like a circus (tent).
>
> C18: It's not a pentagon because it's not in the shape of a pentagon.

The above children used visual reasoning in their justifications. Within the conceptual-embodied world, their reasoning has not advanced past their perceptions. Both C15 and C17 embodied the rather abstract concept of a pentagon into a more familiar physical entity. C14, C16, and C18 have a mental image of a pentagon which does not fit the shape on the card. These justifications accompanied both correct and incorrect identifications and were not accompanied by monitoring.

Some children gave justifications that were a mix of perceptual reasoning along with reasoning based on attributes. Regarding the non-pentagon:

> C19: It's not a pentagon because it has five vertices but it doesn't look like a pentagon.

C19 is a child in transition. Previously, he had correctly identified the pentagon noting only its five vertices. His justification regarding the non-pentagon takes note of the five points (they are not vertices as they do not connect straight lines), but disregards them because the shape "doesn't look like a pentagon." In other words, he realizes that the attribute of "vertices" is worthy of notice but he may not have the knowledge or words to describe that the sides need to be straight lines. Instead, his final justification relies on his visual perception. In a sense, C19 exhibits monitoring. He clearly has a strategy by which he checks if a shape is a pentagon (counting vertices) but "on line" rejects that reason in favor of relying on his mental image of what a pentagon should look like.

DISCUSSION

This paper has shown that young children are able to justify their conjectures by using appropriate mathematical procedures, such as counting, or by reverting back to critical, geometrical attributes. Some children, capable of giving complete mathematical justifications, also exhibited monitoring behaviors. A child who knows a pentagon must have five straight sides as well as five vertices is ultimately better equipped to monitor both his answer, as well as the quality of his justification.

Some of the justifications given by children were based on visual reasoning. For these children, operating at the first van Hiele level of reasoning, a visual justification is a convincing justification. They "see" with their eyes that one bunch has more than another and either feel no further need to verify their perception or do not have the knowledge to do so. Although we may value and encourage visual estimation, justification and proof are about necessary and sufficient conditions that validate or refute a mathematical assumption. Furthermore, children who base their justifications solely on visual reasoning, claiming that something looks like or does not look like something else, have limited recourse when it comes to monitoring their answers.

Referring back to Schoenfeld (1992), this paper suggests that young children are able to plan a strategy in advance (counting the vertices before identifying the shape), monitor their progress "on line" (change from visual reasoning to reasoning based on critical attributes), as well as act in accordance with this assessment. When encouraged to do so, children are able to express their thinking. This paper has also shown that justification and monitoring may have a reciprocal relationship. A request for monitoring may encourage justification which in turn may encourage further monitoring. At the same time, a request for a justification may encourage the child to monitor his actions, which in turn may improve the justification.

In this paper we presented two tasks which acted as springboards for children to monitor and justify their responses. More research is needed to examine how different tasks, activities, and games, and the questions which accompany them, may be used to promote both monitoring and justification among young children. At this young age, we are interested in children developing a proving attitude (Simpson, 1995), where they value the opportunity to convince themselves and others. This paper focused on the relationship between an individual's monitoring behaviors and justification. We call for more research in the area of monitoring and justifications among young children.

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