DIDACTICAL ANALYSIS OF A DICE GAME

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Abstract: *in this paper, we analyse an activity for 1st grade students, taken from the official pedagogical material for mathematics in French-speaking Switzerland. This activity is part of the curriculum about addition and comes in the form of a dice game. After some succinct considerations about games in mathematics education, we give an a priori analysis (according Brousseau's theory of didactic situations) of the activity. We then give account of an experimentation we made in Geneva, first with the teacher in her class and then with two duos of students outside the class. Finally, we suggest some modification in the didactical design in order to make this activity more pertinent.*

INTRODUCTION

In the whole of French-speaking Switzerland, for mathematics teaching , there is a single common official set of pedagogical material, including text-books and files for students and a teacher's book with curriculum and didactical commentaries. Like in many other countries, especially for lower grades, many of the mathematical activities are presented in the form of games.

The interest for games in mathematics teaching is nearly as old as mathematics. Huizinga (1989) refers to Piaget (1945), who put forward the importance of games with rules in opposition to fiction games for education. Caillois (1951) claims that a game is rather a challenge than just an exercise: “A Child does not train for a specific task. He acquires through games a wider capacity for overcoming difficulties.” (p. 319). The virtues of games are widely recognised in mathematics education especially for lower grades (Milliat & Neyret 1990). Nevertheless, some critical voices can be heard about certain excesses (Valentin, 2001). Indeed, games may be a very good means for learners to acquire mathematical knowledge, yet, it is not always easy to match the game's stake with a precise mathematical goal. In this sense, we recall here some basic principles of Brousseau's theory of didactic situations:

> Doing mathematics is only possible by solving problems, yet, it should be reminded that solving a problem is only part of the work at stake; finding good questions is as important as finding their solution. […] In order to make possible such an activity, the teacher should therefore imagine and offer to students, situations that they can apprehend, in which knowledge appears as the optimal reachable solution to the given problem. (Brousseau 1986, 35) or (Brousseau 1998, 49).

Therefore, when setting up a mathematical activity in the form of a game, one needs to analyse the adequacy of the game's finality with the potential for acquisition of the specific intended mathematical knowledge as an optimal solution to win the game.

In a survey about the use of the official pedagogical material by teacher in French-speaking Switzerland, Tièche-Christinat (2001) noticed that games are usually chosen in reference to the pleasure they are supposed to give to students, while the mathematical content is secondary. It is also well-known that some students do not like games at school. In this research work, we analyze and experiment an activity in the form of a game proposed in the official pedagogical material for the first year of primary school in Geneva. Some work in this sense, but about other activities, had already been done during a one-day seminar organised by the Institute for Pedagogical Research (IRDP) in Neuchâtel (Jaquet & Tièche-Christinat, 2002).

A PRIORI ANALYSIS OF THE ACTIVITY "TURN THE DICE"

This activity is part of the official material for 1P (first year of primary school, age 6) in French-speaking Switzerland. It is located in module 3. *Problems to get to know sums*, in a sub-section entitled: *Add and subtract in situation* and refers to the objective: *Getting to 20 by adding numbers*. Here is a translation of the text of the activity as it is found in the teacher's book:

Turn the dice

Description 2 students / One dice

- *Rules* : One student rolls the dice and says loudly how many points he got. The other turns the dice on one of the lateral sides and adds the points to the preceding total. The game follows on this way: each player, in turn, turns the dice on one of the lateral sides and adds the numbers. The first who gets to 20 wins.

Possible extension: starting with 20 to reach 0. The first who overcome 20 wins…

The first goal of an a priori analysis is to look at an activity from a more distant viewpoint in order to localise some blind spots and elucidate some hidden goals. In this sense, Brousseau's theory of didactic situations (see (Bessot, 2003) for a basic yet enlightening introduction) provides some tools in order to interpret an activity as a special case of a more general set of didactic situations. Describing such a set means revealing didactical variables and their different possible values, such that the activity correspond to a particular choice of value for each variable. A didactical variable correspond to a potential (yet often implicit) choice for the teacher that modifies the accessibility of different strategies for solving the problem. Thus, a different choice of value for any didactical variable changes the nature of the learning and correlatively the meaning of the knowledge at stake. Such a methodology consists in revealing implicit choices made against other possible ones. Therefore, it reveals what is usually hidden because implicit. Listing possible students' answers, which is what an a priori analyses is too often reduced to, is only one part of the analysis and is only fully valuable when one knows how to interpret different strategies in the whole set of possibilities. In this sense, the activity "Turn the dice" can be seen as a specific element of the set of situations in form of a game with two players:

In turn, each player chooses or picks up at random (this may vary at each turn) a number in a set Ei (i^th^ turn): The number is then added to the preceding total. The winner is the player who reaches first a certain predetermined value N.

We define six didactical variables:

- *two about the general rule of the game:*
Vov = "yes" or "no", depending whether the final value N can be overcome or not.
VN = N, the value to be reached or overcome in order to win.
- *two variables that can change at each turn:*
Vrand = "yes" or "no" according to the fact that the number is respectively picked up at random or chosen by the player.
VEi = Ei, the set of possible numbers to be chosen or picked up at random at the i^{th} turn.
- *two variables that deals with the material used for the games:*
Vrep: determines, in relation to the material used, the type of representation for the numbers (side of a dice either with dots or numerals, cards with numbers written with letters, numerals or constellations, etc., tokens, spoken numbers…)
Vwrit = "yes" or "no", depending whether the players can write their sums or not.

Of course, this list of variable is only partial and partly subjective. This is why we have to justify our choices by showing how the subsequent a priori analysis is relevant for our observation. We distinguish two levels: the knowledge at stake locally at each turn of the game, and the global strategy of the game.

Making sums (local knowledge)

Regarding competencies for addition in 1st grade, the value of VN cannot really exceed 20, and the numbers in the sets Ei are also limited to 5 or 6. Moreover, in 1st grade, many students still counts on their fingers and make additions by over-counting one-by-one from the first number of the sum (to do 4+3, the student count loudly or in his head raising fingers three times: "five, six, seven"). The memorised repertory is still very limited, which means that very few sums are known by heart. Vwrit is quite important in this game, not only because students can actually make the addition using written devices, but also because writing the sums at each turn reduces the effort of memorisation. In the activity "Turn the dice", the value of this variable is left to the teacher's choice. In our experimentation, the teacher chose not to let students the possibility to write. Furthermore, the various possible values of Vmat modify the possible techniques for making sums. Dice (with spots), cards with constellations, tokens… make possible, even promote, techniques using one-by-one over-counting. On the opposite, numbers in numerals, letters or just spoken promote other techniques like recalling a repertoire or "calcul réfléchi" or necessitates to use fingers or written techniques if Vwrit=yes. In the activity "turn the dice", the type of representation of the numbers on the side of the dice is not specified. In our experimentation, the teacher chose a dice with spots. However, one of the objective in 1st grade is to progressively bring students to abandon techniques using one-by-one over-counting. They should start memorising the repertoire and use "calcul réfléchi".

This first analysis shows that the choices for the activity "Turn the dice" are coherent with the level of 1st grade students. The game is possible. Yet, regarding the learning of addition, there are some contradictions with the goals at this level of education. Moreover, the game does not provide a milieu with possible feedback for the learning of sums. Indeed, nothing in the game offers a possible feedback to a mistake in a sum, except the control of the other player, or the teacher if s/he is watching at the right time. In other terms, if one student gives a wrong result for a sum and if the other player does not react and the teacher is not watching, the game can go on without the mistake being corrected. Therefore, "making sums" is a knowledge necessary for the game to be played, but is not subject to a control and certainly not the main tool for an optimal winning strategy. Therefore, if we refer to Brousseau's quotation given above, we can see that there is an inadequacy here between the game's stake and the didactical objective: *Problems to get to know sums*. In order to play correctly, students have to know how to make sums correctly. If they do not, they may play anyway, but nothing in the milieu organised through the game gives any feedback. Nothing is organised didactically for them to learn sums, they have to know, but they can make errors without being corrected, except if the other player knows better or the teacher is here to correct. Furthermore, we have seen that the use of a dice with spots is likely to promote the basic technique "over-counting one-by-one", which is supposed to be progressively banished in 1st grade. Such an activity is therefore not especially good in order to train 1st grade students to do sums. At most, if they have a reliable technique, this game may help them memorizing sums, but the excitation of the game is likely to overcome this goal!

Game's stake (global strategy)

At this level, the values given to VEi , Vrand and Vov are crucial.

For the choice Vrand = "no" and VEi = {1,2} at each turn and Vov = "no", the game is called the "race to 20" and has been analysed by Brousseau (1998, 25-44). Such a game has a winning strategy, corresponding to the series of winning numbers 2, 5, 8, 11, 14, 17, 20, that can be discovered by subtracting 3 to 20 repetitively down to 2, or by dividing 20 by 3, the rest being 2. Brousseau showed how such a situation can be used to make 4th grade students discover the Euclidean division and debate about a general strategy for being sure to win. In the case of the activity "turn the dice", there is no such strategy. Even if a strategy for winning is possible, it is far from being reachable by 1st grade or even much older students.

In the opposite, if Vrand = "yes" at each turn, this is just a game of chance, which, therefore, doesn't call for any strategy, at least in relation to any mathematical content. Moreover, dices are often related to games of chance, it is therefore likely that students act just as if "turn the dice" is only a question of chance, especially considering the fact that on the first go, the player rolls the dice.

Is there a possible strategy to win the game “turn the dice”? If yes what can 1st grade student catch from it? The main difficulty of this game is that Ei changes at each turn. Moreover Ei depends on the choice made at the (i-1)th turn, therefore by the other player. Two opposite sides of a dice always add to 7. This gives the rule for possible choices with regard to the last chosen number. Each turn can be represented by the number “i” (order of the turns), the name of the player who just played (P1 or P2) and S(n), S being the last sum calculated and n the last side chosen.

For instance [3 , P2, 12(5)] means that it is the 3rd turn, P2 has turned 5 which adds to a total of 12. At the 4th turn, P1 must therefore choose in E4 = {1,3,4,6}.

- If P1 chooses 1, the status is 13(1) and E5 = {2,3,4,5}. if P2 chooses to turn 4, the status is 17(4). Since 3 is not possible, and numbers over 3 are too big, P1 must choose 1 or 2 and P2 wins at the next turn. Thus, 1 is not a good choice for P1.
- If P1 chooses 3, the status is 15(3), and P2 can turn 5 and wins.
- If P1 chooses 4, the status is 16(4), P2 cannot win but if he turns 2, the status is 18(2), so P1 has no other choice than turning 1 and the game is blocked.
- If P1 chooses 6, the status is 18(6), P2 can turn 2 and wins.

This example shows that the strategy is quite complex. A player must anticipate all the possibilities and short time anticipation may be fatal. Moreover if Vov = “no” like in the original game, some games may lead to a dead-end. This is far too complicated for 1st grade students. Indeed, at this level, students are likely to be unable to just anticipate the result of the next turn. Indeed, this requires more than just addition, but also knowledge about complements to 20, which is a first step toward subtraction: “how much is it from 14 to 20?”, etc.

In conclusion, the game’s stake does not have to do just with adding numbers (no more than 6) to reach 20, but also being able to anticipate the next (one possibly two or more) turn(s). One mathematical knowledge needed is then to be able to anticipate the effect of adding a number and knowing the complements to 20, from at least 14. It is therefore impossible to hope that 1st grade students develop a strategy that leads to victory in each case. At most, they can anticipate one or two turns when the sum gets over 12, or a bit more. Therefore some important didactical questions are: “what knowledge can be aimed at through such an activity?”. “Are 1st grade students sufficiently knowledgeable to do their sums without mistakes?”. “Can they do more than play at random and develop some strategy at least towards the end of the game, involving some abilities for anticipation on sums, and complements to 20?”.

In order to answer these questions, we organised an experimentation of this activity in a 1st grade class near Geneva.

EXPERIMENTATION

The class counts 22 students of average level, in a village near Geneva. The teacher has only 3 years of practice and teaches 1st grade for the first time, she also uses this activity for the first time and we did not exchange with her about it before. The experiment took place in March. The teacher decided to explain the game to the whole class for about 10 minutes, before splitting the class in two. One half plays (5/6 duos), while the rest of the class has to do some work individually in autonomy. Each half-class played for about 15 minutes. In the end, a conclusive session with the whole class is organised. Our observation is based on a video recording of the whole class (beginning and end) and for each half-class, on a video recording of one duo, plus an audio recording of another duo.

Devolution

The teacher reads the rules of the game and asks questions. Some students comment with their own words. Then, the teacher chooses two students to play a game, which is summarised in the following table:

Player	Marie	Renan	Marie	Renan	Marie	Renan	Marie	Renan
Number chosen	3	6	2	3	2	1	2	1
Total	3	9	11	14	16	17	19	20

Neither Marie, nor Renan take time to think about what they choose (except Renan at the last turn!). This validates our hypothesis that, for them, it is like a game of chance. At each choice of a new number, the teacher asks for the total and several students raise their hands, and there is a quick general agreement on the result. Renan starts with a big number, in order to get near 20 quickly. Marie is more careful and on the contrary chooses the smallest number she can, in order to prevent Renan from getting too near to 20! At the 5th turn, Renan chooses 3, getting to 14(3). That can lead Marie to win id she chooses 6! Yet, she does not and nobody notices. She chooses 2, getting to 16(2), Renan can win by choosing 4, but he chooses 1 (nobody notices) getting to 17(1). Again Marie can win, but she chooses 2 (nobody notices either), getting to 19(2). Renan cannot do anything else than win!

This shows clearly that the game's stake is not accessible to the students straightaway. They concentrate on their sums and do not see the goal. Getting to 20 is the criterion to stop but not a goal to reach first. The teacher does not try either any devolution of the stake. One student spontaneously says. " Marie always makes 2 and Renan 1". The teacher interprets: "Oh why do they always choose small numbers?" and Marie instantly replies: "This way it is easier to count!". Clearly the students are concentrated on their sums and reduce the difficulty without care for the game's stake. Therefore, in this collective phase of devolution (5 min.), all is about sums and nothing about the game's stake is debated.

The games

In this paper, we cannot analyse in detail all the games we observed, we only give some general comments (see Dorier & Maréchal (in press) for more details). Some students did not understand that they had to choose one side after the first go, instead they rolled the dice at each turn. This validates again our hypothesis that they play like a game of chance. Something we did not anticipate lead to some unnecessary noise and excitation. Indeed, to turn the dice, many students pressed the edge of the top-side. As a result, the dice often rolled several times or even off the table. The students play fast, which is a sign that they do not choose really their numbers. Globally, they tend to choose big number at the beginning and small ones near the end. This is a sign that the game's stake is taken into account at a basic level. However, several times, students made a choice that allowed the next player to win, while it could have been avoided. Some duos did not respect the fact that 20 should not be overcome. Systematically, students looked around the dice to check the possible choices. Most of the time, they over-counted one-by-one, pointing each spot on the side. Some counted on their fingers and very few recalled memorised results. This validates our analysis and shows that the choice of a dice with spots favours an elementary technique for making sums. Some mistakes on the results of sums (even with the elementary technique) occurred and were usually not corrected by the other player. Some duos have great difficulties in memorising the totals or even making the additions. Nevertheless, some duos show that they tried to anticipate the results near the end of the game. However, the complements to 20 did not seem to be known by heart and students usually counted on their fingers or directly on the sides of the dice. No duo anticipated two turns. No example of a game coming to a dead end had been observed. However, the students were happy, they had play!

Conclusive phase – Whole class

Spontaneously, the students tell stories about their games "I won twice and he won three times!" , "we did not manage to finish.."… This has nothing to do with strategies or even sums, it is all centred on social aspects of the game. In order to re-direct the debate, the teacher asks: "Do you think that, in this game, there is a technique to win? Something that would help to win… more easily?". One student suggests that it is good to choose big numbers. A short debate starts on the effects on the game of choosing big or small numbers. After some discussion, Pierre suggests that choosing alternatively big and small numbers allows to win. In response, the teacher asks Pierre to play against her. At the fourth turn, the status is 14(6) and it is the teacher's turn. She realises suddenly the difficulty and ask the students what she should play. 2 and 5 are given as answers, she chooses 5, getting to 19(5), which allows Pierre to win. The teacher's conclusion is that Pierre's technique only works if the other player follows his rule! One of the observer then ask what would have happened if the teacher had chosen something else than 5, like 3. The teacher agrees and turns 3, the status is then 17(3). She asks Pierre what he would do. He stays silent for quite a long time and finally says he would choose 3. Obviously at this stage, the teacher is not quite sure of herself, so she closes the discussion by saying: "Is there only one technique?". There is quite a long silence before a student starts talking again. But even then, nothing really interesting happens. Finally the teacher says: "Is there a time in the game, … maybe one number… from which you know you can win… maybe…for instance if you get to 10, can you be sure to win?". We can hear a few “no”… the teacher goes on: “Is there a number, that you can say: ‘if my friend put this number, I am able to win if I turn the right side?’.” No answer. Then Marie claims that she has a technique: “In fact I… at first, I choose nothing special… and then, toward the end when it is a bit more difficult, eh.. I look around the dice and I count the sums, and…”. The teacher goes on: “You look around the sides and you look which comes to 20. Did you all think about looking at the possible sides before you turned the dice?”. Around 8 students raise their finger. “Did that help you to win?”. One student answers: “There was not the side I wanted, because it was underneath.” At this moment the bell rings and the class is finished.

The conclusive phase shows that the teacher struggles with her goals and the students’ reactions. She had probably under-estimated the difficulty of the game. Of course, this is a lack of questioning from her part, but this is also due to the difficulty of the situation itself and the lack of didactical analysis in the official pedagogical material, in order to help teachers lead this activity. Our a priori analysis shows that the milieu of the situation is not suitable to give sufficient feedback to the students on the validity of their sums. It also shows that, without any other didactical device, students are likely to play by chance and develop very few strategies. At most, they try big numbers at the beginning of the game and small ones at the end. Our observation confirms these conclusions. It also confirms that students use only one-by-one over-counting strategies and do not use more elaborate techniques for their sums. Nevertheless, some students do try to anticipate the results of their choices toward the end of the game and try to guess the complement to 20, mostly by counting on the visible sides of the dice. Yet, without stronger motivation, they fail to really develop a strategy, and do not anticipate more than one turn. Our observation also shows that students do not spontaneously reflect on the reason that made them loose, by analysing the last turns of the game they just played. They do not try other choices, to see what could have changed. In our experimentation, the teacher did not try to make students do so. Moreover, when one of the observers tries to initiate such an analysis in the collective conclusive part, the teacher finally gives up.

New experimentation with duos out of the class

Even if this experimentation allowed us to validate our a priori analysis, we wanted to see what kind of behaviour students may have, if they were asked to reflect on the end of a game they just played, and anticipate the effects of other choices. Therefore, a few weeks later, we asked the teacher if we could work individually with a couple of duos. She accepted and we organised a new experimentation during an hour with two duos of students, in a separate room, while the teacher stayed with the rest of the class. We do not have space here to analyse what happened then, so we will only give a short account (see (Dorier & Maréchal, in press) for more details).

Globally, this experimentation shows that when asked to reflect on the last turns of a game they have just played, the students we observed are able to anticipate the two or even three next turns. They understand that they have to find the complement to 20 and anticipate the possible choice for the next player. Once this type of reflection is initiated, they play more carefully the following games, and develop some anticipating strategies, that make them reflect on the complements to 20 and possible issues. Moreover, this experimentation showed that students knew their sums by heart, and were able to give up the "one-by-one over-counting strategy", if they were asked to, or when they had to anticipate and therefore were not able to use the dice to count. This confirms the fact that in its basic version the activity "turn the dice" promote a technique that students can overcome using a more expert one. It also shows that making them anticipate the next turns, induces them to switch technique.

CONCLUSION

Our observations have been limited, thus, we have to be careful about the conclusions we can draw. Globally, the experimentation in class with the teacher confirms the conclusion of our a priori analysis, that such an activity is likely to be reduced to a game of chance, which means that students do not learn much. The second experimentation shows, on the contrary, that on certain specific conditions, students can be led to reflect on the way they play and develop some more expert strategies, and in particular, acquire some knowledge about complements to 20. In this sense, "turn the dice" may be seen as a consistent mathematical activity accessible to 1st grade students. However, the conditions of our second experimentations are too particular to be reproduced as such in normal conditions. Therefore, we need to find a didactical device in order to make the realisation of this activity possible in "normal conditions" and proper to induce a consistent learning. Using a dice with numbers written in numerals rather than spots, could be a solution in order to block the one-by-one over-counting strategy, but then it is impossible to use it to check sums in case students fail. Therefore, this solution is only possible, if students do know their sums by heart. Therefore, this activity should not be given in the beginning of 1st grade, but rather at a time when most students have memorised sums with little numbers.

Letting the students play a few games at the beginning is quite important in terms of devolution, even if they just play by chance. During this phase of appropriation, it is important to check that all the rules are understood (the dice is rolled only at the beginning, it is forbidden to exceed 20, it is important to control the turning of the dice…). It may also be possible to tell students that they can (should?) use other techniques than one-by-one over-counting on the side of the dice (or this can be debated in the next phase only).

After this first phase (as short as possible) a first time in common can be organised by the teacher. After asking the students what they did, two can be chosen to play a game in front of the class. Then, the teacher can organise a collective reflection on the last turns of the game and analyse the effects of alternative choices. This should produce a change in attitude for most students (like what we observed in our last experimentation). This can be repeated once or twice, before students are asked to play again in duos, 8 games each. It is important to limit the number of games and to give sufficient time, to prevent students from going too fast trying to play as many games as possible, like we observed in the beginning of our experimentation. Each time a player wins he gets one point. The totals are to be compared at the end. This gives a bit of competition in the games, in order to favour the search for a strategy and not just chance. A final collective debate should lead to the institutionnalisation on the strategies as well as complements to 20.

Of course, a new experimentation is necessary to see if this new proposition inspired by our first analysis would lead to a more satisfactory lesson.

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