LEARNING MATHEMATICS WITHIN FAMILY DISCOURSES

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In our research, we are concerned with early mathematical learning processes embedded in family discourses. Thereby, the focus is on interactional patterns which shape the mathematical experiences of preschoolers. What kind of mathematical discourse do preschoolers become familiar with? And what conceptions of mathematics arise from such everyday discourses?
In this paper, the centre of attention is the research design of a study in progress. Thus, we present our theoretical framework and underlying methodological considerations. Additionally, we complete this article with some data from preliminary studies in order to illustrate our approach.

Keywords: home mathematics, support structures, enculturation, acculturation

INTRODUCTION

In mathematics education research, the understanding of mathematics as a human product, which cannot be separated from its cultural context, is more and more prevalent. Regarding this culturality of mathematics, two complementary views of learning mathematics can be recognised. On the one hand, learning mathematics means that one becomes a part of the mathematical culture which permeates one's social environment (Bishop, 1988). On the other hand, mathematical learning processes are also an intended acquirement of an apparently unchangeable faculty culture with its specific set of terms, structures and principles (Prediger, 2003). In our opinion, these two descriptions supplement each other and correspond with the fundamental distinction between enculturation and acculturation (Bishop, 1988 & 2002; Frade & Faira, 2008). In both conceptions, mathematical learning is embedded in discursive processes between one generation and the next.

Against this background, we are interested in early mathematical learning processes. Toddlers and preschoolers already make varied experience with mathematics in different social activities. Thereby, discourses with their parents are of prime importance. Thus, our main research question is: What kind of mathematical discourse from the familial context is familiar to the child entering school? We want to pursue this question in an empirical and qualitatively laid out study, which is in line with the interactionistic research paradigm (Cobb & Bauersfeld, 1995).

In the following pages, we shed light on the picture of mathematics as a cultural property and clarify the implications for our conception of learning mathematics. Subsequently, the methodological approach derived from this framework will be presented and, finally, be illustrated by data from our preliminary studies.

THEORETICAL FRAMEWORK

In looking back at children's experiences with mathematics, we necessarily do so with a certain preconception of mathematics. „Mathematics is an intellectual instrument created by the human species to describe the real world and to help in solving the problems posed in everyday life." (D'Ambrosio, 2001, p. 67) For our theoretical framework, we adopt this idea from the research in ethnomathematics: mathematics is no entity existing outside human experience, but a human product (Prediger, 2003; Street, Baker & Tomlin, 2005).

This assumption about the nature of mathematics affects our conception of learning mathematics. Thus, children do not encounter mathematics itself, but a cultural practice that is recognized as mathematical by capable members of the belonging culture (Sfard, 2002). For this reason, not only is mathematics a social construction, but learning mathematics is as well. Therefore, Bishop demanded as early as 1988: "[…] a mathematical education must have at its core the assumption of being a social process." (Bishop, 1988, p. 13) Consequently, learning mathematics means that a child participates in a practice to an increasing degree. This idea of learning is explicitly exhibited in Sfard's theoretical work. She defines learning mathematics as "becoming fluent in a discourse that would be recognized as mathematical by expert interlocutors." (Sfard, 2002, p. 5) Pursuant to this latter definition, adults are of prime importance for the child's development due to the fact that they can spur mathematical discourses.

In line with this approach to mathematical learning, we focus on the emergence of mutual understanding and coordination in discourses between a child and an adult as expert interlocutor in a certain degree.

Home Mathematics

With regard to early mathematics and its conjunction with school mathematics, van Oers states: "In fact, students are from the beginning of their life a member of a community that extensively employs embodiments of mathematical knowledge. The school focuses attention on these embodiments and their underlying insights, and by so doing draws young children into a new world of understanding." (van Oers, 2001, p. 59) Subsequent to this claim, we focus in our research project on the type of constitution of these "embodiments of mathematical knowledge" emerging in the familial environment of preschoolers. According to our theoretical fundamentals presented above, we assume that the individual conditions under which the children enter the "new world of understanding" are fundamentally different according to their cultural experiences at home.

For children, family is a place of experience beside others such as the nursery school or peer groups. In spite of being just one component of the child's life-world, family has an extraordinary relevance, with its own values, rules and practices.

With regard to our research focus "learning mathematics within family discourses", we refer to Bishop's differentiation between enculturation and acculturation (1988 &

2002). These two conceptions contain two different perceptions of learning mathematics. In the first one, learning mathematics is seen as the induction, by the cultural group, of young people into their culture (Bishop, 1988). Pursuant to this point of view, mathematics is a natural part of the everyday life that is shared with the young. By contrast, Bishop (2002) delineates learning mathematics as a process of acculturation. Following Walcott, he defines acculturation as a "modification of one culture through continuous contact with another" (Bishop, 2002, p. 193f.). So, in this case, mathematics is regarded as a separate culture which is, for a start, disconnected from children's everyday life. With regard to our field of observation, we don't commit ourselves to one of Bishop's opposed conceptions. In fact, we like to identify the degree to which home mathematics learning can be thought as an enculturative or acculturative experience (Fade & Faria, 2008).

Furthermore, mathematical discourses practiced at home are of particular importance not only because they carry certain pictures of mathematics, but because they familiarize children with particular interactional patterns (Street et al., 2005). An empirical study conducted by Street et al. (2005) shows that children's experiences of these discourses are dramatically different. In terms of mathematical discourses at home and at school, the researchers explain that, for some children, there is a gulf between these contexts: "The school replicates the Primary Discourse of middle class homes whilst it presents children from other backgrounds with a Secondary Discourse." (Street et al., 2005, p. 7) At this point, we can clearly see the connection between early mathematics, discourse practices at home and their relation to mathematics education. According to the study just cited, many children are restricted in their prospects to succeed in mathematics education because they are confronted with a problem of language: the switch between home and school discourses can be a source of difficulty because of different values, rules and patterns. In line with those conclusions, but without relating her research to classes, Sfard exposes interactional patterns that are especially similar to school discourses. "This structural similarity can be seen mainly in the type of questions presented to the children, in the parent's fine-tuned scaffolding actions, and in their tendency for repeating one kind of tasks several times, until the children show evidence of some mastery." (Sfard, 2005, p. 249; see also Street et al., 2005).

Support Structures

This view on early learning processes is related to our idea of support structures in child-parent-discourses and to the general discussion about the decisive role of adults for children's development (Vygotsky 1978, Bruner 1983, Rogoff 1989). Vygotsky delineates learning as a process in which children internalize skilled approaches from their participation in joint activities with more skilled partners. These joint activities that would be impossible for the child on its own define the so-called "zone of proximal development" (Vygotsky, 1978). With this theory of development Vygotsky realizes the integration of individual learning in social and cultural context. In another manner, Bruner (1983) does the same. He conceptualises learning with regard to a

support system provided by capable interlocutors. The child is induced in a certain "format", which contains the idea of increasing autonomy and responsibility for the child. An advancement of these two theories was introduced by Rogoff (1989). With regard to Bruner, she pushes the interactional equality of adults and children closer to the spotlight: "The mutual roles played by children and their caregivers rely both upon the interest of caregivers in fostering mature roles and skills and on children's own eagerness to participate in adult activities and to push their development." (Rogoff, 1989, p. 209) According to this basic assumption, she describes the learning process as a "guided participation". Thereby, she replaces Vygotsky's idea of internalization by that of "appropriation". In the process of appropriation, the children "can carry over to future occasions their earlier participation in social activity." (Rogoff, 1989, p. 213) In other words, in her opinion, learning is a process of transformation of individual participation in cultural activities. Because of this analogy to interactionistic fundamentals, we regard the concept of guided participation as especially valuable for our theoretical framework. What kind of guided participation shapes the child's early mathematical experiences? And, in more detail, what picture of mathematics do young children become familiar with?

Pursuing these key questions, we plan to explore the different forms of guided participation in German families between the two poles of enculturation and acculturation.

METHODOLOGY

Our main focus is on everyday mathematical discourses between preschoolers and their parents. In order to achieve a well-rounded picture of early mathematical learning processes in families, we plan to collect different types of data, which will be related to each other via the help of data triangulation. Hence, we will collect basic data of the family (age, siblings, educational background, etc.), data of interaction and data from parent interviews. This need not mean that we use the diversity in data to mutually check their validation, but rather to shed light on the subject matter – namely processes of enculturation or acculturation within the family – and, as such, gain a more multi-faceted than inherently consistent image. We lay out our study as a comparative set of case studies, which means that we will collect data in several families and, after analysing them case by case, we will compare different families on the one hand and insights from different kinds of data on the other.

In the following, we will describe the main data types - "interaction processes" and "guideline interview" - and illustrate them with examples from our preliminary studies.

Interaction processes

To get access to interaction processes which are of interest within the scope of our research project, we have chosen two impulses which we consider as more or less typical for the familial context: picture-books and games. Therefore, we would like to ask a child of preschool age and its parent in each case to take a look at a picture

book, or to play a game together. These situations will then be videotaped for later analysis.

The reason we regard picture books and games as adequate for initiating mathematical discourses is because of their value in the child's everyday life: „The underlying thought of using picture books for mathematics education is that they can offer a meaningful context for learning mathematics and can offer a 'cognitive framework' with 'cognitive hooks' to explore mathematical concepts and skills. Picture books are also ascribed an important role for the development of mathematical language." (Heuvel-Panhuizen, Boogaard, Scherer, 2007, p. 831) In our opinion, games can be of similar relevance for learning mathematics.

In order to initiate mathematical discourses, we chose picture books and games that offer varied mathematical contents. In addition, we will invite the participating families to present a book or game they are familiar with. In each case, the participants may choose the place as well as the book or game and, finally, stop reading or playing whenever they wish to. Thereby, we assume that everyday practices and discourse structures emerge even in contact with potentially strange material. Analysing such discourse structures referring to mathematical learning processes, we focus on emerging support structures.

In order to identify support structures in these initiated discourses, we will conduct an analysis of interaction which refers to the interactional theory of learning (Cobb & Bauersfeld, 1995). This method was devised by a working group around Bauersfeld, in reference to ethnomethodological conversation analysis. Focusing on the evolvement of the topic(s) and patterns of interaction, this analysis serves as a foundation. Thus, an analysis of participation follows which focuses on the issues of "responsibility and originality that one can ascribe to a person's utterance" (Krummheuer, 2007, p. 67; Brandt, 2007).

Interview

These interactional situations are to be complemented by semi-structured interviews taken with each parent at the beginning of the study, thus, nearly a year before the start of school, and also at the end, a few weeks after the child's first day at school.

The interviews are based on problem-centered guidelines (Patton, 2002; Witzel, 2000). The first interview is to shed light on the parents' ideas of mathematics, of mathematical and general learning processes, the families' practices concerning books and games and the preparation for the forthcoming school start. In the final interview, however, different priorities are set. So, the focus is rather on the experiences made with our materials during the preceding months, on the potential impact that the study has on the family's everyday life, and on the experience with school start.

In line with the conception of the problem-centered interview, the respondents are always considered as "experts of their orientations and actions" (Witzel, 2000). For this reason, the interview guidelines just serve as a basic checklist during the

interview to make sure that all relevant topics are covered. In fact, the most important point is that the interview situation provides "a framework in which respondents can express *their own* understandings in their own terms" (Patton, 2002).

In order to find the basic ideas outlined by the parent, we will conduct the qualitative content analysis devised by Mayring (Mayring, 2000). We will use this generally accepted method in a certain form witch includes two central steps: "inductive category development and deductive category application" (Mayring, 2000, p. 3). The scope for the category development will be the distinction between mathematics as a social practice in everyday live and as a fixed faculty culture and in this sense learning mathematics as enculturation or acculturation.

EXAMPLES FROM PRELIMINARY STUDIES

In order to illustrate our research design, we will present examples of the main data types and first conclusions in the following.

Example 1: Florian – mathematical discourse

This first episode is extracted from a reading session with Mrs. Gerlach, her 5-year-old son Florian and her 2-year-old daughter Loni [1]. They look at the picture book "365 Pinguine" [2].

Mrs. G.	`Every morning, a new penguin arrives. How many are there?`
Florian	Hum.
Loni	Two!
Mrs. G.	`31 plus 28 equal?`
Florian	Hum, I don't know.
Loni	(*citing the book)* Ring! Ring!
Florian	Oh.
Mrs. G.	That's rather difficult.
Florian	Yes, but it is... Well, 20 plus 30 equal, oh, 50. Then, plus 8 is 58. Yeah, it is 58.
Mrs. G.	You did it really well. However, you missed one.
Florian	59.
Mrs. G.	Fif, and here is the solution (*points at the solution presented in the book*).

In this short sequence, a mathematical matter arises from reading. Entering into that question, Mrs. Gerlach doesn't push her son for an answer. By emphasising the intricacy of the problem at hand, she opens the situation for him. From now on, he can fail to answer the question without losing face. Against this background, Felix uses the opportunity to exhibit his mathematical capacity. He ventures to enter a

mathematical field with which he isn't familiar yet. Thereby, he decomposes the problem into two steps. The second step of calculation is not affirmed by Mrs. Gerlach. She refers to the solution presented in the book instead. Altogether, Felix is responsible for the solution process; in terms of the analysis of participation, he is the "author" which means that he expresses his own ideas in his own words (Krummheuer, 2007).

Example 2: Linus – mathematical discourse

This second episode is from a reading session with Mrs. Bultmann and her 5-year-old son Linus. They look at the picture book "Es fährt ein Boot nach Schangrila" [3].

Mrs. B.	`At pier 6, the woodpecker starts feeling sick. For this reason, five koalas immediately complain to the captain.` Five bears, small and grey. Do you know where they are?
Linus	(*tips a koala in the picture)*
Mrs. B.	One. Point a finger at the koalas! Look here, one *(points the finger at another koala in the picture)*. With the finger, Linus!
<Linus	*(points at all the five koalas one after another)*
<Mrs. B.	One, two, three, four, five – great!

In this episode, Mrs. Bultmann reads the text out at first. Subsequently, she sets a specific structure, asking Linus to find the koalas. Instead of answering verbally, he points at a koala in the picture. This nonverbal answer is marked as inadequate by Mrs. Bultmann. Thus, she gives the number word and asks Linus to point at the koalas, although he already did the latter. By this means, she specifies how to perform the fixed algorithm she demands: pointing and pronouncing the number words at the same time and step by step. In the following, she initiates the counting process once again, starting with another koala. Linus continues pointing at the koalas, whereas his mother pronounces the number words. Altogether, the mother insists on a specific structure, in which Linus' action is integrated; in terms of the analysis of participation, Linus is a „relayer", which means that he "claims no responsibility neither for the syntactical nor for the semantic aspect of his statement" (Krummheuer, 2007, p. 67).

Example 3: Different ideas of mathematics - interview

In addition to the reading sessions, we interviewed all parents. Here are three answers to the question: What comes first to your mind when you hear the word mathematics?

Mrs. Gerlach: Hum, mathematics? Well, logic, structures. Hum… Hum, and everyday life as well, so, the relevance for the everyday life, thus, there are a lot of things which have to be calculated. So, it is of great importance on all levels and, it is, yes, I think, it is really important.

Mrs. Bultmann: When I think about math? Oh, my God… Everything with plus, I would say. So, spontaneously, I would think about everything with plus.

Mrs. Yoritomo: Mathematics, so, systematic thinking. And very useful. And, for me, with the piano, it is especially important, no, the foundation of course. It's really counting and playing at the same time. This is really of prime importance.

These three answers shed light on the diversity of views on mathematics. While both Mrs. Gerlach and Mrs. Yoritomo spontaneously emphasise rather abstract ideas of mathematics, Mrs. Bultmann names the concrete operation of addition – but as a strange idea without connection to her everyday live. Against the background of the complete interviews, this difference between the answers will be even more obvious. While Mrs. Gerlach and Mrs. Yoritomo regard the mathematical basic operations (like addition and subtraction) as part of their everyday lives, Mrs. Bultmann constricts useful mathematics to counting. Her larger distance from mathematical matters comes to the fore as well, when she describes situations in which her son encounters mathematics within the family's everyday life. In this regard, she speaks about proportionality, whereas her son just copes with counting up to ten in the reading situation. By contrast, Mrs. Gerlach's and Mrs. Yoritomo's examples concerning the same topic are more concrete. They report on kitchen activities, playing shops or games of dice, planning holidays or taking interest in mathematical basic operations. It is an astonishing notice that Marc, Mrs. Yoritomo's 4-year-old son, spontaneously names preparing jam as something with relation to counting. Quite afterwards, his mother explains this concrete kitchen experience and the embedded mathematical activities.

Summary and Conclusions

As a summary, we will relate the presented diversity in the parents' views on (home) mathematics and in forms of support structures to our basic idea of learning mathematic as enculturation or acculturation.

Firstly, the ideas of (home) mathematics, reported in the interview, shed light on different levels of familiarity with mathematics. For instance, Mrs. Bultmann regards even mathematical basic operations aside from counting processes as strange and disconnected from her everyday life. Consequently, her son may adopt this distance to mathematics, experiencing elementary calculations in an acculturation process. The other two families treat mathematical topics as more common and integrated in their everyday discourses. This is discernable in Marc's spontaneous insertion during the interview mentioned above and in the short interaction sequence with Mrs. Gerlach and her two children: Not only Florian's participation, but also Loni's reaction shows understanding of the problem at hand: Although "two" is a wrong answer regarding the number of penguins, the utterance is thematically adequate. In contrast to Linus, the children in these families become familiar with mathematical practices within an enculturation process.

These expositions can be supplemented by a deeper examination of the reading sessions. Within these sequences, different kinds of support structures emerge. More precisely, we can see the space given by the conception of "guided participation" (Rogoff, 1989). While one support structure focuses on the child's involvement in a fixed practice, the other one emphasises the child's role as a competent interlocutor who produces ideas on his own. We assume that, by these different kinds of participation, the children get different ideas of how to learn mathematics: adopting a fixed structure or probing a flexible tool according to individual ideas. On a more theoretical level, the first form conforms to an intended acquirement of an apparently unchangeable faculty culture, thus, to an acculturative experience. By contrast, the second form corresponds the conception of enculturation, which includes mathematics as a natural part of everyday life.

NOTES

1. Transciption rules: `This font marks text read from the picture book.` < marks persons speaking simultaneously.

2. "365 Penguins". Fromental, J.-l. & Jolivet, J. (2008). *365 Pinguine*. Hamburg: Carlsen Verlag.

3. "A boat goes to Shangrila". März, L. & Scholz, B. (2006). *Es fährt ein Boot nach Schangrila*. Stuttgart/Wien: Thienemann Verlag.

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