MATHEMATICAL TASKS AND LEARNER DISPOSITIONS: A COMPARATIVE PERSPECTIVE

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Mathematical tasks in textbooks, their 'mediation' by teachers and the classroom environments in England, France and Germany are the focus of this study. The author claims that the different mathematical tasks in textbooks (in connection with their mediation by teachers) influence, to a large extent, the differences in activities and practices that are going on in mathematics classrooms, and that these in turn mediate different kinds of learner dispositions. The classroom culture, with its differing dimensions, is likely to set the scene for pupil development as 'learners of mathematics'. The web of these connections is studied in this report.

Keywords: Mathematical tasks; learner identity; comparative education; socio-cultural; culturally figured worlds.

INTRODUCTION

Mathematical tasks in textbooks, learning opportunities and pupil dispositions

Students spend much of their time in classrooms working on mathematical tasks chosen from textbooks. In recognition of the central importance of textbooks, the framework of the Third International Mathematics and Science Study (TIMSS) included large-scale cross-national analyses of mathematics curricula and textbooks as part of its examination of mathematics education and attainment in almost 50 nations (Valverde et al, 2002). They claim that

> Textbooks are the print resources most consistently used by teachers and their students in the course of their common work (ibid., p. viii).

Moreover, they comment on different learning opportunities being offered to students in different mathematics classrooms.

> Clearly, one issue of pervading importance to the nations that participated in TIMSS was the quality of educational opportunities afforded to students to learn mathematics and science - and the instruments that optimise such quality (ibid, p. viii).

Textbooks are a major source of provision of these educational opportunities. Romberg and Carpenter (1986), for example, noted that the textbook was consistently seen (in the US) as "the authority on knowledge and the guide to learning". (p. 25)

It appears that tasks in textbooks influence, to a large extent, how students experience mathematics. Textbooks provide children with opportunities to learn, and learn those things which are regarded as important by their government. Teachers mediate textbooks by choosing and affecting tasks, and in that sense student learning, by devising and structuring student work from textbooks.

It can also be argued that tasks, most likely chosen from textbooks, influence to a large extent how students think about mathematics and come to understand its meaning. Indeed, Henningsen and Stein (1997) assert that

> the tasks in which students engage provide the contexts in which they learn to think about subject matter, and different tasks may place different cognitive demands on students …. Thus, the nature of tasks can potentially influence and structure the way students think and can serve to limit or to broaden their views of their subject matter with which they are engaged. Students develop their sense of what it means to "do mathematics" from their actual experiences with mathematics, and their primary opportunities to experience mathematics as a discipline are seated in the classroom activities in which they engage … (p. 525)

Hiebert et al (1997) similarly argue that students

> also form their perceptions of what a subject is all about from the kinds of tasks they do. … Students' perceptions of the subject are built from the kind of work they do, not from the exhortations of the teacher. … The tasks are critical. (p. 17-18)

Moreover, they assert that

> the nature of the tasks that students complete define for them the nature of the subject and contribute significantly to the nature of classroom life …. The kinds of tasks that students are asked to perform set the foundation for the system of instruction that is created. Different kinds of tasks lead to different systems of instruction. (p. 7)

It appears that mathematical tasks are central to student learning, their developing perceptions of what the mathematics is and what doing mathematics entails.

CLASSROOM ENVIRONMENT, MATHEMATICAL TASKS AND LEARNER IDENTITY

According to Lave and Wenger (1991), tools (and artefacts) constitute the resources, and students learn by participating in social practice using the tools. This also relates to 'conceptual tools', most likely reflected and used in tasks. If students use a conceptual tool, as perhaps advised by a worked example, or teacher's exhortations, or an exercise, and if they use the tool actively, they are likely to build an increasingly rich understanding of the 'usefulness' of this tool in their mathematical world, and of the tool itself. Learning how to use a conceptual tool involves much more than the set of explicit rules it may describe. The occasions and conditions for the use arise out of the contexts of tasks and activities that students are expected to do, and they are framed by the ways the members of the community (e.g. textbook authors) see the world of mathematics.

Different practices in mathematics classrooms are likely to influence the development of different learner identities. For example, Boaler et al (2000) investigated the practices of secondary school teaching from a student's perspective "in order to understand how they construct a sense of themselves in relation to mathematics" (p.

4). They argue that in the US and UK classrooms they studied there exists an "unambiguous vision of what it means to be successful at mathematics, and of what it means to be a mathematician" (p. 8).

According to Henningsen and Stein (1997) what it means to 'do mathematics', or to 'behave mathematically', for students, is largely dependent on the nature of the tasks and activities students are engaged in, and these in turn 'colour' their perceptions of the subject. Thus, doing mathematics, and developing certain perceptions of the subject, is likely to 'produce' particular 'mathematical dispositions' or a 'mathematical point of view' (Schoenfeld, 1988), as well as acquiring mathematical knowledge.

As Boaler (2000) emphasises, students do not just learn methods, or how to carry out a task or to apply algorithms, in mathematics classrooms, but they learn 'to be mathematics learners'. Different classroom cultures, different constraints and affordances, provided by different settings and opportunities for engagement in mathematical practices, are likely to influence their perceptions of what it means to learn and do mathematics. Learning how to engage successfully with the mathematics means learning how to and identifying with the norms of the classroom community. Particular tasks in textbooks may reinforce practices initiated and propagate by the teacher, or vice versa.

Furthermore, Boaler and Greeno (2000) use the notion of identity formation in "figured worlds" (Holland et al., 1998) to explore pupil learning and the influence of pedagogies on their learning. Figured worlds are perceived here as places "where agents come together to construct joint meanings and activities" (p. 173). Mathematics classrooms can be regarded as such figured worlds, because students and teachers work together in these environments and construct meanings of the mathematics, and within that of themselves as learners of the mathematics. Holland et al (1998) is cited to draw attention to actors, and to interpretations by actors when asserting that figured words are socially and culturally constructed realms "of interpretation in which particular characters and actors are recognised, significance is assigned to certain acts, and particular outcomes are valued over others" (p. 52).

This is particularly interesting in terms of comparing "figured worlds" in different countries' classrooms. Questions such as the following may arise: What is similar, or different, in mathematics classrooms in England, France and Germany? What are the rituals of practice? What kinds of tasks are pupils expected to perform, what kinds of activities do pupils, and teachers, engage in? What kinds of interpretations are made, what kinds of acts are respected, what kinds of outcomes are valued?

RESEARCH DESIGN

In a previous study (e.g. Pepin, 1999; Pepin, 2002) the author developed an understanding of practices in lower secondary mathematics classrooms in England, France and Germany, concluding that national educational traditions were a large

determinant and influence on what was going on in these classrooms. In a more recent study, Pepin and Haggarty have investigated mathematics textbooks in the three countries, and connected to that, the ways they were used, by teachers (e.g. Pepin & Haggarty, 2003). This not only supported some of the earlier findings, but also suggested that the use of curricular materials (such as textbooks), together with the selection of (mathematical) tasks, impacts to a large extent on the mathematical 'diet' offered to students.

The author thus re-analysed the amount of data collected over the years, in particular mathematical tasks in selected textbooks, in terms of potential pupil disposition and identity formation. Particularly relevant, and useful, was the work of Boaler and Greeno (2000) and the notion of pupil identity formation in 'figured worlds' (Holland et al, 1998). In terms of analysis a procedure involving the analysis of themes similar to that described by Burgess (1984) was adopted, which had already proved useful in other cross-national studies (e.g. Broadfoot & Osborn, 1993). However, due to the additional cross-cultural dimension, it was important to address the potential difficulties with cross-national research, in particular issues related to conceptual equivalence, equivalence of measurement, and linguistic equivalence (Warwick & Osherson, 1973; Pepin, 2002). In order to locate and understand teacher pedagogic practices and the classroom cultures in England, France and Germany, it was useful to draw on knowledge gained from earlier research (see above) which highlighted the complex nature of practices in mathematics classroom environments, and the value of comparing.

The main questions asked was: How may mathematical tasks in textbooks, teacher practices and classroom environment influence pupil identity construction as learners of mathematics in England, France and Germany?

DISCUSSION AND CONCLUSIONS

To connect tasks in textbooks to students' developing identities as learners of mathematics is not a common link made. Textbooks are often frowned upon, and teachers do not wish to be seen to teach 'according to the book'. However, for better or for worse, and as research indicates, textbooks are the main resources used in mathematics classroom all over the world (Valverde et al, 2002).

This is also true for England, France and Germany. Moreover, teachers choose tasks and exercises from those books, for pupils to complete, students learn from the kinds of work they do during class, and the tasks they are asked to carry out shape to a large extent the kind of work they do. Pupils learn the conceptual tools provided by the tasks in textbooks, by 'legitimate peripheral participation' (Lave & Wenger, 1991) in the practice of school mathematics. However, there are particular school mathematics practices in different countries, and within those countries differing practices in different school 'streams' and 'sets' that are supported by different textbooks for those groupings. Moreover, the types of tasks, the mathematical connectivity between tasks, the conceptual tools suggested for solutions, amongst others, reflect and

support a particular school mathematical culture. Pupils are socialised into these cultures, and as members of the cultures, develop dispositions and form identities as learners of mathematics. However, it would be difficult to claim that in each country there exists a homogeneous mathematical culture supported by textbooks. Instead, the developing 'identities' here are seen as those potentially emerging from the analysis of mathematical tasks in textbooks, and the mediation of those tasks by teachers, thus the tools used by teachers in their classroom practice.

What would pupils learn from the tasks provided by the textbooks analysed, and what kinds of work/activities would they do related to the tasks? In order to engage in the mathematics, pupils must find the task intriguing, something they would like to resolve. This assumes that students relate to the task in the sense that the contexts and situations make it real for them. On the basis of results from this study it is argued that in all three countries pupils are likely to be asked to do exercises and to complete tasks (from textbooks) that are presented in context- context embeddedness seems to be important- and these contexts are similar. Whether the contexts are relevant to pupils, whether they connect to their life experiences is beyond the scope of this study. What is different in the three countries is how the mathematics is linked to the contexts and what pupils are asked to do in those tasks. Whereas in German textbooks it appeared that context and mathematical concepts are connected in the tasks analysed, and links are forged between them, in the English textbook chapter pupils are asked to do contextualised tasks where context are chosen seemingly for their own sake, and with little logical progression or connection to the underpinning mathematical ideas. Most exercises could be done without knowing about concepts of the topic area. To what extent students may deduce concepts, by simply doing the exercises, is not clear. Interestingly, French textbook exercises studied appeared to use contexts as a pretence for introducing the mathematics, a Trojan horse to lead students to the 'essential' section, the 'cours', the mathematical concepts.

To ask what students would learn from these tasks also needs a more nuanced perspective. By addressing the mathematics at the conceptual level (e.g. 'oppositeness' in negative Numbers) one could argue that in France and Germany students would get more insights into the conceptual nature of mathematics, and perhaps its structure, than through English textbook tasks. A second type of 'residue' (Hiebert et al, 1997), it can be argued, may be given through the strategies or methods, for solving problems, provided. French textbooks are explicitly addressing this in a separate section ('apprendre a resoudre') and exercises are organised accordingly. Putting the three country's textbooks on a continuum, it is argued that English textbooks leave it to pupils, or their teachers, to devise or identify strategies to solve problems, and this is likely to be with common sense, whereas in particular French textbooks are explicit about how to solve particular problems.

The message that students may therefore get is that (1) mathematics is simply there to be done (e.g. English *KM 7*[2]), and that contexts and concepts do not necessarily 'talk to each other'; that (2) it is not the contexts that matter (e.g. French *Cinq sur Cinq*),

but the underlying mathematical concepts, and that there are strategies to 'reduce' the contextualised problems to 'simple' mathematical tasks; or that (3) concepts and contexts may be connected, and that the formally structured mathematics, including its strategies for solving problems, may be useful in real life problems (e.g. German Grammar school *LS7*).

In terms of teacher mediation of tasks it appears that one of the most important responsibilities for a teacher is to set appropriate tasks. Teachers in all three countries chose those tasks predominantly from textbooks. What was different were to what extent teachers initiated pupils into those tasks and the ways they chose to introduce the mathematical ideas necessary to do the exercises selected from textbooks. The picture that was painted was that whereas in one country (Germany) teachers introduced the mathematical notion in whole-class discussions and chose particular tasks to 'consolidate' the concept, in another (England) teachers gave relatively brief introductions or rules, and wanted a large number of straight forward exercises to practice. In another (France) teachers were provided with activity type tasks, from textbooks, to initiate pupils into the concept, and after explaining the 'essentials' (*cours*) teachers wanted differentiated exercises to attend to the perceived heterogeneous class.

To what extent teachers selected appropriate and related tasks, so that pupils could see the same mathematical idea from a different angle, or to chain tasks in such a way that opportunities are created to gradually increase pupil understanding was not clear. The literature (e.g. Hiebert et al, 1997) claims that tasks that are related in such away increase the coherence of students' mathematical experiences. Coherence here means that students would perceive the sequence of activities and exercises to fit together and make sense. This goes beyond the scope of this study, but it could be argued that, from the analysis of textbook tasks in selected English textbooks, and looking at the sequence of tasks in selected chapters, students are likely to be asked to do a series of individual, nearly random, tasks that are relatively disconnected and appear not to be leading anywhere. French textbooks provide exercises, graded with respect to the level of perceived difficulty and for particular areas within the topic.

In addition, results from a previous study (e.g. Pepin, 1999) show that French, and in particular German Gymnasium teachers chose exercises, that were perceived to exemplify the idea well and to be 'difficult', for solving in class, and sometimes in whole-class discussion, whereas 'easy' routine exercises were assigned for homework. English teachers said that most of their students needed 'much of the same' to practice.

In terms of classroom environment and culture teachers have a great influence, and this was true for England, France and Germany. Within the limits of the system, whether students were taught in mixed classes (*collège* France), whether they were setted (England) or streamed (Germany), teachers had some freedom to select tasks that could potentially guide their instruction and they could mediate those tasks in ways they thought best. To what extent teachers created cognitive conflict, in order to

challenge pupils' ideas, is beyond the scope of this investigation, but in terms of tasks in textbooks this may potentially be provided by selected cognitive activities (*activitées*) in French textbooks (Pepin & Haggarty, 2003). Moreover, allowing mistakes, perhaps even inviting them for pupil learning, or asking open questions would be another way of influencing the mathematics classroom culture. Looking at tasks in textbooks, there were no open questions in the English textbook chapter analysed, and hardly any in the French and German textbooks. Teacher pedagogic practices, however, may be interpreted as going some way towards that goal: all teachers, but particularly German teachers, used mistakes in homework exercises as a site for deepening pupil understanding (Pepin, 1998). These were discussed in detail and at times over an extended period of time.

In summary, it can be argued then, albeit from this limited research, that the dispositions that pupils are likely to develop as learners of mathematics, are linked to the textbook tasks provided by teachers, the practices that pupils are engaged in when doing those tasks, and the environment they work in and experience in class during engagement- and these are different in the three countries. Whereas in all three countries one could argue that pupils are 'conditioned' to become 'conformists'- hardly any negotiation about the mathematics and its learning is provided-, in England the mathematical diet in textbooks may also offer learners to become 'common sensers'. Can one say that in France the 'instrumentalist' identity may be favoured, and in Germany the 'connector', in addition to the 'conformist'? If this link was seen to be strong, one would need to consider to what extent pupils are 'trapped' in these identities, for better or worse, according to what they are offered by their teachers. What kinds of opportunities would need to be provided for change to be possible?

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