MATHEMATICAL BEHAVIORS OF SUCCESSFUL STUDENTS FROM A CHALLENGED ETHNIC MINORITY

Tiruwork Mulat and Abraham Arcavi

The Weizmann Institute of Science, Rehovot, Israel

This study explored the mathematical behavior of resilient students of Ethiopian origin (SEO), members of an underrepresented and challenged ethnic group in Israel. Using qualitative methodologies, we examined six SEO, three in an advanced secondary school mathematics track and three in a pre-academic course while working on non-routine mathematical tasks. The mathematical behaviours and views of these students were found to be highly consistent with their professed beliefs and behaviors, which we explored in a previous study. Success was attributed to beliefs enacted during problem solving and was accounted for by neither giftedness nor special ethnic characteristics, but rather by high motivation, self-regulation, and persistence driven by positive identities, personal agency and ethnic identification.

Key words: Mathematical behavior, beliefs, self-regulation, resilient, ethnic identity.

INTRODUCTION

In many countries all over the world, immigrants and ethnic minorities often face barriers at school resulting from various factors. Many researchers and educators believe that differential student learning, achievement, and persistence along ethnic and racial lines is one of the most troubling issues in mathematics education and in education in general (e.g. Martin 2000, 2003). In the case of Israel, educators and researchers have done much to describe and classify social, cultural, educational, and other societal difficulties encountered by different groups of immigrant Jews and in particular, those students of Ethiopian origin (SEO, more than half of whom are second generation). A range of studies have documented the overall academic underachievement, the relatively high dropout rates, and the high representation of SEO in special education programs (e.g. Lifshitz, Noam, & Habib, 1998; BenEzer, 2002; Levin, Shohami, & Spolsky, 2003; Wolde Tsadik, 2007). In mathematics, SEO are significantly underrepresented in the advanced tracks towards Matriculation. For example, during the years 1999-2003, among all SEO who were eligible for the 'Bagrut', the Matriculation exam taken at the end of grade twelve in different subjects, only 2% studied mathematics in the advanced track [1], compared with 17% of the entire student population.

In different countries, some groups of immigrants and ethnic minorities achieve well academically; sometimes they even outperform mainstream students. Several studies have focused on explaining differential achievements between various minority groups and within certain minority groups (e.g. Ogbu, 1991; Martin, 2000, 2003; OECD, 2006). Most findings challenge the belief that the disadvantages and difficulties created by being an immigrant or a member of a minority prevent students from excelling in education.

Researchers are increasingly linking motivational, cognitive, and social environmental aspects of learning. Many studies have provided new insights into why individuals choose to engage in different learning activities, and how their identities, beliefs, values, and goals relate to their engagement and mathematics achievements (Steele, 1997; Nasir, 2002; Martin, 2000, 2003; Sfard & Prusak, 2005). It is argued that students' problem-solving processes are influenced by beliefs about the self, about the nature of mathematics knowledge, the task at hand, and its context (e.g. Schoenfeld, 1983). Moreover, implementing self-regulation during problem solving is regarded as an important variable affecting the quality of the solving process: self-regulated learners analyze tasks and set appropriate goals to accomplish these tasks, monitor and control their behaviors during performance, make judgments of their progress and alter their behaviors according to these judgments (Zimmermann, 1989). Social cognitive theorists, assume that self-efficacy is a key variable affecting self-regulated learning and performance (Bandura, 1986); self-regulated learning is believed to occur to the degree that a student can use personal (i.e. self) processes to strategically control and direct both his/her behavior and the immediate learning environment (Bandura, 1986; Zimmermann, 1986).

Based on the personal and environmental factors identified by research in mathematics education and especially based on the findings related to the success of individuals from populations at risk of academic failure, we sought to understand the success factors of SEO, students enrolled in the advanced mathematics track towards Matriculation. We focused on these students' views about their personal experiences in learning mathematics and the perceived impact of the personal and environmental variables on their persistence and success [2]. The conceptual framework used to guide our inquiry is based on the assumption that there are certain malleable personal and environmental factors that play significant roles in these students' academic resilience, defiance of the odds and their ultimate academic achievement. We adhere to the claim that, as opposed to studies of failure (regardless of their academic depth), studies of success constitute a more promising way of understanding and eventually increasing the circle of successful students (Garmezy, 1991; Martin, 2003). In our studies we sought to understand what enables some SEO to succeed despite the potential obstacles they face. We attempt to answer the following questions:

1. To what perceived personal/environmental variables do SEO in Israel attribute their success in mathematics?
2. What are the salient mathematical behaviors of SEO when working on mathematical tasks? How do they view, and reflect upon, their own behaviors?
3. How do the perceived variables, the enacted mathematical behaviors, and the students’ views of these behaviors relate to each other?

In a previous study we explored the first question, through students' self-reports obtained using semi-structured interviews (see below a summary of this study). In the present study we present findings concerning the second and third questions.

FINDINGS FROM THE PREVIOUS STUDY: STUDENTS' SELF-REPORTS

A diverse group of SEO enrolled in the advanced mathematics track towards Matriculation were interviewed and followed up. The group consisted of fourteen students aged 17-19 (seven males and seven females), of which nine were high school students from four different cities and the other five were students enrolled in a special pre-academic program in a prestigious technological university in Israel (each from a different city). All were 'solos', i.e., the only SEO in the advanced mathematics track in their cohort at their schools, which is the optimal situation in most high schools. Our goal was to better understand how these students interpret their experiences and academic achievements within the advanced track in mathematics, in high school and in the university preparatory program, where the presence of students of Ethiopian origin is scarce. Using the qualitative methodology of a collective case study (Yin, 1984; Shkedi, 2005), we analyzed interview transcripts using a grounded approach and employing open coding techniques (Strauss & Corbin, 1990). Data were also triangulated with other sources such as classroom observations and interviews with other students, teachers, and parents. The key elements of success we identified were organized under three major categories (Mulat & Arcavi, submitted):

(1) Motivational variables related to mathematics (e.g., mathematics identity, personal agency, productive attribution beliefs, academic goals, ethnic identification, and social goals activated by a positive cultural model)

(2) Actions and strategies – perceived behavior (e.g., fostered use of academic self-regulation and coping strategies)

(3) Immediate environmental variables (mathematics classrooms, teachers, and parental support)

The central finding of the study was that the synergy among students' motivational variables, their academic self-regulation and coping strategies, shaped and supported by their interaction with the environment, appeared as the key to their success in mathematics.

THE PRESENT STUDY

The aim of the study reported here is to explore the mathematical behaviors and the task-related views of a subgroup of the participants in the previous study, and to examine how the findings of the two studies relate to each other.

METHODOLOGY

Subjects: Six SEO from the previous study participated in this study. Three of them (Eden, Melka, and Jacob) were high school students, and the other three (Selam, Ronnie, and Danny; all pseudonyms) were students in the pre-academic program. The selection of these participants depended upon the availability of extensive data relevant to this study.

Tasks: The students worked on five mathematical tasks, selected especially for this study according to the following criteria: The tasks had alternative solutions; they varied in their level of difficulty; their content level was rather basic and accessible to high school students, yet they were non-routine, challenging, and required some planning strategies. The problems were previewed by mathematics educators who agreed on the mathematical appropriateness for high school students.

Data collection and analysis: The data consisted of students' written work, the interviewer's recorded observations, the protocols of the dialogues, questions and reflections that emerged during task completion, and the transcripts from follow-up interviews. In the interviews, all students were asked to describe their solution approaches and their thinking processes in completing the tasks and to describe their perspectives. These tasks were also given to students' peers in the lower mathematics tracks of the secondary schools. A qualitative descriptive methodology was used to analyze the combined data (Shkedi, 2005).

FINDINGS

A description of students' solution processes, along with the observed behaviors and views for three of the tasks are given, followed by a summary of the significant findings.

Problem 1.

> Find the equation of the line parallel to the given pair of parallel lines and that lies exactly midway between them: (1) $3x-2y-1=0$
>
> (2) $3x-2y-13=0$

Task completion: All participants efficiently completed this problem. The task was characterized by all of them as non-routine since its formulation was seen as different from what they usually encountered at schools, yet it was perceived as easy and accessible by available tools or algorithms.

All subjects showed confidence in their ability to complete this task, and had completed it easily; appearing to be satisfied with their ability (two had minor computational errors). However, despite the existence of alternative ways to solve the problem, both the high school and the pre-academic students applied the 'slope-point formula' procedure they learned at school. Accordingly, the common stages in the students' solution procedures were in this order:

- Transformation of the equations to an explicit form
- Identification of the common slope
- Identification of the y-intercepts (some solved for the x-intercepts)

- Finding the midpoint between the intercepts (using formula or graphs)
- Writing the answer - equation of the line

The participants attributed their success to their rich experience and mastery of similar school tasks. Yet, this task was found to be difficult to many students in the lower tracks, who blindly tried to solve the pair of simultaneous equations in search of a point, after they found the common slope, the first two stages above.

Although both the high school students and the pre-academic students were equally successful in solving this task, we detected a difference in their use of a heuristic and the perception of its necessity. Two of the high-school students drew the graphs of the lines to find the midpoint of the intercepts, whereas all of the pre-academic students did not, claiming that they do not need the graphs to solve this problem and if they do, they can imagine them. The following quotations exemplify these differences among students:

> Instead of visualizing in your head, it is already in your notebook and it is hard to get confused that way. (Jacob)

> Here I do it in my head. You see that they have the same slope…when I can't see things with my imagination, I use sketches. But here you know the question leads you to the solution. (Selam)

Problem 2.

ABC is a right-angled triangle, ∠ ABC=90°.

AB=16; BC=12 and BE=9; BD is the median to AC, and

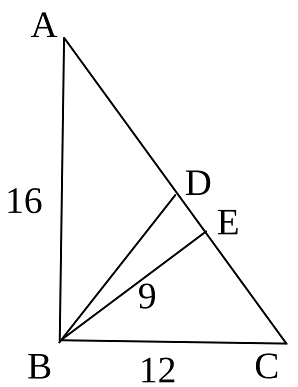
BE is the altitude to AC.

There is an error in one of the given numbers.

(a) Show that there is an error (report all your processes).

(b) Change only one of the numbers (9, 16, or 12)

to correct the error.



Task completion: Students showed different performance levels on this task. All students started by marking the given numbers on a triangle they drew and by calculating the length of the hypotenuse AC=20 (one made a computational error). Five of the students also marked AD=BD=DC=10, referring to the theorem about the median to the hypotenuse in a right triangle, but only three used this information to produce their solutions later. Only three of the students completed both parts of the task independently showing ease and confidence (but one had computational as well as other major errors and thus got a wrong answer). The other three students had difficulties in devising a plan and an effective strategy to proceed with the task; they were stuck for a long time; two of them said that they checked whether there is a side with a length greater than the sum of the other two sides. These students were confused and disturbed since they did not know how to plan their solution procedure and were uncertain about their understanding of the question. After some unsuccessful trials, they quit and proceeded with the other questions and returned to complete the task after receiving supporting clues and prompts from peers and from the interviewer.

In the first part of the task, students used different strategies to show that a triangle having the given sizes is not possible. Two showed that they got two different areas for the same triangle, three showed two different sizes for a side of the triangle; another student showed that the corresponding sides of similar triangles are not proportional. Five of these students used the same strategies they used for the first part to answer the second part of the question. One chose to use a trial and error method. Half of the students mentioned the possibility that the error could be corrected by changing any one of the three numbers. Since there were different ways to show that there is an error, the error could be corrected by changing any one of the three numbers, implying different ways and possibilities to answer the second part of the question. Yet all participants decided to change 9 (which was a good choice); four students (two of them with support) completed the problem successfully. The other two students, one who used a trial and error method and another who made a major error in her computations to change 9 got wrong results.

All students characterized this task as non-routine, saying that it is not like school tasks that they usually solve with great ease and success, and that here they could not just apply known algorithms to obtain a solution. Danny, who completed all the tasks successfully, characterized this task as 'a deceptive question'. Jacob said:

> This is a question in geometry, but never, at least I never encountered questions like this, saying that there is a mistake, correct a mistake. Usually they give you exercises that have solutions at the very beginning, and if you work by the book, you succeed, but here you have to think more.

Melka also referred to her school experiences:

> We are not used to such kind of questions; they never tell us to correct mistakes; they always provide us with given objects and ask to do other things and not to correct mistakes.

In sum all the students (some with probing), completed the first part of this task successfully by using different strategies. While four of them also succeeded with the second part, the other two students used ineffective strategies and got wrong answers.

Problem 3.

Given is an array of natural numbers arranged under four columns, A, B, C, and D, as shown here.

(a) Under which letter does the number 101 appear?

(b) Under which letter does the number 1001 appear?

A	B	C	D
1	**2**	**3**	**4**
8	**7**	**6**	**5**
9	**10**	**11**	**12**
	…	**14**	**13**

(c) Answer questions (a) and (b) above, for a five-column array of numbers with the same pattern

Task completion: This task seemed to be more difficult than the other tasks especially for the high school students. It also took most students more time than each of the other tasks. Only two students (both pre-academic) found effective rules and gave correct solutions with clear explanations.

As a first step towards finding a solution to this problem, all students added more numbers to the list following the given pattern. All of them tried for quite a long time to find a possible pattern or rule to solve this task (unlike students in the lower tracks, who tried to answer it by listing numbers to reach 101 without looking for a rule). As stated above, only two students (both pre-academic) proposed a similar rule: even and odd multiples of 4 can be found in alternate lines of the outer left and outer right columns (A and D), respectively. The other pre-academic student, however, did not recognize the sequences' pattern on the extreme columns and added to the list in a wrong order. Consequently, she did not succeed in completing the task, but she refused to hear a solution method and asked to complete the task at home by herself. Three of the high school students did not write their rule clearly, and their answers were mostly wrong or not justified, although two were certain they had obtained a working rule. The other student, who seemed less confident, said that she solved it logically, using her common sense, and that she did not know how to communicate her method.

This task was characterized as difficult by all participants. One of the students even commented that it is not a mathematical question; the other said it is a 'thinking' question that challenges the mind, and that schools do not offer such questions.

SUMMARY

As stated above, this study explored the mathematical behaviors displayed by successful SEO, and analyzed the relationships between these behaviors and the professed beliefs and reflections found in a previous study in which the students also participated. Some of the findings from the previous study (e.g., ethnic identification, social goals, and parental support) were not salient in the present study due to their very nature; these categories are rarely captured while students work on mathematical tasks. However, in other findings we found consistency between the 'professed' beliefs and behaviours and the 'enacted' mathematical behaviors, as described in the following.

Motivational beliefs: Students showed a variety of behaviors and performances. Although some students lacked confidence when they had no handy effective strategies, their behaviors were consistent with their professed efficacy beliefs and their confidence in their ability to solve the problems. They said that they have the mathematical knowledge necessary for completing the tasks and shared their enjoyment and satisfaction of being engaged in questions that demand thinking. They attributed their difficulties in solving these problems to a lack of previous experience with non-routine questions. They expressed their expectations that schools should provide opportunities to encounter and practice such tasks that require 'thinking'.

Self-regulation strategies: Self-regulation is one of the characteristics that we had identified in the previous study as playing a prominent role in these students' success in school mathematics. The students expressed their belief that what it takes to succeed in school is planning and evaluating their own actions and strategies by investing time and effort to study what is taught at school. When these students failed to solve some of the non-routine tasks of this study, they attributed it to not having the right tools, since their learning efforts were directed to what school had taught them. Thus, cognitive regulation and retrieval of the appropriate knowledge and the strategic tools needed for the tasks in this study were difficult for them. Many of the students quit after some unsuccessful trials, moved on to other questions but still returned to the unsolved tasks later. We took this willingness not to give up as yet another manifestation of these students' good self-regulation strategies applied to difficult situations for which they were unprepared. This strategy was found to pay off for some students, since with some probing they succeeded to complete the tasks.

Solo learning: Though students were told that they can work with their peers (three of them had opportunities to do so) and also that they can ask for support from the interviewer at any time while working on the tasks, they did not use these opportunities productively. Suggestions to support the students when they were stuck at certain stages were all initiated by the interviewer. The preference of students to work alone was also in line with these students' professed 'solo learning' characteristics.

Perceptions about the tasks: The tasks were characterized as non-routine, including the first question that all could easily solve, yet they expressed their enjoyment and satisfaction in performing such tasks. The students were very critical about mathematics lessons at schools that do not offer students opportunities to face challenging tasks.

Differences within groups: Though the tasks are appropriate for any high school student, overall, the pre-academic students showed (a) greater confidence in completing the tasks (even when they were not always successful), and (b) better communication skills to write and explain clearly their solution processes. These differences could be attributed to the pre-academic students' self-reports that in contrast to high school teachers, the teachers in the program have exposed them to meaningful mathematics learning, which also developed their confidence, intrinsic interest in mathematics and mathematics identity.

DISCUSSION AND CONCLUSION

Whereas these SEO's success in school was, to some extent, due to learning by playing well the school rules, which are mostly rehearsing and following algorithms, completing the tasks of this study engaged these students with a quite different experience. Thus, since these students were not especially gifted and their knowledge resources come only from school, their success can be attributed to their mathematics identity, motivation, and self-regulation skills; all these were supported by their other professed beliefs and views in relation to the tasks. Moreover, the heterogeneity of solution approaches and strategies observed in this study is proposed as a further confirmation how resilient and minded to success these students are, each of whom mustered resources and alternatives from his/her own to solve the tasks.

In sum, neither exceptional cognitive ability nor common cognitive characteristics of a certain "ethnic" group are variables that play significant roles in analyzing success (or failure) of these SEO. It is their determination, personal identities and support that shape their self-regulation, persistence and beliefs that shape their behaviours and ultimately their success. From this and related findings, we argue that educational systems that want ethnic minorities to succeed academically have much to learn from these and related findings regarding the roles of identities, self regulation, enhancement of motivation and support of learning which can take place in collaboration with peers.

NOTES

1. In the Israeli education system not all students are eligible for Matriculation; eligibility is determined according to the students' prior achievements. In mathematics, those eligible have taken one of three levels: basic (3 units), intermediate (4 units), and advanced (5 units).

2. In this study success refers to enrolment in the advanced track towards Matriculation

REFERENCES

Bandura, A. (1986). *Social foundations of thought and action: A social cognitive theory*. Englewood Cliffs, NJ: Prentice-Hall.

BenEzer, G. (2002). *The Ethiopian Jewish exodus: Narratives of the migration journey to Israel, 1977-1985*. London: Routledge

Garmezy, N. (1991). Resiliency and vulnerability to adverse developmental outcomes associated with poverty. *American Behavioral Scientist, 34*(4), 416-430.

Levin, T., Shohamy, E., & Spolsky, B. (2003). Academic *achievements of immigrant children in Israel*: *Research report*. Jerusalem: Israeli Ministry of Education, Culture, and Sport [in Hebrew].

Lifshitz, H., Noam, G., & Habib, J. (1998). *Absorption of adolescents of Ethiopian origin: Multidimensional perspective.* Jerusalem: Brookdale Institute, Joint [in Hebrew].

Martin, D. B. (2000). *Mathematics success and failure among African-American youth: The roles of sociohistorical context, community forces, school influence, and individual agency*. Mahwah, NJ: Erlbaum.

Martin, D. B. (2003). Hidden assumptions and unaddressed questions in mathematics for all rhetoric. *The Mathematics Educator, 13*(2), 7-21.

Mulat, T. & Arcavi, A. (Submitted). Elements of success in mathematics among students of Ethiopian origin in Israel: A case study. Submitted to *The American Educational Research Association.*

Nasir, S. N. (2002). Identity, goals and learning: Mathematics in cultural practice. *Mathematical Thinking and Learning, 4*, 213-247.

OECD**,** Organization for Economic Co-operation and Development (2006). Where immigrant students succeed: A comparative review of performance and engagement in PISA 2003.

Retrieved from: http://213.253.134.43/oecd/pdfs/browseit/9806021E.PDF

Ogbu, J. U. (1991). Immigrant and involuntary minorities in comparative perspective. In M. A. Gibson & J. U. Ogbu (Eds.), *Minority status and schooling: A comparative study of immigrant and involuntary minorities* (pp. 3-33). New York: Garland.

Schoenfeld, A. H. (1983). Episodes and executive decisions in mathematical problem solving. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 345-395). New York: Academic.

Sfard, A., & Prusak, A. (2005). Telling identities: The missing link between culture and learning mathematics. In H. L. Chick (Ed.), *Proceedings of the 29th conference of the International Group of Mathematics education: Vol.1*. (pp. 37-52). Melbourne: PME.

Shkedi, A. (2005). *Multiple case narratives: A qualitative approach to studying* multiple populations. Amsterdam: Benjamins.

Steele, C. (1997). A threat in the air: How stereotypes shape the intellectual identities and performance of women and African Americans. *American Psychologist*, *52*(6), 613-629.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory, procedures and techniques.* London: Sage.

Wolde-Tsadik, A. (2007). Ethiopian Integration – Education and Employment: New Findings in Brief. Jerusalem: MYERS, JDC, Brookdale Institute.

Yin, R. K. (1984). *Case study research: Design and methods*. London: Sage.

Zimmerman, U. J. (1986). Development of self-regulated learning: Which are the key Subprocesses? *Contemporary Educational Psychology, 16*, 307-3 13.

Zimmermann, B. J. (1989). A social cognitive view of self-regulated academic learning. *Journal of Educational Psychology*, *81*(3), 329-339.