SPOKEN MATHEMATICS AS A DISTINGUISHING CHARACTERISTIC OF MATHEMATICS CLASSROOMS IN DIFFERENT COUNTRIES

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This paper reports research into the occurrence of spoken mathematics in some well-taught classrooms in Australia, China (both Shanghai and Hong Kong), Japan, Korea and the USA. The analysis distinguished one classroom from another on the basis of public "oral interactivity" (the number of utterances in whole class and teacher-student interactions in each lesson) and "mathematical orality" (the frequency of occurrence of key mathematical terms in each lesson). Our concern in this analysis was to document the opportunity provided to students for the oral articulation of the relatively sophisticated mathematical terms that formed the conceptual content of the lesson. Classrooms characterized by high public oral interactivity were not necessarily sites of high mathematical orality. The contribution of student-student conversations also varied significantly. Of particular interest are the different learning theories implicit in the role accorded to spoken mathematics in each classroom.

Key words: Spoken mathematics, classroom research, international comparisons

INTRODUCTION

The Learner's Perspective Study (LPS) sought to investigate the practices of well-taught mathematics classrooms internationally. Data generation focused on sequences of ten lessons, documented using three video cameras, and interpreted through the reconstructive accounts of classroom participants obtained in post-lesson video-stimulated interviews (Clarke, 2006). The post-lesson interviews address the challenge of inferring student conceptions from video data (Cobb & Bauersfeld, 1994). The LPS approach of conducting case studies of classroom practices over sequences of at least ten lessons in the classes of several competent eighth grade teachers in each of the participating countries offers an informative complement to the survey-style approach of the two video studies carried out by the Third International Mathematics and Science Study (TIMSS) (Hiebert et al., 2003; Stigler & Hiebert, 1999). The criteria for the identification of the competent teachers studied in the LPS were specific to each country, in order to reflect the priorities and values of the school system in that country. In this paper, we report analyses of lessons documented in classrooms in Australia, China (Hong Kong and Shanghai), Japan, Korea, and the USA.

The complete research design has been detailed elsewhere (Clarke, 2006). For the analysis reported here, the essential details relate to the standardization of transcription and translation procedures. Since three video records were generated for each lesson (teacher camera, student camera, and whole class camera), it was possible to transcribe three different types of oral interactions: (i) whole class interactions, involving utterances for which the audience was all or most of the class, including the teacher; (ii) teacher-student interactions, involving utterances exchanged between the teacher and any student or student group, not intended to be audible to the whole class; and (iii) student-student interactions, involving utterances between students, not intended to be audible to the whole class. All three types of oral interactions were transcribed, although type (iii) interactions could only be documented for the selected focus students in each lesson. Where necessary, all transcripts were then translated into English. All participating research groups were provided with technical guidelines specifying the format to be used for all transcripts and setting out conventions for translation (particularly of colloquial expressions).

In this paper, our unit of analysis is the utterance and we distinguish *private* spoken student-student interactions from whole class or teacher-student interactions, both of which we consider to be *public* from the point of view of the student. Our major concern in this analysis was to document the opportunity provided to students for the oral articulation of the relatively sophisticated mathematical terms that formed the conceptual content of the lesson and to distinguish one classroom from another according to the manner in which such student mathematical orality was afforded, promoted, constrained or discouraged in both public and private arenas.

STUDYING SPOKEN MATHEMATICS IN THE CLASSROOM

This paper reports four stages of a layered attempt to progressively focus on the significance of the situated use of mathematical language in the classroom. In our first analytical pass, an utterance is taken to be a continuous spoken turn, which may be both long and complex. We restricted our second-pass analysis to those mathematical terms and phrases that referred to the substantive content of the lesson (usually designated as such in the teacher’s lesson plan and post-lesson interview). The third and fourth passes repeated the focus on utterances and then mathematical terms, but in the context of student-student (private) conversation.

We take the orchestrated use of mathematical language by the participants in a mathematics classroom to be a strategic instructional activity by the teacher. In this paper, we invoke theory in two senses: (i) the (researchers’) theories by which the actions of the classroom participants might be accommodated and explained, and (ii) the (participants’) theories implicit in the classroom practices of the teacher and the students. A particular focus is the role of the spoken word in both. The instructional value of the spoken public rehearsal of mathematical terms and phrases central to a lesson’s content could be justified by reference to several theoretical perspectives. Interpretation of this public rehearsal as incremental initiation into mathematics as a discursive practice could be justified by reference to Walkerdine (1988), Lave and Wenger (1991), or Bauersfeld (1994). The instructional techniques employed by the teacher in facilitating this progression could be seen as “scaffolding” (Bruner, 1983) and/or as “acculturation via guided participation” (Cobb, 1994).

The oral articulation of mathematical terms and phrases by students could be accorded value in itself, even where this consisted of no more than the choral repetition of a term initially spoken by the teacher. Teachers and students in some of the classrooms we studied clearly attached value to this type of recitation. In other classrooms, the emphasis was on the students’ capacity to produce a mathematically correct term or phrase in response to a very specific request (question/task) by the teacher. In such classrooms, both of these activities accorded very limited agency to the learner and the responsibility for the public generation of mathematical knowledge seemed to reside with the teacher. By contrast, in other classrooms, the instructional approach provided opportunities for students to “brainstorm” or to generate their own verbal (written or spoken) mathematics, with very little (if any) explicit cuing from the teacher (e.g. the classrooms in Tokyo).

The role of student-student spoken interactions also varied widely among the classrooms studied. The teacher’s posing of particular mathematical tasks (Mesiti & Clarke, in press) could prompt (and even promote) certain forms of individual, dyadic or small group mathematical behaviour and even monitor and guide that behaviour during classroom activities such as Kikan-Shido (Between-desks-instruction) (O’Keefe, Xu, & Clarke, 2006). However, within these constraints, students have significant latitude and agency in their use of spoken mathematics. The frequency of occurrence of student-student utterances varied from zero in some lessons (eg. Seoul) to as many as 100 distinct student-student utterances per lesson by individual students in classrooms in Australia and the USA. In each classroom, the activity of speaking mathematics was performed differently.

The results that are reported in this paper certainly suggest that the teachers in this study differed widely in the opportunities they provided for student spoken articulation of mathematical terms, whether in public or in private, and in the extent to which they devolved agency for knowledge generation to the students. The demonstration of such differences (and we would like to argue that these differences are profound and reflect fundamental differences in basic beliefs about effective instruction and the nature of learning) in the practices of classrooms situated in school systems and countries that would all be described as “Asian” suggests that any treatment of educational practice that makes reference to the “Asian classroom” confuses several quite distinct pedagogies. This observation is not to deny cultural similarity in the way in which education is privileged and encountered in communities that might be described as “Confucian-heritage.” But, the identification of a one-to-one correspondence between membership of a Confucian-heritage culture and a single pedagogy leading to high student achievement is clearly mistaken, and cultural similarity is not a sufficient indicator of those instructional practices that might be associated with the educational outcomes that we value.

THE USE OF MATHEMATICAL TERMS

In this paper, "utterance" and "mathematical term or phrase" require clear specification (below). Our analysis of public and private classroom interactions has restricted its attention to key and related (primary and secondary) terms, however the analysis of the post-lesson student interviews also considered 'other' terms used by students in interview to explicate the lesson's content or in reflecting on the nature of mathematical activity in general. This paper focuses on analysis of public and private classroom interactions. Consideration of student use of spoken mathematics in the post-lesson interviews will be reported in another paper.

Figure 1 shows the number of utterances occurring in whole class and teacher-student interactions in each of the first five lessons from each of the classrooms studied in Shanghai, Hong Kong, Seoul, Tokyo, Melbourne and San Diego. An utterance is a single, continuous oral communication of any length by an individual or group (choral). Used in this way, the frequency (and origins) of public utterances constitute a construct we have designated as public oral interactivity. This does not take into account either the length of time occupied by an utterance or the number of words used in an utterance (problematic in a multi-lingual study like this one). Figure 1 distinguishes utterances by the teacher (white), individual students (black) and choral responses by the class (e.g. in Seoul) or a group of students (e.g. in San Diego) (grey). Any teacher-elicited, public utterance spoken simultaneously by a group of students (most commonly by a majority of the class) was designated a "choral response." Lesson length varied between 40 and 45 minutes and the number of utterances has been standardized to 45 minutes.

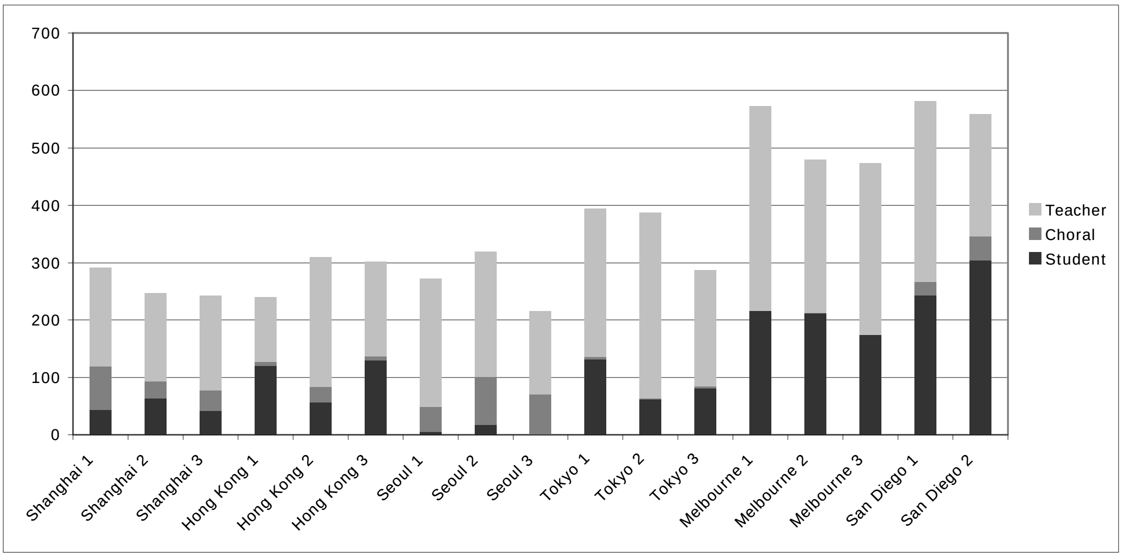

Figure 1: Number of Public Utterances in Whole Class and Teacher-Student Interactions (Public Oral Interactivity)

Figure 1 suggests that lessons in Melbourne and San Diego demonstrated a much higher level of public oral interactivity than lessons in Shanghai, Hong Kong, Seoul, or Tokyo. There were also substantial differences in the relative frequency of teacher, student and choral utterances. It is worth noting that both teacher and student utterances in Shanghai tended to be of longer duration and greater linguistic complexity than elsewhere.

The classrooms studied can be also distinguished by the relative level of public mathematical orality of the classroom (that is, the frequency of spoken mathematical terms or phrases by either teacher or students in whole class discussion or teacher-student interactions) and by the use made of the choral recitation of mathematical terms or phrases by the class. This recitation included both choral response to a teacher question and the reading aloud of text presented on the board or in the textbook. For the purposes of this paper, those mathematical terms were coded that comprised the main focus of the lesson's content.

Figure 2 shows how the frequency of public statement of mathematical terms varied among the classrooms studied. In classifying the occurrence of spoken mathematical terms, we focused on those terms that could be related to the main lesson content (e.g. terms such as "equation" or "co-ordinate"). This meant that our analysis did not include utterances that constituted no more than agreement with a teacher's mathematical statement or utterances that only contained numbers or basic operations that were not the main focus of the lesson.

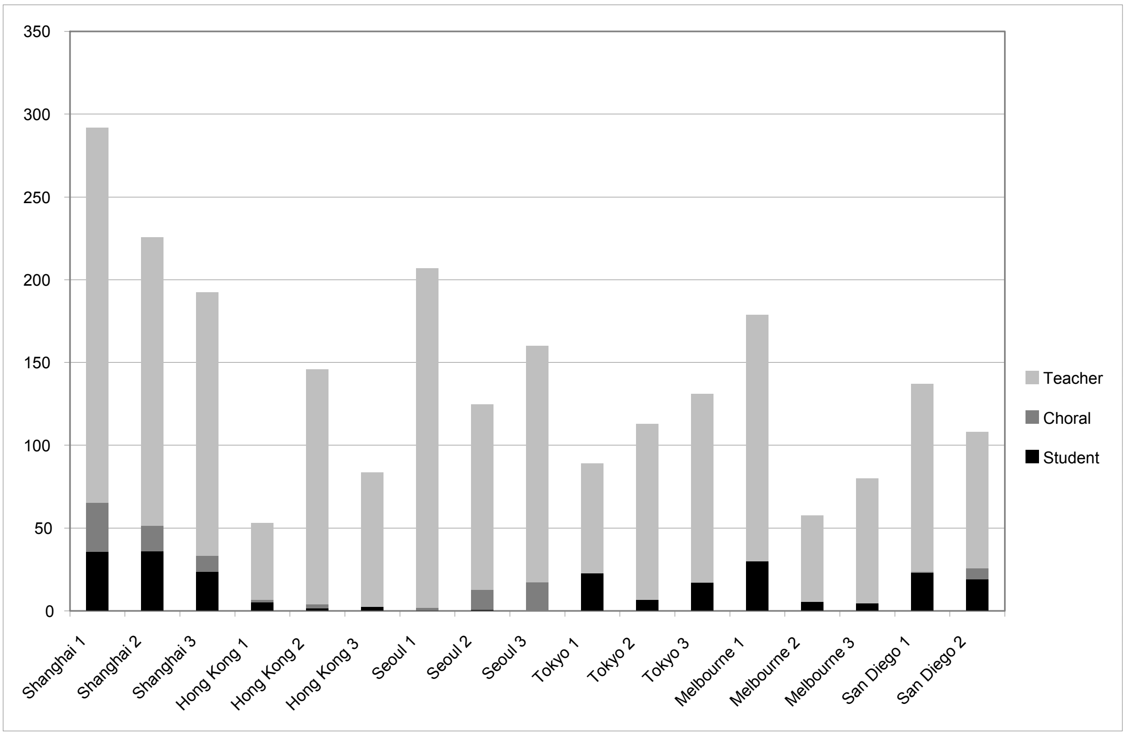

Figure 2: Frequency of Occurrence of Key Mathematical Terms in Public Utterances (Mathematical Orality)

In the case of the Korean lessons, the choral responses by students frequently took the form of agreement with a mathematical proposition stated by the teacher. For example, the teacher would use expressions such as, "When we draw the two equations, they meet at just one point, right? Yes or no?" And the class would give the choral response, “Yes.” Such student statements did not contain a mathematical term or phrase and were not included in the coding displayed in Figure 2. Similarly, a student utterance that consisted of no more than a number was not coded as use of a key mathematical term. It can be argued that responding “Three” to a question such as “Can anyone tell me the coefficient of x?” represented a significant mathematical utterance, but, as has already been stated, our concern in this analysis was to document the opportunity provided to students for the oral articulation of the relatively sophisticated mathematical terms that formed the conceptual content of the lesson. Frequencies were again adjusted for the slight variation in lesson length.

The most striking difference between Figures 1 and 2 is the reversal of the order of classrooms according to whether one considers public oral interactivity (Figure 1) or public mathematical orality (Figure 2). The highly oral classrooms in San Diego made relatively infrequent use of the mathematical terms that constituted the focus of the lesson’s content. By contrast, the less oral classrooms in Shanghai made much more frequent use of key mathematical terms and phrases. Since a single utterance might contain several such terms, and it was terms that were being counted in this analysis, Figure 2 provides a different and possibly more useful picture of the Chinese lessons, where both teacher and student utterances appeared to be longer and more complex than elsewhere.

Comparison between those classrooms that might be described as “Asian” is interesting. Key mathematical terms were spoken less frequently in the Seoul classrooms than was the case in the Shanghai classrooms. Even allowing for the relatively low public oral interactivity of the Korean lessons, the Korean students were given proportionally fewer opportunities to make oral use of key mathematical terms in whole class or teacher-student dialogue. In contrast to the teachers in Shanghai and Tokyo, the teachers in the Hong Kong and Seoul classrooms did not appear to attach the same value to the spoken rehearsal of mathematical terms and phrases, whether in individual or choral mode. It should be noted that Hong Kong 3 used English as the instructional language, while Hong Kong 1 and 2 used Cantonese, so any common features of the Hong Kong classrooms are likely to reflect dominant pedagogical practices, rather than be a specific result of the use of the Chinese or English language. The teacher in Hong Kong 2 appears similar to the three Shanghai teachers in the sense that he conducted his teaching most frequently in the form of whole class discussion. But his lessons show no signs of the pattern, evident in all three Shanghai classrooms, where the students were systematically ‘enculturated’ into the language of school mathematics. In particular, despite similarities between the public oral interactivity of Hong Kong 2 and Shanghai 1 (for example), the frequency of student use of mathematical terms in Hong Kong 2 was much lower.

While the overall level of public oral interactivity in the Tokyo classrooms was similar to those in Seoul, the Japanese classrooms resembled those in Shanghai in the consistently higher frequency of student contribution, but with little use being made of choral response. The value attached to affording student spoken mathematics in some classrooms could suggest adherence by the teacher to a theory of learning that emphasizes the significance of the spoken word in facilitating the internalisation of knowledge. The use of choral response, while consistent with such a belief, could be no more than a classroom management strategy. The Hong Kong classrooms offered students least opportunity to use spoken mathematical terms of all the classrooms studied and student spoken mathematical contribution, whether individual or choral, was extremely low, even though the student component of general public oral interactivity of the Hong Kong classrooms was at least as high as in Shanghai.

THE RELATIVE SIGNIFICANCE OF STUDENT–STUDENT INTERACTIONS

While the private conversations recorded in any one lesson were only those of the Focus Students, it was possible to compare the public oral interactivity of these students with their private oral interactivity and, similarly, their public and private mathematical orality. From the outset, it must be noted that six classrooms stood out because of the virtually complete absence of student-student interaction: those in Shanghai and Seoul. In these six classrooms, student-student conversation can be discounted as an instructional strategy (or as a subversive practice by students). For example, in Seoul classroom 1, there were no instances of student private talk in the first four recorded lessons and only two private utterances from one of the focus students in lesson five. The first utterance was “That’s yours” and the second was “No.” Obviously, neither involved any technical mathematical terms.

In reporting the results that follow, we have put both Shanghai and Seoul to one side. The role played by private student-student interactions in the remaining classrooms is particularly interesting. In Table 1, the figures quoted for both public and private Oral Interactivity and Mathematical Orality are per focus student per lesson and have therefore been averaged over the spoken contributions of around 10 students per classroom. This should minimize the effect of individual student timidity or extroversion, although awareness of being recorded will have been a common characteristic of all focus students (and of their teachers). In reading the ratio columns of Table 1, it is simplest to think of the results as indicating, for example, that focus students in Hong Kong class 1 used a mathematical term on average once every eight public utterances but only once every 48 private utterances.

It seems a reasonable hypothesis that student use of mathematical terms would be less likely in private contexts than in public teacher-orchestrated contexts. For seven of the 11 classes reported in Table 1, this was clearly the case. It is all the more interesting, therefore, that in all three Japanese classrooms and one of the Hong Kong classrooms the focus students were at least as likely to use mathematical terms in private conversation as they were to use them when participating in teacher-orchestrated public discussion. Hong Kong 2 seems anomalous in its very low number of student utterances per lesson, both private and public. With such small utterance numbers, slight variations in count may have the effect of inflating the ratio of private utterances to privately spoken mathematical terms.

Table 1: The use of spoken mathematics by students in public and private contexts

Schools	**Oral Interactivity** (utterances per focus student per lesson)		**Mathematical Orality** (mathl. terms per focus student per lesson)		**Public Ratio** (utts./ term)	**Private Ratio** (utts./ term)
	Public	Private	Public	Private		
Hong Kong 1	4.21	22.59	0.52	0.47	8.10	48.06
Hong Kong 2	2.84	7.15	0.41	1.30	6.93	5.50
Hong Kong 3	2.39	23.80	0	0.83	n.a.	27.67
Tokyo 1	6.13	14.79	0.28	2.24	21.89	6.60
Tokyo 2	2.08	33.85	0.23	9.46	9.04	3.58
Tokyo 3	6.92	11.67	0.61	0.99	11.34	11.79
Melbourne 1	16.16	99.14	2.85	5.59	5.67	17.74
Melbourne 2	14.36	83.75	0.18	0.30	79.78	279.17
Melbourne 3	15.78	73.51	0.17	5.63	92.82	13.06
San Diego 1	12.69	6.64	1.36	0	9.33	n.a.
San Diego 2	9.31	55.33	1.12	3.56	8.31	15.54

The Japanese result remains interesting; suggesting that Japanese students have a fluency in spoken mathematics that persists even across the public/private interface. It is also clear that student-student mathematical exchange was a feature of the Tokyo mathematics classrooms studied to a much greater extent than for the classrooms in Shanghai and Seoul.

CONCLUSIONS

It appears to us that the key constructs Public Oral Interactivity and Public Mathematical Orality distinguished one classroom from another very effectively. Particularly when the two constructs were juxtaposed (by comparing Figures 1 and 2). The contemporary reform agenda in the USA and Australia has placed a priority on student spoken participation in the classroom and this is reflected in the relatively high public oral interactivity of the San Diego and Melbourne classrooms (Figure 1). By contrast, the “Asian” classrooms, such as those in Shanghai, were markedly less oral. However, this difference conceals differences in the frequency of the spoken occurrence of key mathematical terms (Figure 2), from which perspective the Shanghai classrooms can be seen as the most mathematically oral. However, students in the Tokyo classrooms used spoken mathematics in both public and private situations. The relative occurrence of spoken mathematical terms is one level of analysis. We should also distinguish between repetitive oral mimicry and the public (and private) negotiation of meaning (Cobb & Bauersfeld, 1994; Clarke, 2001).

Despite the frequently assumed similarities of practice in classrooms characterised as Asian, differences in the nature of students' public spoken mathematics in classrooms in Seoul, Hong Kong, Shanghai and Tokyo are non-trivial and suggest different instructional theories underlying classroom practice. Any theory of mathematics learning must accommodate, distinguish and explain the learning outcomes of each of these classrooms. Consideration of the non-Asian classrooms is also interesting. With frequent teacher questioning and eliciting of student prior knowledge, the students in the Melbourne classrooms were given many opportunities to recall and orally rehearse the mathematical terms used in prior lessons. In terms of overall public mathematical orality and level of student contribution, Melbourne 1 resembles Shanghai 1 (without the use of choral response). In Melbourne 1, this public orality was clearly augmented by small group discussions, in which students drew upon their mathematical knowledge to complete tasks at hand. Such student-student conversations occurred much more frequently in the Melbourne classrooms. Student use of mathematical terms in situations not directly orchestrated by the teacher can be taken as a reasonable indicator of both the perceived need and the capacity for the purposeful employment of the technical language of mathematics. The relative infrequency of mathematical terms in student-student interactions in Melbourne 2 compared with the other two Melbourne classrooms suggests that these indicators are reflective of teacher influence.

To summarise: Students in the mathematics classrooms in Seoul have few opportunities to speak in class (either privately or publicly) and seldom employ spoken mathematics. Students in the Hong Kong classrooms are publicly and privately vocal, but make very little use of spoken mathematical terms in either context. Students in the mathematics classrooms in Shanghai are guided through the public orchestrated rehearsal of mathematical terms by their teachers, but seldom speak to each other in private during class time. Students in the mathematics classrooms in Tokyo participate orally in both public and private discussion and employ mathematical terms to a significant extent in both. By comparison, the students in Melbourne classroom 1 are highly vocal in both public and private contexts, and make more frequent public use of mathematical terms than any of the three Japanese classrooms, but less frequent use of mathematical terms in their private conversations. These different combinations of oral interactivity and mathematical orality represent at least five distinct pedagogies.

The next question is, of course, whether or not students are advantaged in terms of their mathematical achievement and understanding by classroom practices that afford the opportunity to develop facility with spoken mathematics. The implicit assumption in the classrooms studied in Hong Kong and Seoul seems to be that the employment of spoken mathematics is not to the students' benefit. Classrooms studied in Melbourne, Tokyo and Shanghai, despite differences in implementation, seem to make the opposite assumption. The post-lesson interviews may provide evidence of a connection between classroom mathematical orality and student learning outcomes.

This analysis is currently underway. We suggest that the empirical investigation of mathematical orality (in both public and private domains) and its likely connection to the distribution of the responsibility for knowledge generation are central to the development of any theory of mathematics instruction.

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