URGING CALCULUS STUDENTS TO BE ACTIVE LEARNERS: WHAT WORKS AND WHAT DOESN'T

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We report an on-going design experiment in the context of a compulsory calculus course for engineering students. The purpose of the experiment was to explore the feasibility of incorporating ideas of active learning in the course and evaluate its effects on the students' knowledge and attitudes. Two one-semester long iterations of the experiment involved comparison between the experimental group and two control groups. The data were collected from observations, research diary, course exams, attitude questionnaire and two additional questionnaires designed to explore patterns of students' learning behaviors. The (preliminary) results show that active learning can have a positive effect on the students' grades on condition that the students are urged to invest considerable time in independent study.

Key words: active learning, achievements, attitudes, calculus, design experiment

THEORETICAL BACKGROUND

Research on undergraduate mathematics education convincingly argues that active learning is more beneficial for students than learning in traditional mode (e.g., Artigue, Batanero & Kent, 2007). Following Sfard (1998), we refer here to *active learning* as learning through participation based on engaging in problem solving and collaborative activities, and to *traditional learning* – as learning through acquisition based on listening to a teacher exposing theoretical material or demonstrating problem-solving approaches. We learn from the research literature that active learning can help either in developing positive attitude to mathematics (e.g., Tall & Yusof, 1999) or in improving students' grades in undergraduate calculus, algebra and statistics courses (e.g., Burmeister, Kenney & Nice, 1996).

Teaching in accordance with the principles of active learning is not an easy endeavour. There is a growing body of research that explores pitfalls of active learning, either from academic staff' or students' perspectives. For instance, Pundak & Rozner (2008) reviewed the reasons why academic staff frequently resists innovative teaching and suggest that adopting by the lecturers and TAs active learning paradigm heavily depends on:

> ...(1) teaching staff readiness to seriously learn the theoretical background of active learning, (2) the development of an appropriate local model, customized to the beliefs of academic staff; (3) teacher expertise in information technologies, and (4) the teachers' design of creative solutions to problems that arose during their teaching" (p. 152).

Solow (1995), cited in Roth-McDuffie, McGinnis & Graeber (2000), found that active learning oriented faculty were anxious about resistance and negative reaction from their students who did not want their teachers "to shake their comfortable relationship with math, no matter how distasteful that relationship may be" (p. 226). In summary, existing students' and teachers' beliefs and perceptions about mathematics teaching and learning are pointed out as the major barriers to spreading active learning methods (e.g., Roth-McDuffie, McGinnis & Graeber, 2000).

Are there more barriers? Apparently, yes, and it seems reasonable that some of them are embedded in the current educational system. For instance, the aforementioned study of Yusof & Tall (1999) reported success in implementation of active learning in a problem solving course with a flexible syllabus, in which some topics could apparently be omitted, and the released time could be used for learning in more depth the remaining topics. Such flexibility is rarely allowed. In another aforementioned study reporting success, by Burmeister, Kenney & Nice (1996), the students were provided practically unlimited assistance, and, even more importantly, they were ready to accept it. Again, such a situation is rather a lucky exception from what is observed in many colleges and universities.

We found rather a surprising lack of research that takes into account the apparent tension between what active learners are expected to do and what they can do, given the entire burden of college study. Our on-going study contributes to addressing this lacuna. In this paper, we describe an experiment aimed at incorporating active learning in a compulsory calculus course for engineering students and focus on the following questions:

1. How do engineering students cope, in terms of time and effort, with requirements of calculus course, in which tutorials and assignments are organized to promote active learning?
2. How does the promotion of active learning, under given constraints, affect the students' grades and attitudes towards the subject?

METHOD

The research setting

The experiment is conducted at ORT Braude Engineering College, in the contest of a multi-variable calculus course given for second-semester undergraduate students. The syllabus of the course consists of the following topics: vector-valued functions, differentiation of scalar functions, maxima and minima, double and triple integrals, integrals over paths and surfaces, the integral theorems of vector analysis and applications. The course is compulsory for the students; its syllabus is compulsory for the teachers. The students take the course in continuation of a one variable calculus course. We will refer to the first-semester course as CAL1, and to the second-semester course as CAL2. CAL2 is taught 6 hours a week: four hours of lectures in groups of 40-60 students and two hours of tutorials in groups of 20-30 students.

The study design

The study was initially designed as a one-semester quasi-experiment with a control group (Cook & Campbell, 1979). It then evolved into a design experiment (Cobb, 2000; Cobb et al., 2003) of several one-semester long iterations. This paper is written after the second iteration and before the third one. The purpose of a quasi-experiment was to find out the effect of implementation of active learning ideas, in terms of the course grades. The need in continuation of the study in the form of design experiment emerged from the lack of satisfaction from the results of the first semester and from our thinking how to refine the teaching and to capture various effects of active learning. For these reasons we decided to keep comparing the experimental group (G1) and the control groups (G2 and G3) within every iteration.

Participants

Overall numbers of students (NS) in G1, G2 and G3 groups and the numbers of tutorial classes to which each group was divided (NTC) are given in Table 1. The groups G1 and G2 consisted of all second-semester students of the Department of Software Engineering. At the beginning of every semester, the students were given brief information about two different styles of tutorials, active and traditional. Based on this information, some students chose to join G1, and the rest – G2. Group G3 consisted of all the students of the Department of Electrical and Electronic Engineering. They were not given the choice and were taught in a traditional mode (see *Theoretical Background* section).

	G1		G2		G3	
	NTC	NS	NTC	NS	NTC	NS
Iteration 1	1	25	2	40	3	62
Iteration 2	1	20	2	46	4	94

Table 1: The sample

Groups G1and G2 were taught by Ludmila Shvartsman, one of the authors of this paper, who conducted both lectures and tutorials. Group G3 was taught by a team of lecturers and TAs, including another author of this paper, Buma Abramovitz. All the lecturers and TAs involved in the experiment were of comparable teaching experience and of similar level of teaching achievements. Specifically, their past students, on average, achieved similar grades in the course and gave similar feedback.

The mathematical content of the lectures, as well as the problems and exercises given to the students in the tutorials, were the same in all the groups. All the students had access to the same theoretical materials and examples published at the course website. Also, the students were given the same midterm and final exams. The difference between G1 and the rest of the groups was in the styles of conducting tutorials and in the use of homework assignments, as will be described below.

The research tools

The experiment is described in detail in the research diary written by Ludmila. It includes descriptions of and reflections on all tutorials in G1, a protocol of a lesson in G2 compared with a lesson in G1 based on the same problems, and protocols of more than 10 meetings of the research team. One lesson in G1 was videotaped. The information about teaching in G3 was collected from Buma who taught there and from many meetings and conversations with the other lecturers and TAs of G3. We also developed and run a student questionnaire in all the groups. We call it Tutorial Styles Questionnaire (TSQ). The questions concerned the students' opinions about tutorials and patterns of their participation in the tutorials. The questionnaire was validated in 8 interviews with G1 students at the end of the first iteration.

During the first and the second iterations, G1 students' final grades in CAL2 and CAL1 were compared with grades of G2 and G3 students. The variance in CAL2 final grades was explained using stepwise multiple regression analysis, in which CAL1 grades and the variables indicating to which group a student belonged served as independent variables.

After the first iteration we developed and implemented two additional multiple-choice questionnaires. The first one concerns the students' attitudes to multi-variable calculus and solving problems. It is adapted from Yusof and Tall (1999). We call it Attitudes Questionnaire (ATQ). The second one was developed to estimate effort that students invest, or can invest, in studying the course before and after the lessons. We call it Effort Distribution Questionnaire (EDQ).

RESULTS AND ANALYSIS

Iteration 1

During the first semester active learning in the experimental group was promoted, but not urged. The G1 students were required to read relevant theoretical material and to approach problems, published on the course website, before every tutorial lesson. The solutions were also published. In addition, all the students were invited to get help from Ludmila during her office hours. The tutorials' content and conduct were built on the assumption that the students would come to the lesson being familiar with the basic problems.

During the lessons, the students were given more advanced problems than those published on the web. The students were given some time to think and discuss these problems in small groups, and then their ideas were presented to the whole class. Finally, the solutions emerged from these discussions and presentations. The teacher acted more as a mediator of the discussions than as an authority providing the solutions. The G1 classroom supported such interactive and collaborative activities (see Pundak & Rozner, 2008, for a detailed description of this special classroom).

All G1, G2 and G3 students were given an optional once-a-week Webassign homeworks of 4-5 exercises, the answers to which were to be submitted and checked electronically (see www.webassign.net for details). G1 students in pairs were also offered an opportunity to solve additional, more challenging, homeworks. These homeworks were commented and graded by the teacher every week. The purpose of these additional homeworks was to further promote interactive and cooperative learning. We call the former type of homework *Webassign homeworks*, and the latter one – *Commented homeworks*. Both types of homeworks could be resubmitted for one time to improve the grades.

The components of final course grades are presented in Table 2.

Group	Final exam	Midterm exam	Webassign homeworks	Commented homeworks
G1	70%	20%	5%	5%
G2, G3	70%	20%	10%	

Table 2: The structure of final grades in the first semester

Midterm exams, Webassign homeworks and Commented homeworks were optional, that is, it was up to the students to include or not the homework grades into a final course grade. The final grade of the students who did not take part in midterm exam and/or did not submit homeworks was fully determined by the final exam.

The reality appeared to be more complicated than our expectations. Most of G1 students appreciated the new for them style of the tutorials, but only about half of the group actually followed the requirements (it was evident from TSQ, the diary and the interviews). We observed that some G1 students indeed came prepared for the tutorials, and others did not. Some were engaged in cooperative problem solving, and some remained the consumers of the solutions demonstrated by others. Some students had benefited from the feedback on the homeworks, and others had ignored them.

Ludmila became more satisfied with the conduct of the tutorials and the students' collaboration at the second half of the semester. Generally speaking, the desired style of the tutorials has been finally achieved in G1, and it indeed was different from the traditional style in G2 and G3. This was evident from the comparative analysis of two lesson protocols and TSQ. However, the desired change in out-of-class study was not achieved. In particular, G1 students devoted less time to homework than it was expected: from 30 to 60 min instead of 2 hours a week. G3 students, on average, also invested in the homeworks from 30 to 60 min a week, and G2 students – less than 30 minutes.

Comparative analysis of the final course grades was also not in favour of G1. The mean and SDs were: 63.9 (19.5), 66.0 (22.7) and 76.0 (15.9) for G1, G2 and G3, respectively. A stepwise multiple regression analysis revealed that belonging to G3 was beneficial even after neutralizing the fact that, on average, CAL1 grades in G3 were higher than in G1 and G2 (72.15 (11.98) in G3, 70.32 (12) in G1, and 69.72 (12.34) in G2). Let us remind that G1 and G2 were taught by the same teacher, and G3 was taught by other teachers.

At the end of the semester, we summarized the findings and designed the second iteration. We decided:

- To urge students to work more out of the class by changing the structure of the course final grade.
- To control more aspects of the experiment. In particular, we decided to measure the students' attitudes towards the subjects (see the Research Tools section).
- To check feasibility of the requirements to learn actively by taking into consideration the students' overall burden of study.

Iteration 2

The second iteration was started six month after finishing the first one. The in-between time was used for validating TSQ, developing EDQ, piloting new elements of teaching and refining the evaluation tools.

First, challenging preparatory problems were published on the web without solutions. These problems were discussed at the beginning of each tutorial during 10-15 min. The rest of the lesson was conducted as in the first iteration.

Second, Webassign homeworks that included technical exercises were cancelled for all the students. The Commented homeworks became compulsory for G1 students, and remained optional for G2 and G3 students.

Third, a new compulsory test was offered in addition to an optional midterm exam and a compulsory final exam. This test was composed from two out of about 150 preparatory problems and the problems that appeared in the Commented homeworks; we call it *Homework test*. All the students were aware of its structure and the source from which the tasks were to be chosen. The components of a final grade of the course are presented in Table 3.

Group	Final exam	Midterm exam	Homework test	Commented homeworks
G1	65%	20%	10%	5%
G2, G3	65%	20%	15%	

Table 3: The structure of final grades in the second semester

For those students, who decided not to take the midterm exam, the weight of the final exam was 85%.

These changes worked as follows. At the beginning of the semester, about three quarters of G1 students were ready for the tutorials and actively participated in the discussions. Less than half of the students remained active learners in the middle of the semester. They explained that they merely did not have enough time to properly prepare themselves for the tutorials, so we decided to try something else. Ludmila started asking different pairs of students to take a lead during the lesson. Naturally, the leading students had to invest more time in preparations. This made the lessons more interesting and, in a way, showed the rest of the class that they can do the same.

As in the first iteration, TSQ results enlightened the difference between tutorial styles in G1 and the other two groups, however, the levels of satisfaction of G1, G2 and G3 students from the tutorials were about the same. The attitudes towards the subject, in terms of ATQ, were also not different in all the groups.

EDQ data showed that G1 students devoted more time to out-of-class study than G2 and G3 students (on average, 6.24 (2.43) hours in G1 vs. 4.98 (1.75) hours in G2 and G3 a week, $t=1.97$, $df=41$, $p<0.05$); about 60% of the time was devoted to doing the homework in G1, and 47% - in G2 and G3. Note that, according to our estimation, an average student needs about 8 hours a week to fully cope with the requirements. EDQ also showed that G1 students studied systematically during the semester, whereas G2 and G3 students increased the time of independent study towards the end of the semester.

In addition, the students were asked in EDQ: "Given the general load of your study and time constraints that you have, which minimal grade in CAL2 course would you accept as satisfying?" and then "How much additional time are you ready to invest per week in study in order to obtain a 10% higher grade than that you have indicated in the previous question?" Surprisingly, the responses of G1, G2 and G3 students to these questions were very close. We interpret this finding as follows. First, learning motivation of G1 students was not significantly higher than that of G2 and G3 students. Second, the expectation that an average student should invest about 8 hours a week in out-of-class study was not beyond of what the students said they could do (on average, the students of all the groups were ready to invest 4 additional hours).

This time G1 students did better than their peers in terms of the course final grades. The mean and SDs were: 71.5 (16.3), 52.4 (26.6) and 65.2 (26.7) for G1, G2 and G3, respectively. A significant regression equation showed that belonging to G1 was beneficial in comparison with belonging to either G2 or G3, even after neutralizing the differences in CAL1 grades (71.6 (12.7) in G1, 64.4 (9.2) in G2, and 73.8 (11.4) in G3).

Thus, we can report success, in terms of course grades, of an experimental style of conducting tutorials. However, the students' attitudes to the subject did not change and remained relatively low. It should also be noted that our expectations about the students learning behaviors were only partially fulfilled. Specifically, we succeeded more in urging the students to do their after-the-lesson homeworks than in convincing them to solve recommended problems before the tutorials.

We are going to deal with these issues in the future iteration(s). In particular, we consider publishing more problems on the course website before the lesson, and asking students to choose which problems they are interested to discuss during the lesson. We hope that the students will take more responsibility for their learning outcomes (cf. Brousseau, 1997). This may encourage them to invest more time in preparation for the tutorials and have more influence on the content of the course. In turn, this may affect their attitudes to the subject.

DISCUSSION AND CONCLUSIONS

The main lesson that we have learned from the first two iterations of the experiment can be put in words of Latterell (2008): "Students do what is expedient, and not necessarily what professors think they should" (p. 12). So, for us, the crucial issue was how to make active learning of calculus expedient for the students. The first iteration of the experiment showed that conducting tutorials in interactive and cooperative mode is not sufficient in order to obtain traceable improvements in the students' achievements and attitudes. It has become evident that fulfillment of our expectations requires changes also in the students' learning behaviors out of class, and that these requirements should be supported by appropriate modification of the structure of a course grade. This idea was realized during the second iteration and appeared feasible, in terms of time and effort, for the students. The second iteration resulted in significant advantage of the experimental group in comparison with two control groups. Is the observed effect due to incorporated innovations? We believe that it is, for the following reasons:

- The experimental group did better not only in comparison with G2 control group, taught by the same teacher, but also in comparison with G3 control group taught by the others. The teachers were aware of competitive nature of the experiment. They all were of comparable experience and past achievements in teaching, so it is unlikely that the observed advantage of the experimental group can be just attributed to the differences in the teachers' professionalism or enthusiasm.
- The mathematical content of the course was exactly the same in all three groups.
- We admit that random assignment of students to the experimental and control groups would be preferable. Even though it could not be realized under the conditions embedded in practice of college education, the achieved effect cannot be attributed just to the differences in students' learning motivation or mathematical background. This claim is supported by EDQ data and by the regression analysis. Note that our way of dealing with the issue of non-random assignment is in line with what is done in some other studies (cf. Schwingendorf, McCabe & Kuhn, 2000).

We are aware, of course, that the reported effect may be due to some combination of the aforementioned factors or to some uncontrolled in our experiment ones. This adds us motivation to keep running the experiment. Currently, we see the process of educating undergraduate students to learn actively as a multi-stage enterprise, in which many factors are involved. Some of them, for instance, beliefs of students and teachers, are extensively explored (Pundak & Rozner, 2008; Roth-McDuffie, McGinnis & Graeber, 2000). Others only recently deserved attention of the mathematics education research community.

The distinction that Harel (2008) made between intellectual and psychological needs involved in learning mathematics is particularly relevant to discussion of our findings. The intellectual needs, such as the need to construct new knowledge in response to a perturbing problem that otherwise cannot be solved, are in the focus of contemporary mathematics education research. Psychological needs, such as the need to be competent and secure in relationships with others, frequently remain peripheral. However, the latter needs are crucially important in our and our students' real lives and must be taken in consideration when one requires his or her students to be active learners, and thus, to put more time and effort in study. As a matter of fact, one difference between the first and the second iteration of our experiment can be explained in these terms: the first iteration was focused on intellectual needs of the students, whereas the second one was organized so that the students could be more successful when conforming to the requirements of active learning. In a way, this distinction calls for balance between active and traditional learning modes, as suggested by some theorists (e.g., Sfard, 1998) and practitioners (e.g., Tucker, 1999) since the active learning mode relies mostly on the students' intellectual needs, and the traditional mode – on their psychological needs.

The last comment is about content dependency of the presented findings. Because of our intention to outline a long study in a brief paper, examples of calculus problems from the tutorials and examples from the questionnaires are not included. It may create an impression that the reported findings are not exclusive for the chosen mathematical context. Perhaps, they are not indeed. We hope to discuss this topic in the oral presentation and in the future publications.

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