PROBLEM POSING BY NOVICE AND EXPERTS: COMPARISON BETWEEN STUDENTS AND TEACHERS

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Lately, problem posing gained terrain in mathematical education research due to its connection with mathematical understanding and thinking. Still, comparisons between novice' and experts' problem posing are still scarce. In this paper we compare students' and teachers' generated problems on three aspects: variety of problem types and of tasks, and quality of questions. We found that teachers use their pedagogical knowledge to constrain problem types and tasks, and that teachers' classroom experience shapes their view on difficulty. In conclusion, teachers are always guided by the audience they have in their mind in contrast with what can be observed at students.

INTRODUCTION

Research on problem posing can be structured along several lines. First, there is a research trend on relating problem posing to instruction: by which means a problem posing approach can be beneficial in the classroom. Studies that can be subscribed to this category look at the relation between problem posing and problem solving (in case of pre-service teachers – Crespo, 2003; in-service teachers – Chang, 2007; both – Silver et al., 1996; students – Imaoka, 2001), international comparisons (Cai, 1997) or problem posing and mathematical understanding, modelling and open ended problems (Lin, 2004; Pirie, 2002). Another line of research focuses on enhancing problem posing skills: in traditional (Yevdokimov, 2005) or by development of computational settings (de Corte *et al.*, 2002). There are also a series of studies that relate problem posing to individual attitudes towards mathematics and affect (Akay & Boz, 2008). A fourth line of research connects problem posing to creativity and evaluates the posing process and results from creativity point of view (Silver, 1997). However, comparisons between novices (from some particular point of view) and experts are scarce and there is no commonly agreed framework that would allow this.

One explanation to such a situation is the fact that mathematical problems need a rich characterization of them. However, such an inquiry leads to questions like: when a situation turns into a problem, what makes it to belong to a particular topic, which of the problems elements (like given, asked for) should be considered and which meta-characteristics are important (like solvability, cognitive resources involved in

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solution, etc.). In conclusion, researchers need to take into account the particular topic, beside general aspects, in order to define their evaluation criteria.

In the present paper we intent to contribute on this line by proposing a framework for the evaluation of problems and apply it to compare problems posed by university students (pre-service teachers, considered as novice from the point of view of classroom teaching) and in-service teachers (considered as experts). The categorization into novice and expert is done on terms of pedagogical, mathematical knowledge and classroom teaching experience.

METHODOLOGY

In the present study, 88 persons from Romania (25 first year or second year mathematics students, 41 middle school teachers, and 22 high school teachers) completed a problem posing task. Students were of 18-20 years old and entered to university after completing an admission exam. Teachers had more that 5 years teaching experience. Participants were selected randomly, without any reference to their professional or scientific performance. None of them has been subject of training in problem posing, however it is possible that some of them would have experience in Olympiads as students or teachers.

The participants had to generate three sequence problems (as home assignment task) such that to have an easy, one of average difficulty and a difficult problem. They had a week at their disposal to finish; at the end, they responded a questionnaire regarding their problem posing process. It was requested to hand in not only the final formulations, but also the scratch work. The questions were about the following aspects of the problem posing process: the existence of an initial idea (for each problem of different difficulty), change of the idea during generation, problem types from which to start the generation process, a theorem or generalization as from where to trigger the problem posing process and difficulty criteria they used.

ANALYSIS OF THE POSED PROBLEMS

It has to be mentioned, before the presentation of results, that we found two situations along with the expected one: first, not all participants posed problems for each difficulty level and, second, some of them, posed more than one problem for a specific difficulty level. The problems were analyzed from three perspectives: variety of problem types and of questions, and problem formulation .

Problem - type analysis

The problem typology for sequences was taken from Pelczer and Gamboa (2006). Theoretical problems are the ones in which there is no quantitatively specified sequence, but rather a generic sequence is specified as the mathematical object under inquiry. The term "contextual" was employed as in Borasi (1986), meaning the situation into which the problem is embedded. The rest of categories refer to the way in which the general term is specified. Table 1 contains the results concerning problem types, in percent (E – easy, A – average, D - difficult). The total number of problems appears in the last line of this table.

Table 1. Statistical results on problem types. For each problem type we specify, in parenthesis, as a triplet the number of problems posed by students, secondary and high school teachers.

Problem types	Students			Secondary			High school		
	E	A	D	E	A	D	E	A	D
Theoretical	-	-	-	-	-	-	-	-	-
Contextual (8,-,-)	12	12	10	-	-	-	-	-	-
Explicit (13,42,40)	28	4	5	41	38	27	73	67	43
Implicit (15,6,1)	12	36	14	5	2	8	4	-	-
Linear Recurrence (27,4,5)	44	36	33	-	5	5	-	16	10
Non-linear Recurrence (8,3,3)	4	12	18	-	2	5	-	-	14
Enumeration (2,37,5)	-	-	10	40	30	25	19	4	-
Sum, Product (2,26,11)	-	-	10	14	23	30	4	13	33
Total nr. of problems	**25**	**25**	**21**	**41**	**40**	**36**	**22**	**22**	**21**

We can observe from table 1 that at students recurrence problems dominate; at high school teachers prevails the problem in which the general term is expressed explicitly by a formula and at secondary teachers the “enumeration” type (sequence specified by the enumeration of few initial terms) is the most frequent. The dominance of enumeration type at secondary teachers can be explained by the curricula: the accent is on identifying and formalizing the sequence’s patterns and moving between different representations of the sequences (geometric, analytic, formal and recurrence).

The observation holds for high school teachers, too, with the remark that in their case there is an increase also in non-linear recurrence problems. In case of high school teachers, the dominance is one of the explicit problems – situation which, again, can be explained by the curricula. High school teachers concentrate on clarifying basic calculus concepts, like limit, convergence, monotony and for all these explicit problems are proper. As the difficulty of the problem has to increase, they move towards the types “sum” and “non-linear recurrence”. These problems, when analyzed, showed that teachers still focused on theorems and criteria present in textbooks (just as in case of easy problems with explicit general term), but asking for skillful application of them. By “skillful application” we mean that no advanced techniques are needed, but rather good knowledge of algebra (identities, inequalities) or typical examples and sequences (like in case of applying the majoring criteria).

This later is the main aspect that differentiate students' and teachers' problems. As it can be seen in the above table, students prefer implicit or recurrent definitions of the sequences. It is also interesting that many students pose "contextual problems", that is problems in which sequences appear as a collateral issue: the main focus is on another mathematical object so that the problem can't be seen as strictly relating to introductory analysis.

These results suggest that students see problem posing as a self-referenced activity focused on problems and with no specific audience. Problem difficulty is judged based on the ability to solve the problem and use of techniques, meanwhile teachers build their problems with their students in their mind. When speaking about the problem posing process they mention that the addressee is their classroom and difficulty is judged based on curricular indications and classroom experience. The case of the (posed) difficult problems is interesting: where students ask for specific transformations (usually beyond the textbook's reach) or use non-familiar contexts, teachers concentrate on situations about which they know that the application of the usual theorems can be problematic. Therefore, they prefer problem types (like non-linear recurrence or explicit) that can be solved with text-book theorems and the difficulty relies in identifying the instances that satisfy the conditions of application. In these terms, teachers problem posing can be seen as a constraint based process, where constraints arise from their classroom experience.

Questions' analysis

Some interesting conclusions about the posing process were reached by the analysis of the task specified by the problem, that is, by the analysis of the problems' questions. We defined four principal categories. In the first category we included questions related to the *verification of the concepts*, that is the question refers to the statement of some definitions or theoretical results, recognition of some property, construction of examples or counter-examples. In the second category are the *demonstration* tasks, those that ask for justification (through mathematical reasoning) of some facts of algebraic or analytic nature. In these cases, the problem statement is imperative and the facts to be demonstrated are explicitly stated. A third category contains *exploration* tasks. These can ask for the verification, study or observation of a property, identification of a sequence's pattern given by some terms and/or generation of following terms, discussions of the results on the value of parameters or different representations of a mathematical object. The questions from this category are characterized by doubt, meaning that *a priori* one can obtain several answers. The last category of questions – of *computations* – include tasks that ask for the application of some formula (in case when the expression of the general term is given), computation of the general term, of a limit, sum, or the determination of a parameter's value such to have some conditions satisfied.

In table 2 the statistical results are shown (in percentage for the questions types), for the four category of questions (tasks) and the three category of participants. The total number of problems and questions appears at the end of the table and a ratio of question/problem is computed.

Table 2. Statistical data on questions

	Students			Secondary			High school		
	E	A	D	E	A	D	E	A	D
Verification	-	3	3	-	-	-	-	-	-
Proof	29	26	23	22	22	27	3	11	24
Exploration	10	20	23	35	26	20	48	22	30
Computation	61	51	52	43	53	53	48	67	47
#Questions	31	35	31	63	58	59	29	27	34
#Problems	24	25	21	42	41	37	20	20	22
Ratio	1.29	1.4	1.48	1.5	1.41	1.6	1.45	1.35	1.55

The data from table 2 leads to some interesting conclusions. A first one is that none of the participant categories seems to be interested in problems that aim the verification of concept understanding. There are only two problems asking for construction of examples, but these are in a special context in which very complex properties are required. A possible explanation of such situation can be the fact that these types of questions are not very common in textbooks, evaluation exams, although probably they are quite common in everyday class activities. Still, teachers and students do not seem to give them importance as stand-alone problems.

A second, surprising, conclusion is that high school teachers seem to be less interested in demonstrations and exploration in favour of computation, when compared with the other two participant category. More, high school teachers, tend to put problems of demonstration type more as difficult ones (24% in difficult against 3% of easy problems). In the meantime, the distribution of demonstration type questions is more equilibrated in case of students and secondary school teachers. Such results can be related to the tendency toward an algorithmic training, as preparation for end school exams, observed in the Romanian education lately (Pelczer *et al.*, 2008).

We also identified a certain disposition of teachers (independently of the school level that teach) for questions that refer to passing sequences from one representation into another, aspects lacking from students' problems. This suggest that teachers know and pay attention to the importance of multiple representations of a concept; passing a sequence between different representational forms has a high pedagogical value. It is interesting that teachers consider exploration as proper, mostly, for easy problems.

As far the ratio between questions and problems is concerned, we see a small tendency of teachers to pose more questions than students. The tendency is even more visible when we count all the questions (even those that are of the same type). Such situation is explained by the fact that teachers generate problems with an audience in their mind (their own class), an audience that is made up of problem solvers; therefore, their tendency for multiple questions reflects their way of acting in the class. We even found problems with more than 5 questions for it. In conclusion, we see that teachers create, through the posed problem, a context for learning in which, on the same problem statement multiple skills can be practiced.

Problem formulation

The first aspect refers to the adequacy of the question with the context of the problem and the difficulty level. In any context there are several questions that can be asked; the context with the question gives a particular instance. By considering that we are interested in classroom problem posing, we study these instances from the point of view of their pedagogical value (Baker, 1991). This attribution is subjective, based on the experience of the authors of the present article. Adequacy with the difficulty level refers to the correspondence between the attributed difficulty and the elements of the problem. In particular, it means to analyze the selection of the question (from a possible set of questions that can be formulated in that context) and whether there were better alternatives. Then, problems are analyzed from the point of view of well-formulatedness: are all the elements necessary for solution mentioned in the problem? The last aspect refers to the solvability of the problem: can the problem be solved under the given specifications?

As pedagogical value of the problems is concerned it can be told that there are some common goals between the three categories of participants, for example, the verification/ application of concepts of monotony, boundedness or convergence. However, there are two interesting results. First, no student posed a problem that would require the identification of the sequence's pattern nor asked for exploration of different situations. Second, students tend to pose problems (especially, when it comes to difficult ones) that require the application of algorithms or techniques that are not in the textbook. This tendency is explained by their vision of difficult problem: one that is out of their own (or most students) reach. However, it is important to underline that such a perception goes beyond of difficulty appreciation; it reflects, partially, their view of a well-prepared student: one that has an extensive knowledge of algorithms and techniques.

It has to be remarked that neither teachers pose problems that aim to check whether there is a deep understanding of the concepts involved with sequences. Above, we already described a possible explanation for this situation. Still, teachers tend to ask for exploration and their problems can be solved just by methods shown in the textbook. This aspect turns us back to the difficulty issue: students make more difficult problems by involving techniques that are beyond the textbook or by transforming the context of the problems, meanwhile teachers involve algebraic knowledge in the expression of the problem such to remain strictly related to the topic. With regard to difficulty, students also have problems in finding the proper question in a context – the question that would turn a problem in a difficult one. Teachers' problems are more typical, the questions that could be asked in a specific situation (and the mathematical object on which focuses the question) are the standard ones, so they choose from a more restricted set of questions and are more familiar with the setting. Students, meanwhile, often create richer settings, but do not necessarily know how to choose a good question.

In other situations, students do not formulate properly the question. We give two examples from students.

Example 1. Let $(a_n)_n$ be a sequence given by $a_1 = 1$, $a_2 = 1$, and $a_{n+1} = \sin(a_n) + \cos(a_{n-1})$. Study if this sequence has a finite limit.

Example 2. Let $(a_n)_n$ be the sequence defined by $a_1 = 12$, $a_2 = 288$, and $a_{n+1} = 24a_n - 144a_{n-1}, n \geq 2$. Calculate $b_n = \sum_{k=1}^{n} a_k$ and examine the monotony and the convergence of the sequence $(b_n)_n$.

In the first example (Example 1, given as difficult problem), the student's question (the "finite" word) suggest that he had not paid enough attention to the expression of the general term: the limit, if it exists, obviously it can't be infinite. In the second example (given also as difficult problem), the second question refers to the monotony and convergence of a sequence defined from the previous one. Once the general term a_n is determined, it is "obvious" the monotony and the divergence of the second sequence (its general terms is positive and major to 1).

Our main conclusion to this first part of the analysis is that teachers' problems are typical ones that require only textbook material for solving and have specific pedagogical goals; their approach is shaped by their classroom and teaching experience: they pose problems having a specific audience in their mind (their own classroom) and think of curriculum as the main guide for the type of knowledge that must be used.

By well-formulated problem we mean a problem in which all the elements necessary for solution are given and there is no contradiction between the given elements. Textbooks, problem books always contain well-formulated problems, a situation which at its turn can lead to the case that students don't know what it is and how they could check a problem from the point of view of formulation. Exactly this situation make well-formulatedness an important factor in the evaluation of the problem posing results.

Solvability, another characteristic, refers to the possibility of finding a solution for the problem with a certain set of knowledge. As in the case of well-formulatedness, students experience in classroom is limited to solvable problems, which gives them a bias when it comes to evaluate the posed problem: often this aspect will not be considered. However, it is true that students frequently do not know to decide whether a problem is not solvable or is just that they can't solve it. Still, in the problem posing context it is natural to expect to pose problems that are solvable, even if not by the author of the problem. It also needs to be underlined that well-formulatedness affects the solvability of the problem, therefore there will be always less solvable problems than well-formulated ones.

In the analysis we carried out there were no cases of ill-formulated or non-solvable problems at teachers. However, at students this appears in few cases. Ill-formulated problems can be grouped as problems that have not enough elements in their statement (like "under formulated") and ones that have contradictory information in their statement (in some cases, over-formulated). We consider two relevant examples.

Example 3. Consider the following recurrence formula: $a_{n+1} = 2a_n - a_{n-1}$. Calculate the general term a_n.

Example 4. If $(a_n)_n$ a sequence such that $\frac{a_n}{a_{n-1}} > 1$ and $\frac{a_n}{a_{n+1}} > 1$, decide if it is convergent.

In example 3 we illustrate the case of under-specification: without specifying the first terms, the general term can't be computed. Example 4 shows a case of contradictory information, that makes that the problem has no sense under the current specification.

Why do teachers create well-defined and solvable problems? We argue that these problems can serve to reach the pedagogical goals they envision, and that they have the mathematical knowledge and teaching experience that allow them to verify their posed problems (or, from the beginning, to restrict themselves to problems that are "worthy" to be done). Whether teacher's choice for well-defined problems is result of the use of textbooks and exams practices or, rather, it is a conscious decision remains a question on which we shall not delve in this paper. On the other hand, students often are not aware of this aspect or are not considering it when reviewing their own problems – a fact that can be (partly) explained by the fact that since they had no particular receiver in their mind during the generation they didn't "looked" at the problem form the solvers' point of view.

As overall conclusion, we can say that differences between teachers' and students' generated problems can be identified at every level (problem types; questions types; meta-characteristics of the problems – well-formulatedness, solvability and adequacy) and the differences can be explained by teacher's classroom and pedagogical experience, on one hand, and mathematical knowledge, on other hand.

CONCLUSIONS

The analysis of the posed problems leads to the conclusion that there is a specific trait for each participant group. This can be underlined by different ways.

In the first place, teachers (secondary and high school) seem to be strongly influenced in the choosing of the problem type and question formulation by the curriculum and the subject usually given at final exams (mostly national scale examinations). High school teachers seem to concentrate on the development of computing abilities, meanwhile secondary teachers pay equal attention to demonstrations, exploration and calculations. Students seem to be interested in extra-curricular contexts and solution techniques. We explain this situation by the fact that teachers have permanently an audience in their mind at the moment of generation and they employ their pedagogical and mathematical knowledge such to adapt the problems to an envisioned concrete classroom situation (known from their classroom experience).

The explanation is congruent with the next conclusion, too. Teachers seem to be guided by diverse pedagogical goals and take into consideration their class when adapting the difficulty level. On contrary, students see problem posing as a self-referenced activity focused on the problems with no specific audience. There are two further arguments in this line. On one hand, a teacher starts, in general, from a specific idea of problem generation and formulates (in average) more tasks (or questions). On other hand, teachers pay much more attention to the formulation of the problem, in comparison with students: many of students' generated problems have an unclear statement or the proposed solutions are erroneous which very rarely occurs at teachers.

The analysis we carried out has several benefits. First, sheds light on what students and teachers do perceive as important in teaching, evaluating and knowing about sequences. Second, the analyses proves interesting for pre-service teacher education. Some time after beginning their careers as teachers, these students will start to choose or pose the problems with a focus on their audience, but maybe it would be beneficial to explicitly train them, before getting into the classroom, to think on meta-characteristics of the problems and to identify and use techniques that help building them. We consider that our conclusions are in favour of using a problem posing approach or training in pre-service teacher education.

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