MODELING WITH TECHNOLOGY IN ELEMENTARY CLASSROOMS

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In this study we report on an analysis of the mathematical developments of twenty two 11 year old students as they worked on a complex environmental modeling problem. The activity required students to analyse a real-world situation based on the water shortage problem in Cyprus using Google Earth and spreadsheet software, to pose and test conjectures, to compare alternatives, and to construct models that are generalizable and re-usable. Results provide evidence that students successfully used the available tools in constructing models for solving the environmental problem. Students' mathematical developments included creating models for selecting the best place to supply Cyprus with water, finding and relating variant and invariant measures such as tanker capacity, oil consumption, and water price. Finally, implications for further research are discussed.

Keywords: Modeling, technological tools, environmental modeling problem.

INTRODUCTION

The importance of modeling and applications has been well documented and a significant number of researchers discussed the impact of modeling in the teaching and learning of mathematics (Pollak, 1970; Blum & Niss, 1991; Lesh & Doerr, 2003). Additionally, professional organizations, like the National Council of Teachers Mathematics (NCTM, 2000), recommended that the inclusion of real world based problems in the curriculum can capture students' interest and students will gain mathematical problem solving skills, as well as an appreciation of the power of mathematics and some essential mathematical concepts and skills (NCTM, 2000).

Students, even at the elementary school level, need to be able to successfully work with complex systems that daily appear to the mass media (English, 2006). More than ever before, the nature of the mathematical problem-solving experiences has to be changed, if we want to prepare students to adequately deal with the complexity of the rapidly changing world (English, 2006; Lesh & Zawojewski, 2007). Traditional forms of problem solving constrain opportunities for students to explore complex, messy, real-world data and to generate their own constructs and processes for solving authentic problems (Kaiser & Sriraman, 2006). In contrast, mathematical modeling provides rich opportunities for students to experience complex data within challenging, yet meaningful contexts. Students' interactions within these experiences can assist them in building mathematical understandings and in developing their problem solving skills (Mousoulides & English, 2008).

In this attempt, given the potential value of technology for enhancing learning, it is imperative that students undertake realistic modeling problems and appropriately use technological tools for developing their ideas about and their understandings of related mathematical concepts (Mousoulides, Sriraman, & Lesh, 2008; Mousoulides, 2007). Although the increased interest on modeling and applications, even at the elementary school level, only a limited number of researchers focused their agendas on investigating the role of technology in mathematical modeling, on exploring how spreadsheets are used in constructing models (Blomhøj, 1993; Mousoulides et al., 2008), and on identifying how dynamic geometry software features might influence the modeling process (Christou et al., 2005).

This paper reports on the mathematical developments of one class of eleven year old students, as they worked on an environmental modeling problem that involved interpreting a real world situation and dealing with digital maps, tracing ship routes, working with tables of data, exploring relationships among data, and representing findings in visual and written forms. We were particularly interested in exploring the ways in which the students used the available tools (Google Earth and spreadsheets) in constructing the necessary mathematical developments for solving the problem.

MATHEMATICAL MODELING AND TECHNOLOGY IN THE ELEMENTARY SCHOOL

Mathematical models and modeling have been defined variously in the literature (e.g., Greer, 1997; Lesh & Doerr, 2003). In this paper, models are defined as "systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behavior of some other familiar system" (Doerr & English, 2003, p.112). A definition of modeling, as a problem solving approach, is presented in Lesh and Zawojewski (2007): "A task, or goal-directed activity, becomes a problem (or problematic) when the "problem solver" (which may be a collaborating group of specialists) needs to develop a more productive way of thinking about the given situation" (p. 782).

Research studies have shown that mathematical modeling can be considered as an effective medium to improve students' problem solving abilities in working with unfamiliar complex real world situations by thinking flexibly and creatively (Haines, Galbraith, Blum, & Khan, 2007; English, 2006). One approach to having students solve complex problems is through team oriented activities, called model eliciting activities (MEAs). These activities are based upon the models and modeling perspective (Lesh & Doerr, 2003), and they are designed to document students' thinking. MEAs, therefore, provide an ideal setting to assess the knowledge and the abilities that students express during the modeling process (Lesh & Doerr, 2003). MEAs usually consist of three sessions. The first session provides the problem statement and introduces students to the modeling activity. Students define for themselves the problem, assess the problem situation and create a plan of action to successfully solve the problem. During the problem solving session of the modeling problem students work in small groups and go through multiple iterations of testing and revising their solution(s) to ensure that their solution(s) is the best possible for the problem situation. In the third session of the modeling activity each group of students present their solution(s) to the rest of the class for constructive feedback and discussion of the mathematical ideas presented in the modeling activity (Mousoulides, 2007; Lesh & Doerr, 2003).

Modeling activities, set within authentic contexts, engage students in mathematical thinking that extends beyond the traditional curriculum, as they embed the important mathematical processes within the problem context and students elicit these as they work the problem (English, 2006). Problems presented in modeling activities are not carefully mathematized for the students, and therefore students have to unmask the mathematics by mapping the problem information in such a way as to produce an answer using familiar quantities and basic operations (English, 2006). The problems necessitate the use of important, yet underrepresented in traditional mathematical curriculum, mathematical processes such as constructing, describing, explaining, predicting, and representing, together with quantifying, coordinating, and organizing data (Mousoulides, 2007). Key mathematical ideas that appear in the modeling problems can be accessed at different levels of sophistication and therefore all students through questions, revisions and communication can have access to the important modeling and mathematical content. This can result in improving competencies in using mathematics to solve problems beyond the classroom (English, 2006; Kaiser & Sriraman, 2006; Mousoulides et al., 2008).

Recent research studies focusing on mathematical modeling at the elementary school level indicated that students can build on their existing knowledge and develop their mathematical ideas and modeling competencies that they would not meet in the traditional school curriculum (English, 2006; Mousoulides & English, 2008). Students' informal knowledge and ideas assist students in understanding the problem presented in the modeling activity, in identifying variables and constrains, and in building mathematical models for solving the modeling problem (Mousoulides, 2007). The framework of modeling activities does not narrow students' work in only performing calculations or working with ready made models; on the contrary, students need to construct models in a meaningful way for solving a real problem and this approach can lead to conceptual understanding and mathematization (Greer, 1997; Mousoulides et al., 2008; Mousoulides & English, 2008). Conceptual understanding was also reported as students worked in modeling activities in exploring quantitative relationships and in comparing varying rates of change (Doerr & English, 2003), in probabilistic reasoning (English, 2006), and in geometric reasoning and spatial abilities (Mousoulides et al., 2006).

The availability of technological tools is one factor that might influence students' work and outcomes in working with modeling activities (Mousoulides et al., 2006). Recent research studies indicate that appropriate use of technological tools can enhance students' work and therefore result in better models and solutions. In Blomhøj's (1993) research, students successfully used a specially designed spreadsheet for setting models and for expressing relations between variables in spreadsheet notation. More recently, Mousoulides (2007) reported that school and undergraduate students successfully used spreadsheets in developing simple and more complex models for connecting the real world problem with the mathematical world. The contribution of technological tools in modeling problems was also examined in the areas of geometry and spatial geometry. Christou and colleagues (2005) reported that students, using a dynamic geometry package, modelled and mathematized a real world problem, and utilized the dragging features of the software for verifying and documenting their results. In line with previous findings, Mousoulides and colleagues (2007) reported that students' work with a spatial geometry software broadened students' explorations and visualization skills through the process of constructing visual images and these explorations assisted students in reaching models and solutions that they could not probably do without using the software. As a concluding point, it is important to underline that the inclusion of appropriate software in modeling activities can provide a pathway in better understanding how students approach a real world problem and how they might develop technology-based solutions for these problems.

THE PRESENT STUDY

Participants and Procedures

One class of 22 eleven year olds and their teacher worked on an environmental modeling problem as part of a longitudinal study, which focuses on exploring students' development of models and processes in working with modeling problems. The students are from a public K-6 elementary school in the urban area of a major city in Cyprus. The students only met such modeling problems before during their participation in the current project, as the mathematics curriculum in Cyprus rarely includes any modelling activities. Students were quite familiar in working in groups for solving more complex problems than those appear in their mathematics textbooks. However, this was the first time students had the opportunity to work with spreadsheets and Google Earth for solving a real world modeling problem.

The data reported here are drawn from the problem activities the students completed during the first year of the project. The *Water Shortage* modeling problem (appears in the appendix) entails: (a) a warm-up task comprising a mathematically rich "newspaper article" designed to familiarize the students with the context of the modeling activity, (b) "readiness" questions to be answered about the article, and (c) the problem to be solved, including the tables of data (see Table 1). This environmental modeling problem presented in the activity asked from students to help the local authorities in finding the best country for supplying Cyprus with water. Water shortage is one of the biggest problems Cyprus face these days. As a result, students were very familiar with the problem, since almost everyday there are discussions on TV about the possible solutions to the problem.

Table 1: The Water Shortage Problem Data

Country	Water Supply per week (metric tons)	Water Price (metric ton)	Tanker Capacity	Oil cost per 100 km	Port Facilities for Tankers
Egypt	3 000 000	€ 3.50	30 000	€ 20 000	Good
Greece	4 000 000	€ 2.00	50 000	€ 25 000	Very Good
Lebanon	2 000 000	€ 4.00	50 000	€ 25 000	Average

The problem was implemented by the authors and the classroom teacher. Working in groups of three to four, the children spent five 40-minute sessions on the activity. During the first two sessions the children worked on the newspaper article and the readiness questions and familiarize themselves with the Google Earth and spreadsheet software. Introduction to Google Earth focused on the following commands: “Fly to” for visiting a place, “Add Placemark” and “Ruler” for calculating the distance between two points, and “Path” for drawing a path between two points. In contrast to regular maps, Google Earth can help students in making accurate calculations, being more precise in drawing the tanker routes, in “visiting” the different countries for exploring their major ports and finally in observing country’s landscape. In the next three sessions the children developed their models, wrote letters to local authorities, explaining and documenting their models/solutions, and presented their work to the class for questioning and constructive feedback. A class discussion followed that focused on the key mathematical ideas and relationships students had generated.

Data Sources and Analysis

The data sources were collected through audio- and video-tapes of the students’ responses to the modeling activity, together with the Google Earth and spreadsheet files, student worksheets and researchers’ field notes. Data were analysed using interpretative techniques (Miles & Huberman, 1994) to identify developments in the model creations with respect to the ways in which the students: (a) interpreted and understood the problem, (b) used and interacted with the software capabilities and features in solving the environmental problem, and (c) selected and categorized the data sets, used digital maps and applied mathematical operations in transforming data. In the next section we summarize the model creations of the student groups in solving the *Water Shortage* activity.

RESULTS AND DISCUSSION

Group A Model Creations

Group A started their exploration by visiting Lebanon, a nearby country, using the "Fly to" command. This approach helped students in identifying that there were many mountains and therefore Lebanon could supply Cyprus with water. In their final report, students documented that: "Lebanon has a high percentage of precipitation, because there are many mountains there. So, they will probably sell water to Cyprus". They then "zoom in" for finding a port. They decided that Tripoli was a major port and their next step was to add a placemark to Tripoli. Students then "zoom out" from Lebanon and gradually moved to the west for finding Cyprus. Students in group A directly focused on Limassol, the major port in Cyprus and added a second placemark. Group A then used the "ruler" feature of the software for calculating the distance between Tripoli and Limassol.

Students followed the same approach for placing placemarks in Pireus (in Greece) and Cairo (Egypt), and for finding the distances between Cyprus and the other three countries. Since the data table (see Appendix) was supplied in spreadsheet software, students added one column presenting the distances between the three different countries and Cyprus. Students explicitly discussed about oil price, and they reached the conclusion that buying water from Greece would be more expensive than buying water from Lebanon or Egypt due to the greater distance between Greece and Cyprus. Students, however, failed to successfully use the provided data and they finally based their choice (Lebanon) partly on the provided data and on their calculations, without providing a coherent model.

Group B Model Creations

Similar to the work of Group A, students in this group quite easily visited the three countries and added placemarks in their major ports. They drew precise paths between each country's port and Limassol and used ruler to calculate the distances (see Figure 1). They reported that: "It is not easy to decide from which country Cyprus should buy water. Lebanon for example is closer than Greece, but water from Greece is much cheaper than water from Lebanon. After calculating the distances between the countries using Google Earth, they moved into the spreadsheet software and added one column in the provided table, presenting the distances. They, however, failed to incorporate into their model the provided data about oil cost, tanker capacity and water price.

Group C Model Creations

This group commenced the problem by finding a major port in each one of the three countries and by drawing paths from these ports to Limassol. Students in this group then calculated the distances between the ports and continued in calculating oil and water cost for *each* tanker trip. In contrast to Groups A and B, students in this group incorporated within their model one more factor; instead of calculating the total cost for each trip and then ranking the three countries, they decided to calculate the cost

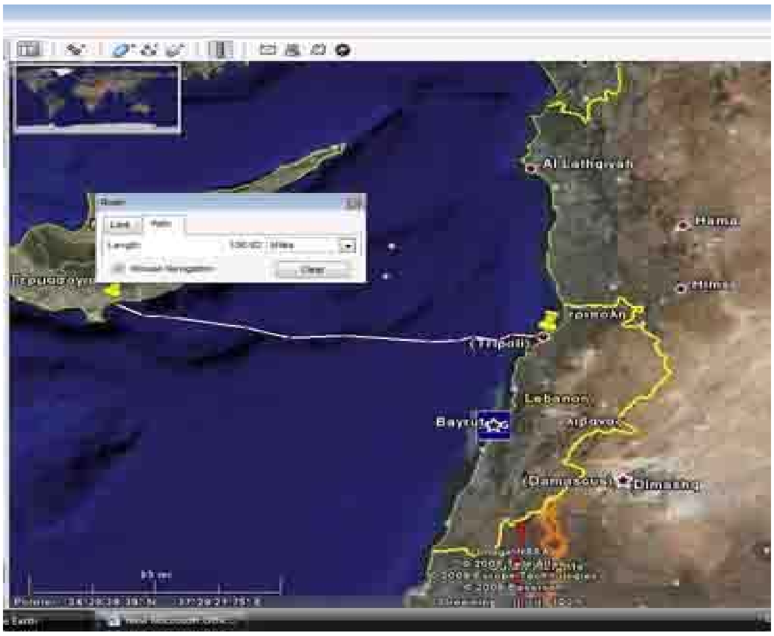

Figure 1: Finding the distance between Tripoli and Limassol.

per water metric ton and based their ranking on this factor. As a result, this model ranked Lebanon as the best possible choice, since the average cost per water ton was only €4.20. On the contrary, the average costs for Egypt and Greece were €6.70 and €7.00 respectively. Student calculations and final selection are presented in Table 2.

Table 2: Group C calculations and final model

Country	Distance	Oil cost	Water cost per tanker	Total cost	Average water cost per ton
Egypt	480	€ 96000	€ 105000	€ 201000	€ 6.70
Greece	1100	€ 275000	€ 75000	€ 350000	€ 7.00
Lebanon	240	€ 60000	€ 150000	€ 210000	€ 4.20

Although this group differed from other groups in that they used a more refined model, they also failed to apply in their model factors such as port facilities for tankers and each country's resources for supplying water to Cyprus. Students in this group, similar to group A and B did not use in their calculations round trips but they rather based their calculations on single trips.

Remaining Groups' Model Creations

Students in the remaining four groups faced a number of difficulties in ranking the different countries. In the first component of the problem, using Google Earth for finding appropriate ports and calculating the distances between Cyprus and the three countries, two groups focused their efforts only on Greece, by finding the distance between Pireus and Limassol. Some other groups faced a number of difficulties in using the software itself.

In the second component of the problem, transferring the distance measurements in the spreadsheet software and calculating the different costs, the students faced more difficulties. Most of their approaches to problem solution were not successful. Many students, for example, just made random calculations, using partially the provided data, and finally making a number of data misinterpretations. One group, for example reported that buying water from Greece is the best solution, since the water price per ton from Greece was only €2.00 (see Table 1).

CONCLUDING POINTS

There are a number of aspects of this study that have particular significance for the use of modeling in mathematical problem solving in elementary school mathematics. First, although a number of students in the present study experienced some difficulties in solving the problem, elementary school students can successfully participate and satisfactorily solve complex environmental modeling problems when presented as meaningful, real-world case studies. Second, our findings show that the available software broadened students' explorations and visualization skills through the process of constructing visual images to analyze the problem, and by using appropriately the spreadsheet's formulas they performed quite complex calculations.

The students' models varied in the number of problem factors they took into consideration. Interestingly, at least three groups succeeded in identifying dependent and independent variables for inclusion in an algebraic model and in representing elements mathematically so formulae can be applied. A number of groups of students made the relevant assumptions for simplifying the problem and ranking the three countries. Finally, the first three groups (as presented in the results session) successfully chose the technological tools/mathematical tables to make precise graphical models in Google Earth and to enable calculations in spreadsheets.

Substantial more research is clearly needed in the design and implementation of technology-based modeling problems and in studying the learning generated. Of interest are, for example, the developments in elementary school students' learning in solving technology-based modeling problems, the ways in which the features of the technological tools can assist students in broadening their explorations and in constructing better models for solving modeling problems, and the teacher professional development training programs that are needed to facilitate mathematical modeling as a problem solving. In concluding, using computer based learning environments for mathematical modeling, at the school level, are a seductive notion in mathematics education. However, further research towards the investigation of their role is needed, to promote both students' conceptual understandings and mathematical developments.

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APPENDIX

Water Shortage Problem: Cyprus will buy Water from Nearby Countries

Background Information: One of the biggest problems that Cyprus face nowadays is the water shortage problem. Instead of constructing new desalination plants, local authorities decided to use oil tankers for importing water from other countries. Lebanon, Greece and Egypt expressed their willingness to supply Cyprus with water. Local authorities have received information about the water price, how much water they can supply Cyprus with during summer, tanker oil cost and the port facilities.

Problem: The local authorities need to decide from which country Cyprus will import water for the next summer period. Using the information provided, assist the local authorities in making the best possible choice. Write a letter explaining the method you used to make your decision so that they can use your method for selecting the best available option (The following **table** was supplied).

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