STUDY OF A PRACTICAL ACTIVITY: ENGINEERING PROJECTS AND THEIR TRAINING CONTEXT

Avenilde Romo Vázquez

Université Paris 7 – Denis Diderot

This paper deals with the question about place that should be given to mathematics in engineering training. In particular, we analyze a practical activity: engineering projects. This activity intends to reproduce the working context in industrial engineering. Our research is developed in the frame of the Anthropological Theory of didactics (Chevallard 1999). We use the Expanded Model of Technology (Castela, 2008) to analyze the engineering project. In this paper, we present the analysis of a task of modelling developed in the projects context

Background

What place should be given to mathematics in the training of engineers? Which contents should be approached in this training? How should it be articulated with other domains of the training?

These question have already been asked and treated in different institutions and different times. For example, Belhoste et al. (1994) who studied the formation given by the French *Ecole Polytechnique* between 1794-1994, have shown that different models of training have arisen during XIX century: Monge's model, Laplace's model and Le Verrier's model. These questions which underlie the establishment of training's models and the changes of model, from Monge to Laplace then to Le Verrier, are the fundamental questions of relation between science and application, relation between science and technology.

Nowadays, these questions are modified by the technological development, technology taking an increasing place in the engineers' work:

> Before the advent of computers, the working life of an engineer (especially in the early part of his or her career) would be dominated by actually doing structural calculations using pen-and-paper, and a large part of the civil engineering degree was therefore dedicated to giving students an understanding and fluency in a variety of calculational techniques. For the majority of engineers today, all such calculations will be done in practice using computer software. (Kent, 2005)

In other words, the development of powerful software changes the mathematical needs because this software encapsulates some of the usually taught mathematics. Mathematics may even appear to be useless to some engineers.

During last years, various researches concerning the nature and the role of the mathematical knowledge in the workplace have been realized (Noss et al., 2000; Kent & Noss, 2002; Magajna & Monagan, 2003; Kent et al. 2004). These works point out the existence of gaps between the educational programs and the real world in which the engineers work. For example, the institutional speech asserts that undergraduate engineers need a solid mathematical education, but the researches show that for graduate engineers mathematics is of little use in their professional work.

> Once you've left university you don't use the maths you learnt there, 'squared' or 'cubed' is the most complex thing you do. For the vast majority of the engineers in this firm, an awful lot of the mathematics they were taught, I won't say learnt, doesn't surface again. (Kent and Noss, 2002)

In our research we intend to contribute to the analysis of the observed gaps and to investigate the role that educational practices and technology play in these gaps. We especially study how one innovative practice in a French engineering Institute intends to articulate theoretical and practical knowledge.

Theoretical Framework: The Anthropological Theory of Didactics ATD (Chevallard, 1999)

The general epistemological model provided by the ATD proposes a description of mathematical knowledge in terms of mathematical praxeology $[T/\tau/\theta/\Theta]$

The praxeology has four components: the first type of tasks T or problems T, the technique is a way to solve the problems, the technology is a theoretical discourse to describe, explain and justify the techniques and the theory is also a theoretical discourse to describe, explain and justify the technologies. The praxeology has two blocks:

Practical block or "know-how" (the praxis) $[T,\tau]$ integrating types of problems and techniques used to solve them

Theoretical block or "knowledge" (the logos) $[\theta,\Theta]$ integrating both the technological and the theoretical discourse used to describe, explain and justify the practical block. (Bosch, Chevallard & Gascón, 2002)

As part of ATD, study is seen as construction or reconstruction of the elements of a mathematical praxeology, with the aim to fulfil a problematic task. To represent finely these processes of construction or reconstruction, ATD offers a model of the study of mathematical praxeology. This model so-called: Moments of the study distinguishes six moments or phases. In this paper we only consider the moment of institutionalization: this moment has the object to specify what is "exactly" the worked out mathematical praxeology. It appears de facto that there are not kept in general in the technology "purified" the elements which are not justified or produced by a theory of empirical knowledge they are rather related to the concrete conditions than the usage of techniques.

Castela (2008) proposes that in the technology of praxeology there are two components: theoretical θ^{th} and practical θ^{p}.

> "…the technology of technique is the knowledge orientated to the production of an efficient practice, which has functions to justify and legitimize the technique but also *to equip* and *to make easier* the implementation with it. Beside possible elements of knowledge borrowed from certain appropriate theories (we shall speak following "*the theoretical component"* of technology, noted θ^{th}) this knowledge appears in technology which, according to research domains, is qualified as operative, pragmatic, practical. Collective work was forged in experience; this *practical component* plays technology (noted afterwards θ^{p}) express and capitalize the science of the community of the practitioners confronted in the same material and institutional conditions with the tasks of type *T*, it favours the diffusion within the group." (Castela, 2008, p.143)

There are six functions associated with the practical component of *praxéologie* θ^{p}*:*

1. **To describe** the technique. The verbal description of the series of steps that make up a technique is an important step in the process of institutionalization.

2. **To motivate** the technique and the different gestures which compose it.

To explain why, in which aims. It describes the aims expected by the technique and analyzes the effects, consequences, different gestures and the difficulties that its absence could provoke.

3. **To promote** the technique's utilization. It considers that knowledge allows users to use the technique with effectiveness but also with a certain comfort.

4. **To validate** the technique: it works, it does what is said. It is main goal is to guarantee the technique, when this is used completely it produces a valid solution and the elements were it belongs achieve the expected aims.

5. **To explain** why does technique work? Is about being interested in the causes of effectiveness. Contrary to the second function, the objective is to detail the mechanisms that make that the technique and its components have the desired effect.

6. **To evaluate** the limits, conditions of effectiveness of the technique. The function of validation is positioned on the side of the truth and justified by a theory. In a practical context this function will consider the efficacy.

The institutionalization within different institutions

There are different institutions which maintain a report with a given praxeology. We shall differentiate the institutions with a function of production *P(K)* of knowledge. And the user *UI* institutions of this praxeology, in sense where subjects of *UI* have to accomplish tasks of type *T*. The aim of *P(K)* institutions is to produce and validate the different components of praxeology. But, we asserts that to a praxeology used in a user Institution; this is a part of technology isn't justify for a theory. The technological knowledge validated by an institution *P(K)* do not exhaust technology, which includes in general a component θ^p for which it is also necessary to examine social modes of validation. The question is therefore to reflect upon construction practises as part of UI, tested through the multiplicity of effective achievements and institutionalization (understood as stabilization rather than explicit recognition by a given institution) of know-how and knowledge.

The **Expanded Model of Praxeology** (Castela, 2008) can be simplified in the following way:

$$\left[T, \tau, \begin{matrix} \theta^{th}, \Theta \\ \theta^{p} \end{matrix} \right] \begin{matrix} \leftarrow P(K) \\ \leftarrow \; UI \end{matrix}$$

Arrows represent social practices of validation in work in the one or other one of the institutions *P(K)* and *IU* carrying respectively on the block $[\theta^{th}, \Theta]$ and on θ^p.

Dynamics of mathematical praxeologies

In our work, we focused on mathematical praxeology present in the engineering projects. To account for the way followed by a praxeology from mathematical origin which has to reach the project, we consider different institutions:

Production Institutions

- P(M) Production institution of mathematics
- P(ID) Production of intermediate disciplines

Institutions inside at Vocational Istitute at the University (IUP) (1)

- T(M) Training of mathematics
- T(ID) Training of intermediary disciplines
- Ep Engineering projects

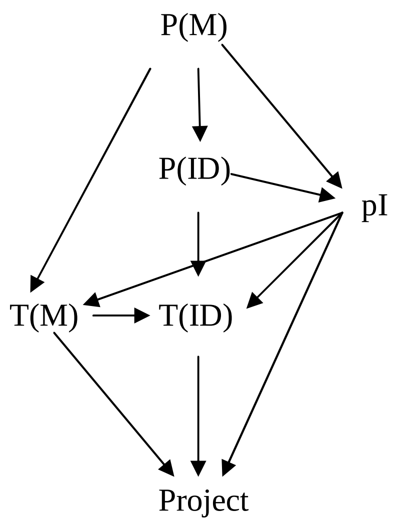
The mathematical praxeologies from production institutions progress to the projects in different ways:

1. **P(M)→T(M)→Ep**
   The first one is from production mathematics to training mathematics until the projects.

2. **P(M)→T(M)→T(ID)→Ep**
   The second one is from production mathematics to training mathematics through training intermediary disciplines and projects.

3. **P(M)→T(ID)→T(ID)→Ep**
The last one is still production mathematics to intermediary disciplines through training intermediary disciplines until projects.

In our context a vocational training, we shall consider also the profession (professional institution pI). The praxeologies presents in the latter institution are also transposed. These have a specific component θ^p, to promote the use in the professional contexts. We shall take into account the influence from profession to training of mathematics T(M), training of intermediary disciplines T(ID) and Engineering projects Ep. The following schema exhibits the links between the previous components:



Context and methodology of research

In order to realize our study, we have chosen the Vocational Institute at University of Evry (IUP). This Institute uses an educational model of practical education closely related to the industrial world: the university training is combined with training in firm; professional practice takes place during twenty weeks (minimum) over the three years of training. But, the mathematical training is solid, it remains classical at university

The question is: How is the IUP model, which is characterized by a strong nearness with professional middle, inserted in a mathematical training which seems to be designed by this classical model? To answer to this question, our study is focused on an innovative practice, the so-called Projects. These projects intend to connect the official universe of educational disciplines and the professional world of engineers.

The aim of this study is devoted to identify the mathematical praxeology present in the realization of projects and linked with technological tools (TEN).

Therefore, we use this study of praxeology to question the institutional mathematical living in intermediate disciplines or lessons of mathematics.

The projects

The projects are realized by a group of three or four students, very independent, respecting a didactical organization which tries to reflect the real organization in workplaces.

The **engineering projects** are carried out by teams of students in their fourth year of engineering school, over five weeks. The subject of every project is open; there is no previous requirement established by client. The final production and the route towards it have to be built together in the same process. Therefore students have to organize and plan their work, to look for solutions; this generally supposes that they adapt or develop their knowledge.

The projects are realized in two phases. After the first one the students write an intermediary report; in this report they describe the pre-project which is in general justified by a study of the subject. They present the technological solution they have chosen among those they have found during their exploratory work. In the second phase the pre-project must lead to a concrete product.
In this kind of projects, the manager is a college teacher, who plays the role of a client who requests a product from a student's group. All the terms and conditions of the project are described in the schedule of conditions (cahier des charges) which is negotiated between the client (teacher) and the distributor (students). The students are supposed to work on their own to come up to the client's request. The project is assessed from on a double point of view, combining workplace and engineering school requirements. The client must be convinced that the technological solution is the best. But this evaluation is also academic; the students present their work to a jury composed of college teachers. The jury evaluates the use of tools in relation whit knowledge taught in the engineering college. Moreover the students are often asked to justify some of their claims.

Projets Observation methodology

We have realized two observations of the projects. To realize the observation of projects, we used Dumping methodology. In the first phase of project (two weeks) we carried out questionnaires and semi-structured interviews with the students and the clients – tutors. After this phase, we collected institutional data, specifications (document), intermediary reports and documents used for the development of projects. This allowed us to get familiar with projects.

For the second phase we chose only three projects, our aim to be able to realize a deeper and precise observation. To select these projects, we based on the intermediate reports following two criteria: 1) the presence of explicit mathematical knowledge and 2) the project domain such as aeronautics, mechanics, electronics, etc.

In the third week of the project, we met with the students' teams (three teams for three projects) for an interview about the intermediary report; the aim of this interview was to understand the project and to investigate on the role of the identified mathematical contents. We asked the students to do a brief exposition of their project. The aim of this exposition was to identify the role that they were giving to the mathematical content expressed in their intermediary report.From this, we identified the work division inside the team, and we realized that only one student has the responsibility to develop the mathematical activity. After these meetings, interviews were realized with each student individually.

Praxeological analysis of projects

We carried out a praxeological analysis of the projects. In this paper, we present the analysis of one task accomplished in one of the projects: the Development of a conveyor belt for the aerodynamic study of a light ultra vehicle. The aim of this project was to build a conveyor belt to reproduce the velocity floor. For this, it was necessary to model functioning of the motor and simulates it in Matlab (software).

Task: Modelling of motor

The task is to build a model of the motor trough the block diagram. This diagram will allow us to simulate this motor in the Matlab software.

Technique: The modelling of the motor pass by two steps.

1) Mathematical model. The differential equations modelling the electrics and mechanics functioning.

Electrics functioning $u(t) = e(t) + Ri(t) + L\frac{di(t)}{dt}$

Mechanics functioning $C_m(t) - C_r(t) = J\frac{d\omega(t)}{dt} + f\omega(t)$

The electrics and mechanics functioning are linked by two equations. Every single equation contains a flow and couple constant k: $e(t) = k\omega(t)$ and $C_m(t) = ki(t)$

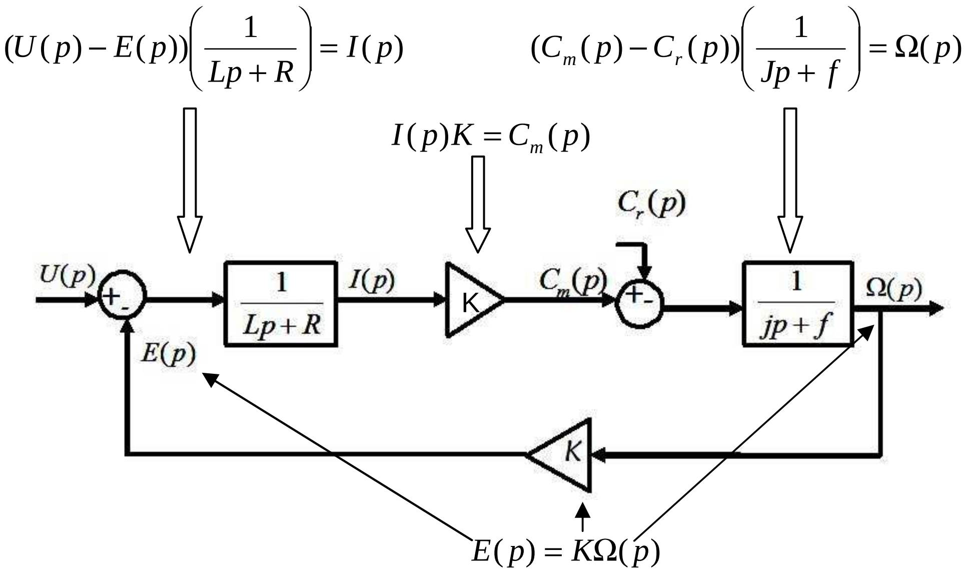
Next, we apply the Laplace transform to every equation:

$$I(p) = \frac{U(p) - E(p)}{R + Lp} \quad (1) \qquad \Omega(p) = \frac{C_m(p) - C_r(p)}{Jp + f} \quad (2)$$

$$E(p) = K\Omega(p) \quad (3) \qquad I(p)K = C_m(p) \quad (4)$$

2) Construction of block diagrams

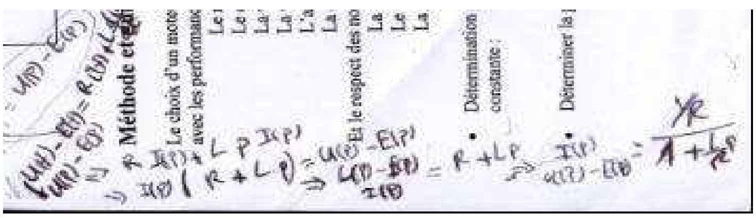
These equations allow us to construct the following block diagrams. Every one element of the equation is represented in the block diagram.



Student techniques:

The student describes the technique utilized to accomplish this task.

> "For example if we take this equation (showing $u(t)-e(t)=Ri(t)+L\frac{di}{dt}$) […] and if we apply Laplace transform we shall have $U(p)-E(p)=RI(p)+LpI(p)$, if we make this (factorize $I(p)$) we shall have this $I(p)(R+Lp)=U(p)-E(p)$, this means that $\frac{U(p)-E(p)}{I(p)}=R+Lp$ and if we make the inverse we shall have $=\frac{I(p)}{U(p)-E(p)}=\frac{1}{R+Lp}$ […] this equation is modelled by this part" (oral explanation)



Written traces accompanying oral explanation

Technology:

In the description of technique, the student shows the aim of task is to express the "transfer function" of the system. The Laplace transform is for the student a tool which allows to treat an electrical equation as a transfer function. At the same time, Laplace transform allows to pass from temporary domain (algebraic) to a non temporary domain (differential equation).

> "[…] we have $U(p) = E(p) + I(p)R + LpI(p)$ and if we transform $pI(p)$, we apply the inverse Laplace transform, then we obtain the derivative of a temporary function" (Oral explanation)

We see here that motivation appears (function 2 θ^p) by the utilization of the Laplace transform. The student focuses in the derivate term $LpI(p)$, showing interest in using the Laplace transform to pass from differential equation (temporary domain) to transfer function (algebraic domain) or the block diagrams.

From the mathematical point of view, there is a notion justifying the block diagram: the transfer function. This notion considers that the physics systems are described by the differential equation:

$$b_n \frac{d^n y}{dt^n} + b_{n-1} \frac{d^{n-1} y}{dt^{n-1}} \ldots + b_1 \frac{dy}{dt} + b_0 y = a_m \frac{d^m u}{dt^m} + \ldots + a_1 \frac{du}{dt} + a_0 u$$

> "If we apply Laplace transform to the differential equation and assume the initial conditions to be null, then the rational fraction which links the output Y(p) to the input U(p) is the transfer function of the system.
>
> $$L\left(\frac{dy}{dt}\right) = p.Y(p) \Rightarrow L\left(\frac{d^2 y}{dt}\right) = p^2.Y(p) \Rightarrow \ldots \Rightarrow L\left(\frac{d^n y}{dt}\right) = p^n.Y(p)$$
>
> $$\Rightarrow b_n p^n Y(p) + \ldots + b_1 pY(p) + b_0 Y(p) = a_m p^m U(p) + \ldots + a_1 pU(p) + a_0 U(p)$$
>
> $Y(p) = H(p).U(p) = \dfrac{a_m.p^m + \ldots + a_1 p + a_0}{b_n.p^n + \ldots + b_1 p + b_0}.U(p)$" (Automatics course: Introduction à l'Automatique des systèmes linéaires, pp.7 -8)

This notion is part of the Automatics course (intermediary discipline).

Conclusion

The task modelling of the motor is the reproduction the existent model. The students are not created a new model. They adapted a type models a specific situation. This adaptation need mobilize the technological elements. These elements are from different institutions: teaching institution of intermediary disciplines T(ID), teaching mathematics T(M) and practical institution pI. We see the processes of transposition of the praxeologies, which pass from one institution to other institution and are transposed. The functions of the practical component θ^p, allows us to analyze the praxeologies used in the projects. To understand the technologies linked to the students techniques, it is necessary to take in account the intermediary disciplines. These disciplines are intermediary between mathematics teaching and mathematics used in practise.

Bibliography

Belhoste, A. (1994) Un modèle à l'épreuve. L'Ecole Polytechnique de 1794 au Second Empire. In Belhoste, B., Dalmedico, A., and Picon, A. (Eds.), *La formation Polytechnicienne* (pp. 9-30). Paris: Dunod

Castela, C. (2008) Travailler avec, travailler sur la notion de praxéologie mathématique pour décrire les besoins d'apprentissage ignorés par les institutions d'enseignement. *Recherches en didactique des mathématiques, 28*(2), 135-179.

Chevallard, Y. (1999) La recherche en didactique et la formation des professeurs : problématiques, concepts, problèmes. Actes de la X Ecole d'été de Didactique, 98-112. Académie de Caen, France.

Kent, P. (2007) Learning Advanced Mathematics: The case of Engineering courses contribution to the NCTM Handbook chapter: Mathematics thinking and learning at post-secondary level. In Lester, K., F. (Ed.), *Second handbook of research on mathematics teaching and learning: a project of the National Council of Teachers of Mathematics*. (pp. 1042-1051). Charlotte, NC: Information Age Pub

Kent, P., & Noss, R. (2001) Investigating the mathematical components of engineering expertise. In Marja van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International group for the Psychology of Mathematics Education Conference*, *1,* 402 Utrecht, The Netherlands.

Kent, P., & Noss, R. (2002) The mathematical components of engineering expertise : The relationship between doing and understanding mathematics. *Proceedings of the IEE Second Annual Symposium on Engineering Education: Professional Engineering Scenarios 2* (pp. 39/1 -39/7)

Noss, R., Hoyles, C., & Pozzi, S. (2000). Working Knowledge: Mathematics in use, In A. Bessot & J. Ridgway (Eds.), *Education for Mathematics in the workplace*, (pp.17-35). Dordrecht: Kluwer Academic Publishers.

Magajna, Z., & Monaghan, J. (2003). Advanced Mathematical Thinking in a Technological Workplace. *Educational Studies in Mathematics, 52*(2), 101-112.