MATHEMATICAL MODELLING IN CLASS REGARDING TO TECHNOLOGY

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Based on the well known modelling cycle we develop a concept of modelling in mathematics education using technology. We discuss the specifics of modelling with computers and handhelds and show some technical possibilities of software tools for mathematics classes. Exemplary we show different modelling cycles using technology based on the three major types of software tools for mathematics.

INTRODUCTION – MODELLING WITHOUT HELP OF TECHNOLOGY

The concept of modelling can be found as a basic concept in some areas of natural sciences, especially mathematics. Therefore it is not remarkable that this basic concept can be found in several curricula all over the world. In mathematics the concept of modelling and the application of real-life-problems in education has been discussed intensively over the last years – see for example Kaiser & Sriraman (2006, p. 304), Siller (2006). It is possible that students of all ages are able to recognize the importance of mathematics through such problems because real-life problems...

- ... help students to understand and to cope with situations in their everyday life and in the environment,
- ... help students to achieve the necessary qualifications, like translating from reality to mathematics,
- ... help students to get a clear and straight picture of mathematics so that they are able to recognize that this subject is necessary for living,
- ... motivate students to think about mathematics and computer-science in a profound way so that they can recall important concepts even if they were taught a long time ago.
- ... allow the teaching of mathematics with a historical background.

If we look at the concept of modelling (figure 1) designed by Blum & Leiß (2007) in mathematics education we will be able to find three important points:

- Design & Development: Comparable to "Finding the real model" and to the step

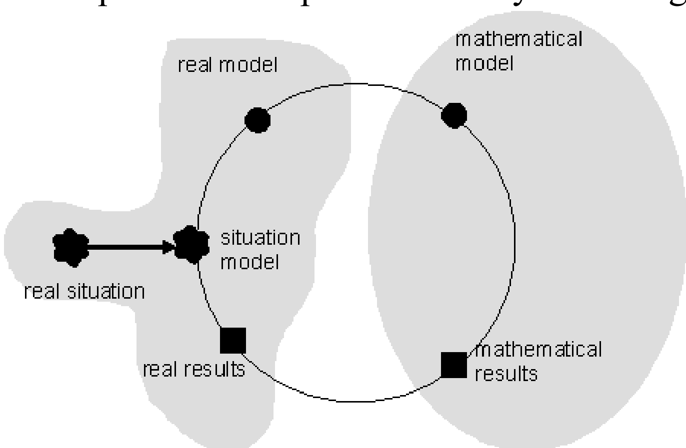

Figure 1: Modelling cycle of Blum & Leiß (2007)

of “Translation” – Real situation to real model by including the situation model.

- Description: Comparable to “Finding the mathematical model”.
- Evaluation: Comparable to “Finding (Calculating) mathematical results” and to the step of validating.

In curricula the usage of technology and the aspect of modelling very often is demanded. For example you can read in the Austrian curriculum (2004):

“An application-oriented context points out the usability of mathematics in different areas of life and motivates new knowledge and skills. [...]

The minimal realization is the acquiring of the issue of application-oriented contexts in selected mathematical topics; the maximal realization is the constant addressing of application-oriented problems, the discussion and reflection of the modelling cycle regarding its advantages or constraints. […] Technologies close to mathematics like Computer algebra systems, Dynamic Geometry-Software or Spreadsheets are indispensable in a modern mathematical education. Appropriate and reasonable usage of programs ensures a thorough planned progress. The minimal realization can be done through knowing such technologies and occasional applications. In a maximal realization the meaningful application of such technology is a regular and integral part of education.”

So each of us has to ask where the usage of technology can be best implemented. The integration of technology in the modelling cycle can be helpful by leading to an intensive application of technology in education. We have thought about a way that the use of technology could be implemented in the modelling cycle. Our result can be seen in figure 2. The “technology world” is describing the “world” where problems are solved through the help of technology. This could be a concept of modelling in mathematics as well as in an interdisciplinary context with computer-science-education.

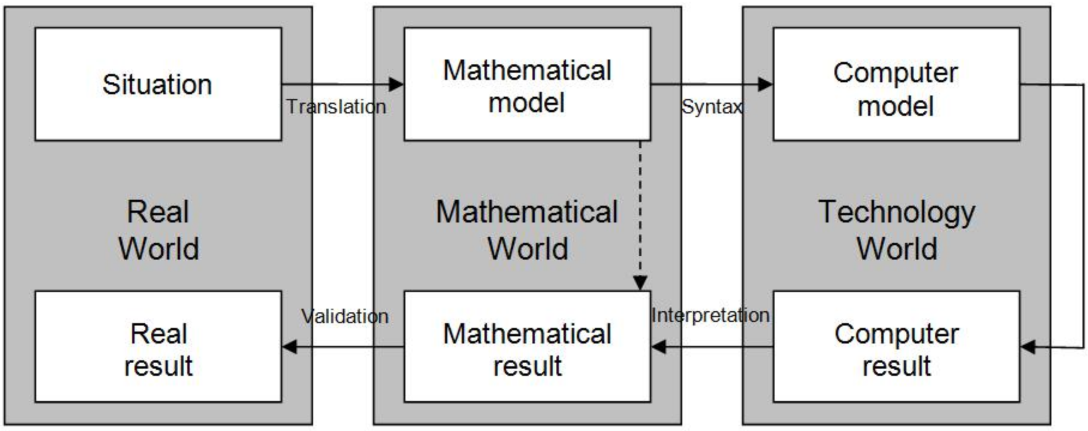

Figure 2: Extended modelling cycle – regarding technology when modelling

The three different worlds shown in figure 2 are idealized; they influence each other. For example the development of a mathematical model depends on the mathematical knowledge on the one hand, on the other it is affected by the possibilities given in the technology world. Using technology broadens the possibilities to solve certain mathematical models, which would not be used and solved if technology would not be available. At this point we want to mention, that successful modelling demands mathematical knowledge and skills in certain software tools.

Based on this graphical illustration we have to discuss the use of technology in terms of modelling in a more detailed way.

MODELLING WITH THE HELP OF TECHNOLOGY

Through the usage of computers in education it is easier to discuss problems which can be taken out of the life-world of students. Through such discussions the motivation for mathematical education can be effected because students recognize that mathematics is very important in everyday life. If it is possible to motivate students in this way it will be easy to discuss and to teach the necessary basic or advanced mathematical contents such as finding a function or calculating the local extreme values of a function.

Unfortunately a lot of teachers and educators prefer not to work with real-life problems. The reasons for this are manifold, e.g. teachers do not want to use CAS or other technology in class or the preparation for such topics is very costly in terms of time. There are however, lots of reasons to combine modelling and technology. Fuchs & Blum (2008) quote the aims of Möhringer (2006) which can be reached through (complex) modelling with technology:

- Pedagogical aims:
  With the help of modelling cycles it is possible to connect skills in problem-solving and argumentation. Students are able to learn application competencies in elementary or complex situations.
- Psychological aims:
  With the help of modelling the comprehension and the memory of mathematical contents is supported.
- Cultural aims:
  Modelling supports a balanced picture of mathematics as science and its impact in culture and society (Maaß, 2005a, 2005b).
- Pragmatically aims:
  Modelling problem helps to understand, cope and evaluate known situations.

As we can see the use of technology can help to simplify difficult procedures in modelling. In some points the use of technology is even indispensable:

- Computationally-intensive or deterministic activities,
- Working, structuring or evaluating of large data-sets,

- Visualizing processes and results,
- Experimental working.

With technology in education it is possible not only to teach traditional contents using different methods but it is also very easy to find new contents for education. The focus of education should be on discussion with open, process-oriented examples which are characterized by the following points.

Open process-oriented problems are examples which …

- … are real applications, e.g. betting in sports (Siller & Maaß, 2008), not vested word problems for mathematical calculations.
- … are examples which develop out of situations, that are strongly analyzed and discussed.
- … can have irrelevant information, that must be eliminated, or information which must be found, so that students are able to discuss it.
- … are not able to be solved at first sight. The solution method differs from problem to problem.
- … need not only competency in mathematics. Other competencies are also necessary for a successful treatment.
- … motivates students to participate.
- … provokes and opens new questions for further as well as alternative solutions.

The teacher is achieving a new role in his profession. He is becoming a kind of tutor, who advises and channels students. The students are able to detect the essential things on their own. Therefore we want to quote Dörfler & Blum (1989, p. 184): "With the help of computers (note: also CAS-calculators) which are used as mathematical additives it is possible to reach a release of routine calculation and mechanical drawings, which can be in particular a big advantage for the increasing orientation of appliance. Because of the fact, that it is possible to calculate all common methods taught in school with a computer, mathematics education meets a new challenge and (scientific) mathematics educators have to answer new questions."

ENABLING TECHNOLOGY

The use of technology in mathematical education always depends on the enabling technology. For mathematical education there are many different hard- and software tools. The three major types which have emerged in this area are - see for example Barzel et al. (2005, p. 36):

- Computer algebra system (CAS): With the help of such a tool it is possible to work symbolically, algebraically and algorithmically.

- Dynamic Geometry Software (DGS): With the help of such software it is possible to create geometrical constructions interactively and work with digital work sheets.
- Spreadsheet Program (SP): With its help it is possible to organize and/or structure data for easier handling, calculating in tables and common analysis.

New developments in the area of technology try to combine these three aspects, although it is difficult to combine all three points and form unique software for each characteristic.

For example the CASIO Classpad, respectively the associated software package Classpad Manager, offers a real interactivity of geometry and algebra.

The simultaneous application of CAS and DGS of the Classpad is, in our opinion, also a useful application. With the help of CAS it is possible to calculate for example non-linear equations symbolically and at the same time the geometrical aspects can be shown through dynamical geometry.

For these purposes equations have to be transformed from the CAS-part to the geometry part. But this method is – until now – not as effective as it should be. After such a transformation the equations cannot be changed interactively. But this problem is not really important, because such examples can be handled easily with other tools, e.g. Geogebra. In Geogebra it is not possible to use a real CAS-part, but the interactivity can be done easily. And a new feature, which is currently available in a Beta-version, is the implementation of a spreadsheet-tool. With its help it is possible to combine interactivity with numerical solutions, calculated in a spreadsheet. To sum up there are several tools combining two or three of the major types CAS, DGS and SP.

Example

The following example which could be discussed with students can be found in everyone's life-world:

Dangerous intersection:

Two cars with different velocities are driving on two different streets towards an intersection where those streets meet. One car is going 60 kilometres per hour; the other has a velocity of 50 km per hour. Try to think about the situation at the intersection – is it possible that an accident can happen? It is given that both cars are running with the constant velocities towards the intersection.

The example can be discussed now under the aspect of different didactical principles:

- Haptical discussion: Students model the given situation, for example with some toy cars, and try to find a solution. This could be a starting point for cross-disciplinary teaching with physics (without computers).
- Graphical discussion: Students have to draw a chart or diagram of the given situation, and/or modify a given chart (with paper and pencil or DGS).

- Symbolical discussion: Students have to describe the situation for both cars with the help of a function or functional dependency (with paper and pencil or CAS).
- Numerical discussion: Students compute lots of data to solve the problem (with a scientific calculator or SP).

It is not that important which method students' first use to solve this problem. An important point is that students are working based on experience and the methods used are kept sustainable. But it is important for the students to see the different approaches for this problem. In our course we used the following problem:

A picture which describes the given situation visually can be found in figure 5.

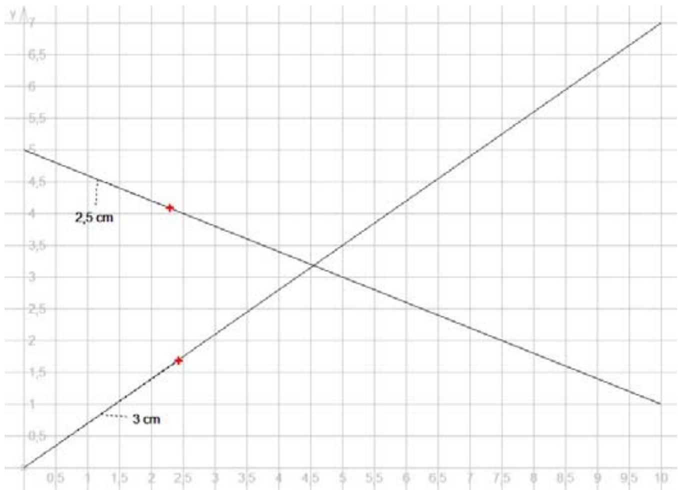

Figure 3: Graphical visualization of the problem

This problem – here in an adequate norm - can be solved in completely different ways. If we use the help of dynamical geometry software, we can move a point for the second car by moving the point for the first car automatically in the right scale and see what happens at the intersection. If we have a closer look at our concept "Modelling with the help of technology" and try to translate the steps which are necessary for solving the problem into our model, it could appear as presented in the following figure (figure 6):

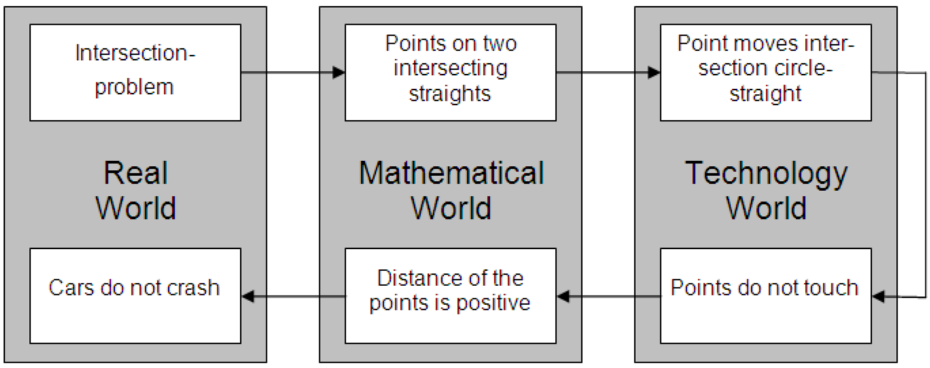

Figure 4: Extended modelling cycle for the problem "Dangerous Intersection"

Alternatively the solution can be calculated with the help of a CAS.

#1: $f(t) := \dfrac{t \cdot [10,\ 7]}{\sqrt{(10^2 + 7^2)}}$

#2: $g(t) := [0,\ 5] + \dfrac{\dfrac{5 \cdot t}{6} \cdot [10,\ -4]}{\sqrt{(10^2 + 4^2)}}$

#3: $d(t) := |f(t) - g(t)|$

#4: $\mathrm{SOLVE}\left(\dfrac{d}{dt}\, d(t),\ t,\ \mathrm{Real}\right)$

#5: $t = \dfrac{261540 \cdot \sqrt{149}}{1037531} + \dfrac{495300 \cdot \sqrt{29}}{1037531}$

#6: $t = 5.647806845$

#7: $d(5.647806845)$

#8: 0.2573238092

#9: $f(5.647806845)$

#10: $[4.626863648,\ 3.238804553]$

#11: $g(5.647806845)$

#12: $[4.369880841,\ 3.252047663]$

Note: For easier readability we have decided to present the solutions in the CAS in decimal notation.

Figure 7 shows the same mathematical model, but a different computer model in the technology world. A CAS works algebraically so we cannot use a geometrical construction to work on the mathematical model. Therefore we decided to use derivation and distance to solve the problem.

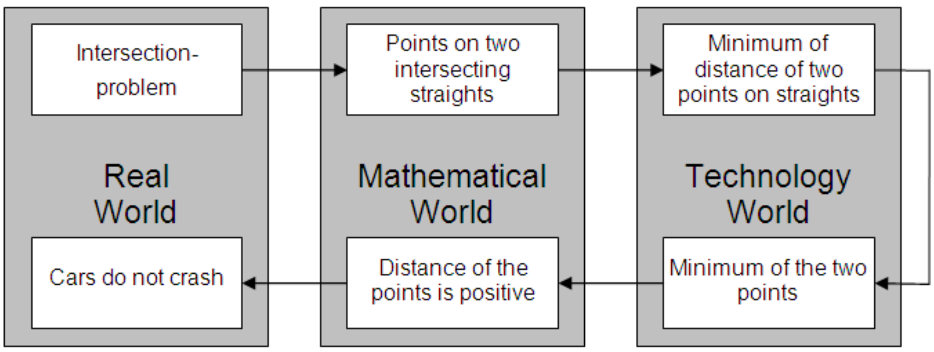

Figure 5: Extended modelling cycle with a different part in the technology world

The solution can also be calculated with the help of a spreadsheet. We will just document this possibility without discussing it. There will of course be a third model in the technology world.

t	x1	y1	x2	y2	dx	dy	d
5,6	4,6	3,2	4,3	3,3	0,3	-0,1	0,2608
5,61	4,6	3,2	4,3	3,3	0,3	0,0	0,2595
5,62	4,6	3,2	4,3	3,3	0,3	0,0	0,2585

5,63	4,6	3,2	4,4	3,3	0,3	0,0	0,2578
5,64	4,6	3,2	4,4	3,3	0,3	0,0	0,2574
5,65	4,6	3,2	4,4	3,3	0,3	0,0	0,2573
5,66	4,6	3,2	4,4	3,2	0,3	0,0	0,2575
5,67	4,6	3,3	4,4	3,2	0,3	0,0	0,2581
5,68	4,7	3,3	4,4	3,2	0,3	0,0	0,2589
5,69	4,7	3,3	4,4	3,2	0,3	0,0	0,2600
5,7	4,7	3,3	4,4	3,2	0,3	0,0	0,2614

Table 1: Worksheet in SP for the example "dangerous intersection"

These three possible solutions (by DGS, CAS, SP) are prototypes for student solutions which represent different mathematical concepts and models. In all three models the assumptions concerning the position of both streets, represented by straight lines, are equal.

The model designed with the help of dynamical geometry software uses only the implicit representation of parameterised straight lines. The main mathematical concept is studying the distance of two (moved) points in the plane. Designing the model as it is shown, presumes the understanding in analytical geometry and connections between the two moving points. The ratio of the velocities of both cars, idealized as points, influences the movement of one point depending to the other. The dynamical visualization allows pupils to experiment with the model (e.g. changing the position of both cars). Thereby possibilities for further developing the model are given (e.g. including the length of the cars).

The models designed by CAS and SP are using parameterised straight lines as algebraic expressions. The distance of both points can be calculated with the help of Pythagoras' theorem. In the CAS model the minimum is calculated with the help of differential calculus, whereas in the SP model the minimum has to be found numerically. One possibility of the CAS model is adding other variables (e.g. different velocities for the cars, changing the starting point of one car) for experimenting or developing the model further. Here more possibilities are imaginable. All of them are very ambitious.

TEACHER EDUCATION

The use of technology in mathematical education does not only depend on technology but also on the knowledge and beliefs of the teacher concerning the different types and usages of technology.

The work with computers in teacher training sets the stage for use in schools. At the beginning and at the end of the course "Computer for Mathematics in School" students who attended were asked about their opinion on the use of computers in class for education. 10 students were present in both interviews comparably. Every question (shown in the diagram of figure 9) had four possible answers: yes, rather yes, rather no, no. In certain cases the beliefs using computers in class changed after attending this course. The topics of the course are the use of CAS, DGS and SP for mathematics in school.

The interviews show a possible change of beliefs while working on topics with computers in class. Some of the positive results concerning computer use can be seen in figure 9, whereas a small bar is closer to the answer "Yes" a bigger bar closer to "No".

The students are asked to say what changes occur using computers in mathematics classes.

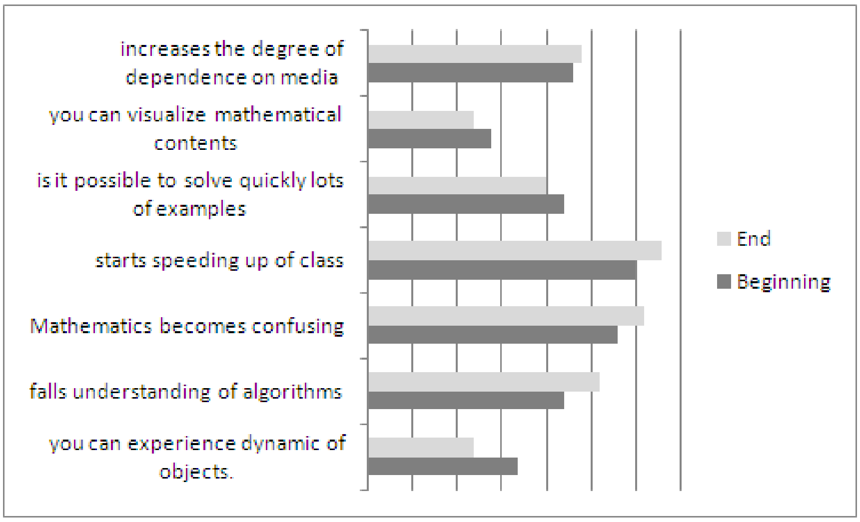

Figure 9: Results of the interview about changes by using computers

This first results show, that it would be interesting to have a closer look at the different strategies of students while modelling with a digital tool. For this research it is necessary to find more examples like "dangerous intersection" with relevance to real life and with different approaches.

Even in teacher education it would be a possible way to discuss examples like "dangerous intersection" by focussing on different computer models. To help the students to reflect upon the role of mathematical software in mathematical modelling processes criteria should be developed and applied (e.g. mathematical content, level of difficulty, possibilities of further developing).

Recapitulatory the use of computers in mathematical education can support and create understanding, in order to improve motivation. The role of technology in the modelling cycle has to be pointed out and examples in education have to be adapted and even created. To implement these points more research in this field needs to be done.

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