ASPECTS OF VISUALIZATION DURING THE EXPLORATION OF „QUADRATIC WORLD" VIA THE ICT – PROBLEM „FIREWORKS"

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Abstract

This paper deals with several proposals for the modelling of physical phenomenon of projectile motion (angled-launched projectile) in the Earth gravitational field. The problem, which we solve in this article named "Fireworks", is situated in the discipline intersection of mathematics, physics and informatics at the secondary schools. We compare the utilisation of the graphic calculator, the mathematical software WinPlot and spreadsheets in the solving process of this problem. It offers a large space for the unconventional approaches of teaching, for the use of information technologies and for the creation of interdisciplinary relations. The paper lays emphasis on the innovative process in mathematics teaching in Slovakia that incites stimulating discussions in this field new modern methods, ICT and e-learning.

Key words: modelling, quadratic function, graphic calculator, spreadsheets, teaching.

1 INTRODUCTION

> *Some Mathematics becomes more important because technology requires it.*
> *Some Mathematics becomes less important because technology replaces it.*
> *Some Mathematics becomes possible because technology allows it.*
> *Bert K. Waits [1]*

There are many arguments for and against the use of Information and Communication Technology (ICT) in mathematics teaching. This paper sets out some aspects of visualization, which is favourable to the exploration in mathematics learning.

The most considerable didactic aspects of the utilization of ICT in mathematics teaching are [3]:

- Aspect of *visualization* that relieves the conception of thinking process and keeps the learning process shorter,

- Aspect of *process simulation* that enables to create an adequate model on the basis of diverse input values (parameters) as well as to understand their hierarchy,
- Aspect of *interaction between an IC technology and a user* that represents one of the most important attributes of multimedia.

In the following text, we would like to focus primarily on the aspect of *visualization (demonstration)* in mathematics teaching. Problematic of the demonstration in mathematical research and also in mathematics teaching is considered to be one of the most important in the development of mathematical thinking. In relation to that, the literature remarks the notion of *visual thinking*. It is well known that the development of human cognition in the certain field relies on the groups of specific separate models of a future notion or knowledge [10].

Mental operations with the images can be complemented by real experimental manipulations and they lead to the concrete practice manipulation. In the frame of visual thinking, we can assert not only the *algorithms*, but also *heuristics*. *Visualization* represents one of the fundamental strategies in the field of creativity, discovering, inventions and abilities for problems solving. The importance of visualization is affirmed by the fact that the biggest part of brain cortex is aimed at *vision* and *visual analysis*.

Today there is no one to argue about the importance and significance of the development of visual thinking for the school mathematics. In spite of this, the visual methods of problems solving are moved at periphery and they are rare in school mathematics teaching. This reflexion is also underlined by the statement of contemporary mathematician and known popularizator of mathematics, Ian Stewart: *„Images transfer much more information than the words can transfer. Many years, we tried to unteach our students to use the images, because „they are not exact". It is the sad misunderstanding. Yes, the images are not exact, but they help to think and we could not despise this aid. "*[3]

The main objective of the innovative process in the mathematics teaching in our country is to show the pupils that the mathematics education is not purposeless. The mathematics is the science, which has various important applications in real life that are inevitable for the development of other scientific and technical disciplines. The process planted into the long term horizon must respect the pupils´ mental abilities oriented at the discovering and the cognition of mathematical notion, the development of pupil's creativity, critical thinking and team-work, but also the need of scientific discussion in the class. The international comparative studies TIMSS and PISA show the actual deficiency of these pupils´ abilities in Slovak school system [5]. That is the reason why in this paper we would like to offer one physical problem together with the possibilities of its solutions including the utilisation of mathematical knowledge and convenient ICT that is accessible to schools. First section outlines the central problem of this article named „Fireworks". The next sections detail the ideas

of the central problem solution with the help of the mathematical software WinPlot, the graphic calculator TI 83+, and MS Excel (spreadsheets).

2 PROBLEM „FIREWORKS"

This part of article was inspired by the mathematics teaching at secondary schools in USA, especially by the implementation of IMP (Interactive Mathematics Program). The aim of this program is to teach the mathematics differently and to prepare a pupil in the constructive way to encounter the world where he lives. The objective is not to let the pupil receive the knowledge in the inactive way, but above all to let him experiment, search, ask, look for the answers, create and test his own hypothesis, consider, work in teams, share and communicate his ideas and inventions.

The principal topic of the following sections is a quadratic function, whose concept is presented from the several points of view (functional, algebraic and geometric). We consider the choice of the „Fireworks" problem as very suitable, because it includes not only the mathematical problem, but also the physical problem, which the pupils are able to solve effectively by the aid of ICT [4].

Problem definition

High school football team has just won the championship. To celebrate this triumph, the young football players want to put on a fireworks display. They will use rockets launched from the top of a tower near the school. The height of the tower is 50 metres off the ground. The automatic mechanism will launch the rockets with the initial velocity 28 metres per second.

The team members want the fireworks from each rocket to explode when the rocket is at the top of its trajectory. They need to know how long it will take for the rocket to reach the top, so they could set the timing mechanism. Also, they need to know the best place for spectators to stay (they need to know how high the rocket will go).

The rockets will be oriented to an empty field and shot at an angle of 65 degrees above the horizontal. The team members also want to know how far from the base of tower will the rockets land, so that they can fence off the area.

Theoretical background (several formulations)

The problem includes the physical phenomenon of projectile motion named angled-launched projectile [6]. This motion consists of a uniform rectilinear movement in the direction of axes x with velocity v_1 and a vertical displacement with initial velocity v_2. Vector of initial velocity v_0 and direction of projectile motion contain the angle α which is named *elevation angle*. The horizontal distance of the projectile range depends on this angle (the distance is biggest when $\alpha = 45°$). The range distance depends also on the initial velocity v_0.
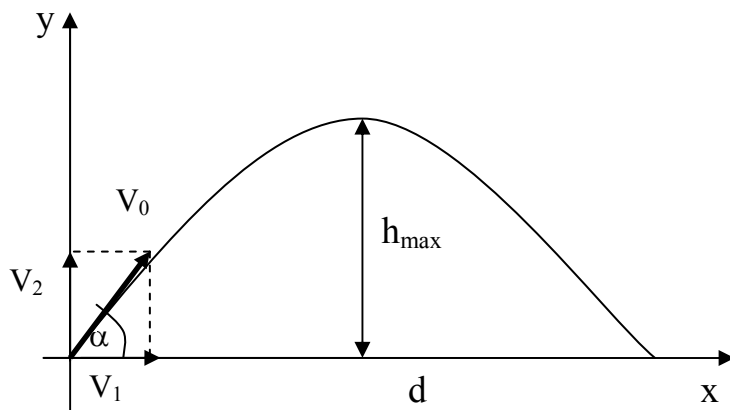
Figure 1 Scheme of angled-launched projectile

$v_1 = \cos(\alpha).v_0$

$v_2 = \sin(\alpha).v_0 - g.t$

$x = v_1.t$

$y = v_2.t - \frac{1}{2}.g.t^2$

David is member of the football team. He is also high school student and he is good in mathematics and physics. He would like to help his team to solve the „Fireworks" problem. He says that there is a function *h(t)* that gives the relation between the rocket's height off the ground and the time *t* elapsed since launch. This relation can be represented by the equation (in metres and seconds): **$h(t) = 50 + 28.\sin 65^o\, t - 5t^2$, $h(t) \cong 50 + 25\, t - 5t^2$.**
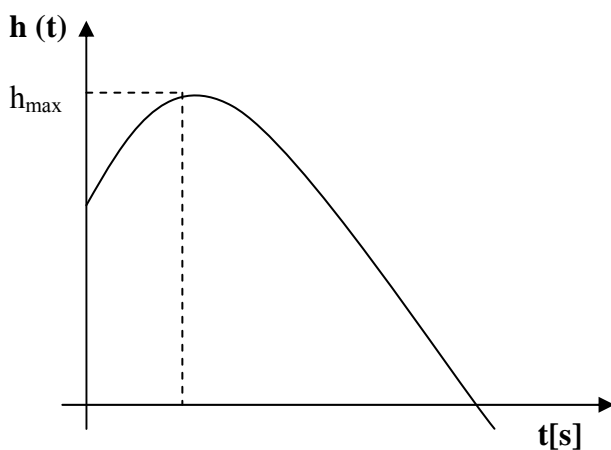

Figure 2 Sketch of the problem situation

We can probably see where the numbers 50, 28, 65 come from. The coefficient -5 in the quadratic component $-5t^2$ coheres with the force of gravity done by the relation: $G = \frac{1}{2}gt^2$.

David also says that it is possible to find a relation describing horizontal distance. The rocket travels with this function: **d(t)= 28 t .cos 65º.**

Again, *t* is the time (in seconds) since the rocket was launched and **d(t)** is the distance (in metres).

Tasks:
1. To draw a sketch of the situation.
2. To find answers to the partial questions of the football team players:

A) What time does the rocket need to reach the top of its trajectory (to find the point where does the function **h(t)** reach its maximum)?

B) Where (horizontal distance of the rocket from the tower) does the rocket reach its maximum height?

C) How far (horizontal distance) from the base of the tower does the rocket land?

3 PROBLEM SOLUTION BY THE AID OF THE SPECIAL MATHEMATICAL SOFTWARE WINPLOT AND THE GRAPHIC CALCULATOR

Software WinPlot enables to create an interactive programme, which describes our „Fireworks" problem. The pupil can change the parameters of the tower height, the initial velocity and the elevation angle. He can observe how these parameters influence the trajectory of the rocket motion. The image underneath represents the trajectory of the rocket since its launch from top of the tower until its landing. The ordered pair **[d(t), h(t)]** express the coordinates of the projectile (fireworks rocket) moving in the frame of its trajectory in terms of the time *t*.

Variable parameters:

H – Height of tower
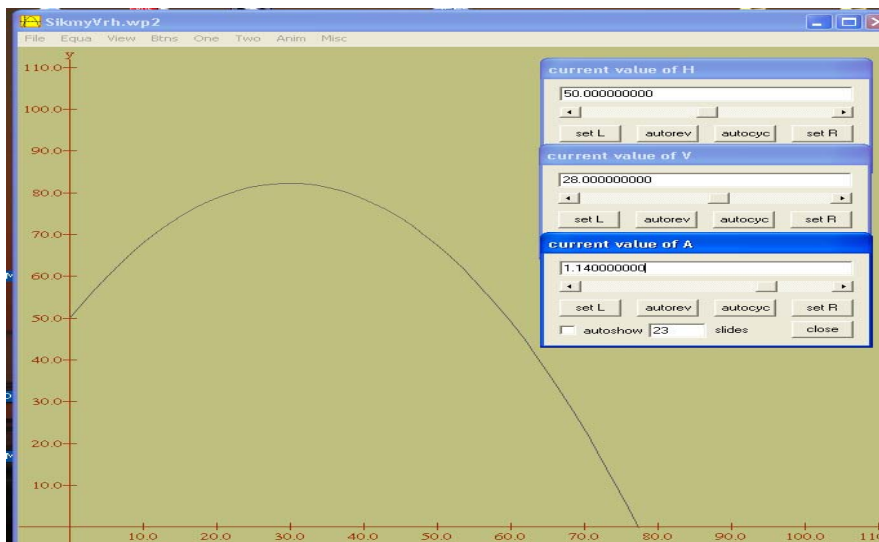V – Initial velocity
A – Elevation angle

**Figure 3 Trajectory of the rocket's motion made in WinPlot**

Solving of the „Fireworks"problem by the aid of graphic calculator (TI 83+)

To solve the problem, we can chart the graph of the quadratic function $h(t) = 50 + 25$ $t – 5t^2$, which represents functional dependence of the launched rocket height h on the time t, with the help of graphic calculator. The task is solved graphically [2].

In order to graph the quadratic function, firstly we have to insert its formula to the function editor **Y=** (a). We must also adapt the window editor to see the whole graph of the quadratic function (b). Than we can let the calculator draw the graph of the function (c).

(a)
(b)
(c)
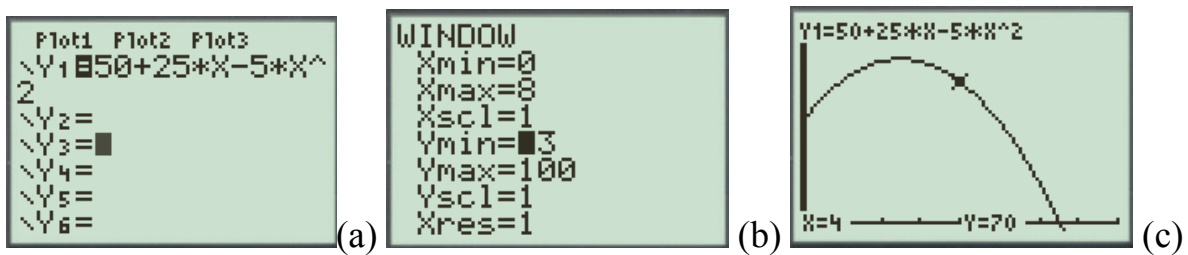
We can answer the question 2A) "What time does the rocket need to reach the top of its trajectory?" by finding the top, the highest point of the graph of quadratic function (it means to find a point where the function reaches its maximum). The calculator function 2nd [CALC] 4: maximum, enables us to count the maximum of the quadratic function h(t) with the corresponding value of t, so we get e.g. the maximum height **hmax = 81,25 m** and the time when the rocket reaches this height **t = 2,5 s**.
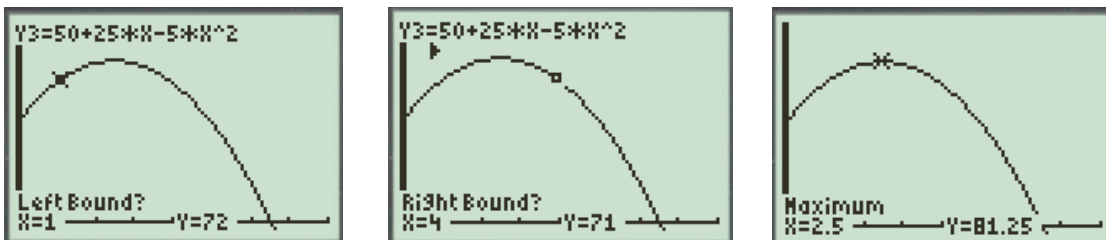
In the task 2B) we have to find the place (horizontal distance of the rocket from the tower), where the rocket reaches its maximum height. We can calculate this position simply by putting the obtained value **t = 2,5 s** into the equation *d(t)= 28* t*cos 65°*, so we receive: d(t) = 28*2,5*cos 65°. Therefore, the place where the rocket reaches the maximum height off the ground level is approximately **29,6** metres far from the tower.

The following calculation will answer the last question 2C): where should we look for the area (place, point) of the rocket's landing.
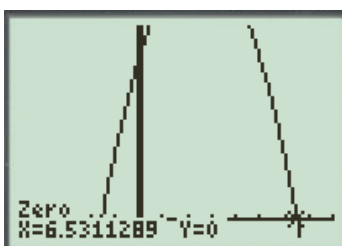
At first, we enumerate the time of rocket landing on the ground (it is one convenient positive root of the equation h(t)=0, or the intersection of the quadratic function graph with the x coordinate axe).

By this calculation, we obtain the time **t = 6,53 s** of rocket landing. Finally, we are able to take the time t = 6,53 s and insert it into the function pattern ***d(t)= 28*t*cos65º***. We acquire the distance of the rocket landing, **d = 77,28 m** far from the base of tower.

The graphs we have demonstrated by the aid of graphic calculator can offer the image and a lot of information about the rocket movement. However, they do not simulate the trajectory of this movement. For this purpose, it is better to use the interactive program made in WinPlot.

4 PROBLEM SOLVING WITH THE HELP OF SPREADSHEETS

During teaching, it is suggested to utilize a spreadsheet processor as a tool to model various possibilities that could occur and to analyze data. The spreadsheet processor and graphical processing of data can be used during education as tools to model and simulate the dynamic processes. These tools are known to the students as quite standard. By the application of spreadsheet programs' features, we can gain quantitative modeling tools, which are suitable for the use during elementary and high school education.

The spreadsheet programs allow us to use one of their important features – an ability to put calculations' results into the graphs [8]. An adequate example could be a modeling of mathematic functions x^2, sin(x), cos(x) or modeling of the angled-launched projectile.

Using the formulas for individual parameters, we put values of α (an angle), v_0 (initial velocity) and constant g (gravitational acceleration) into the cells with absolute addresses. Then we generate a table of calculations for the sequence of values of time parameter t. With the help of functions, we find the values of the maximal height and distance of fall.

Together with the students, we can experiment with the model by changing the angle of throw or initial speed and then observe how it affects a trajectory. The results are visually displayed on the graph.

For the "Fireworks" problem, we use David's equations d(t) = 28*t*cos 65° and h(t) = 50+25*t-5*t^2. We create a table of time values together with functions $h(t)$ and $d(t)$. Based on values we graphically represent relation of the time t to the height function h and distance function d. The students can use the graph to approximate the maximal values of height and fall together with the corresponding time moment. These values can be determined also by utilization of the spreadsheet calculator's function for the maximum.

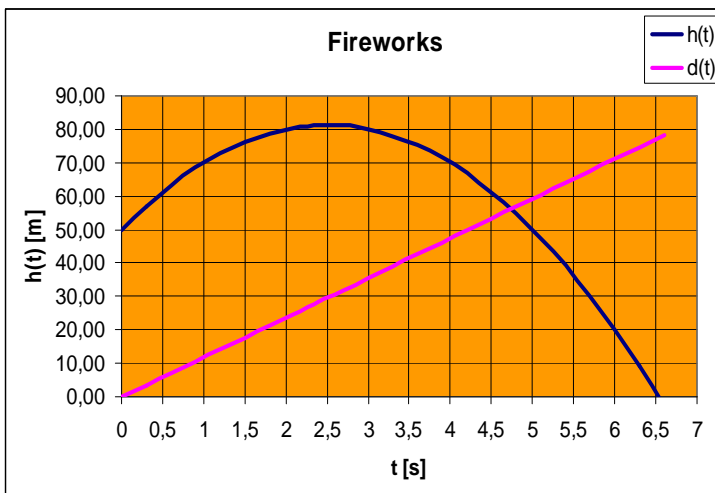| t [s]	h(t)	d(t)
0	50,00	0,00
0,3	57,05	3,55
0,6	63,20	7,10
0,9	68,45	10,65
1,2	72,80	14,20
1,5	76,25	17,75
1,8	78,80	21,30
2,1	80,45	24,85
2,4	81,20	28,40
2,5	81,25	29,58
2,7	81,05	31,95
3	80,00	35,50
3,3	78,05	39,05
3,6	75,20	42,60
3,9	71,45	46,15
4,2	66,80	49,70
4,5	61,25	53,25
4,8	54,80	56,80
5,1	47,45	60,35
5,4	39,20	63,90
5,7	30,05	67,45
6	20,00	71,00
6,3	9,05	74,55
6,5	1,25	76,92
6,6	-2,80	78,10

Table 1 Values of variables

Figure 4 Graphs of functions in the MS Excel program

A representation of the calculations' results by the column of numbers together with the graph allows students to get a deeper insight into the observed phenomenon. The advantages, which result from this kind of spreadsheet programs' utilization in education, are the following ones:

- complicated, repeating calculations are cut down to minimum
- more models of "what happens, if" type can be checked out
- models can be tested by the greater amount of data
- it's possible to graphically represent the examined relations

5 CONCLUSION

Creation and application of the models for the purpose of real world's phenomena demonstration is the subject of teaching process. These models take a significant part in application of didactic principles of science, demonstration and activity [9]. Scientific knowledge is related not only to the content of teaching process, but also it represents the method of its acquirement. Modeling and simulation of the systems, as a scientific method, helps students to gain new information by examining the various systems, based on their models [7].

The graphic calculator, software WinPlot and the spreadsheet processor could be the appropriate tools for the creation of visual and graphically high implemented animation models. A very important function of the models is an enhancement of visual demonstration. A purpose of this demonstration is to create the conclusive ideas for the student. At the age of 12 years, when students acquire an ability to accomplish the formal operations and to think abstractedly, it is desirable to arouse their visual feeling of abstract representations and descriptions of real processes and devices. Various symbolic models, such as diagrams or graphs, can also be used during the mathematics teaching.

A model used for the didactic purposes helps us to demonstrate and discover all the significant features of examined phenomenon. It is appropriate for students not only to get prepared models of the reality, but also to create some themselves. Thus they have to reproduce a structure of the model and to reveal all of its features. Consequently they can improve it or work it over. As a result, the students can learn in a more creative way. This approach creates an area for the use of educational software and tools, which gives us an opportunity to teach the students a given topic with the help of ICT.

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