THE USE OF MOTION SENSOR CAN LEAD THE STUDENTS TO UNDERSTANDING THE CARTESIAN GRAPH

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Abstract. *This paper shows the experimental results of a didactical lesson conducted in three classes of Upper Secondary School using motion sensor. It is an example of modelling practice, in which the students are involved in mathematics representations of real phenomena. Our research corroborates works about the use of MBL-tools, according to which the use of motion sensor allows the students to reading, understanding and interpreting kinematics graphs. Besides our analysis shows that the students acquire these competence respect to graphs of other type too. These results emerge from the implicative statistical analysis of the pre-test and the post-test and from the qualitative analysis of the lessons.*

Key words: *teaching, learning, Cartesian graph, motion sensor, modelling*

INTRODUCTION

This research work consists of the analysis of a didactical situation conducted in three classes of Upper Secondary School. The didactical activities were developed using a motion sensor to visualize, to understand and to interpret space-time and velocity-time graphs, representing moving bodies. Motion sensor is one of MBL-tools (Microcomputer Based Laboratory). In the late 1980's, these tools were produced by the project ''Tools for Scientific Thinking'' in the Center for Science and Mathematics Teaching at Tufts University. The central objective was to help students in order to recognize the connections between the physical world and the abstract principles presented in the classroom (Krusberg 2007). Motion sensor is used in Physics laboratory to study rectilinear motion of bodies moving in front of it.

Our research proves that not only the students improved reading, understanding and interpreting motion graphs but they also improved these graphing practices (Roth 2004 p.2) in other types of Cartesian graphs. We believe that this is an interesting result because learning mathematics means that a person acquires aspects of an intellectual practice, rather that just acquiring any information and skills (Roth 2004 p.7). These interdisciplinary activities give the opportunity to optimize available time in classroom and to increases the student's motivation.

We chose this argument of research because graphing practices are part of the mathematics curricula of all school levels. Moreover, they can become prerequisites for other mathematical subjects. For instance, Cartesian graph is one register of semiotic representation of a function. Besides, graphing practices are central to scientific communication and to the scientific enterprise more broadly (Roth 2004 p.2). Moreover graphing practices have many applications in everyday life as the comprehension of an economy graph printed on a newspaper, the understanding of a temperature graph hanged on a hospital bed, etc.

RESEARCH QUESTIONS

Research hypothesis: *Motion sensor is a learning tool to reading, understanding and interpreting kinematics graphs.*

Research questions:

1) *Using motion sensor to reading, understanding and interpreting kinematics graphs, do students learn to reading, understanding and interpreting other types of Cartesian graphs and, in particular, function graphs representing a statistical phenomenon?*
2) *How can modelling activities aid for the understanding of Cartesian graphs?*

THEORETICAL FRAMEWORK

MBL tools collect physical data and allow visualizing them in tables and Cartesian graphs in real time (Thornton & Sokoloff, 1990). So MBL tools can facilitate the comprehension of abstract representations of physics phenomena and can give long lived conceptual understanding (Bernhard, 2001). Besides collected data can be manipulated, analyzed and fitted, studying the characteristics of the phenomena and testing the relationships between the variables. The efficiency of motion sensor compared to traditional methods for helping students to learn basic kinematics concepts has been proved by several researches, as Thornton & Sokoloff (1990), Redish et all. (1997), Liljedahl (2002), Arzarello & Robutti (2004). Our research wants moreover to show that when the students are involved in activities with sensor motion they become able in graphing practices, not only in kinematics field.

The idea of using motion sensor to improve graphing practices finds strong theoretical support in the cognitive theories *of the Embodiment of the mind,* for which «the detailed nature of our bodies, of our brains, and of our daily functioning in the world structures human concepts and reasoning» (Lakoff & Núñez, 2005, p.27). So it's fundamental in this kind of activity as the students can visualize and analyze in real time the graphs of bodies. Beside according to *Metaphorical Thought* «for the most part, human being conceptualize abstract concepts in concrete terms, utilising ideas and models of reasoning founded on a sensor-motor system» (Lakoff & Núñez, 2005, p.27). Particularly «the functions on the Cartesian plane are often conceptualized in terms of motion on a route» (Lakoff & Núñez, 2005, p.70) and motion sensor induces this type of conceptualization as the students see the graph constructed under their own eyes as *motion of a point that leaves a wake*. It can be explained through a historical-epistemological analysis of the concept of function, which finds its origins in the ambit of kinematics and geometry.

This analysis shows that the representations of the function are: verbal, Cartesian, analytical and tabular (for numerical values). So the laboratory activity with sensor motion could be utilized as kinematics approach to the concept of function (Arzarello & Robutti 2004) because it allows studying all the representations of a kinematics function and to pass from one kind of representation to another. A representation cannot describe fully a mathematical construct and each representation has different advantages, using multiple representations for the same mathematical situation is at the core of mathematical understanding (Duval 2002). The representations of the function developed in different historical periods. Before tables of functions appeared (2000 B.C.), then geometrical representation (middle of the 14th C.) and later analytical form (17th C.) (Youschkevitch, 1976). Using motion sensor the chronological introduction of the representations of the function is respected (Piaget & Garcia, 1985). Besides it involves the students in a historical process that conducted to the function concept: modelling process. In fact it allows analyzing the motion of a body as a point in moving along a straight line respect to the reference point, studying all its mathematical representations (Gilbert, 1998).

In this activity the modelling is a transversal objective, reached by the study of other matters of the mathematics curriculum (Lingefjärd, 2006). Modelling practise can be a way to increase *thinkers*, who can use their mathematics for their own and for society's purposes (Burkhardt, 2006). To conclude we want to point out that motion sensor is an artefact. As referring to mathematical meanings it may be seen as «tool of semiotic mediation» (Bartolini Bussi & Mariotti 2008). The role of the teacher becomes fundamental in the use of this tool to reach the graphing practices.

Some didactical considerations

To clarify the connection between the graphing practices in motion graphs and in any Cartesian graphs, we made the following comparison between competences:

C	MATHEMATICS	PHYSICS
C1	Reading the coordinates of a point of the graph	Reading the values of a kinematics variable in relation to the values of the temporal variable
C2	Reading the extremes and the size of intervals	Reading space and time of departure and arrival, the covered space and the spent time
C3	Distinguishing among increase, decrease and constancy of a function	Distinguishing between motion of approach, motion of separation and still bodies
C4	Individuating absolute maximums and minimums of a function	Individuating absolute maximum and minimum distance with respect to the position reference system
C5	Individuating relative maximums and minimums of a function	Individuating relative maximum and minimum distance with respect to the position reference system
C6	Confronting the different degrees of rapidity of increase or decrease of tracts of a curve	Confronting the velocity of differing tracts of motion
C7	Forming hypothesis and conjecture	Forming inferences on experimental data

EXPERIMENTAL WORK AND RESEARCH METHOD

The experimental work consisted of two laboratorial lessons[1] of two hours each one. It was leaded in three Italian classes[2] of Upper Secondary School (43 students). It is a homogeneous sample because before the experimental work they possessed the same competences in graphing practices and necessary prerequisites for this activity:

- Knowing the real number field and representing them on a straight line
- Representing points on the Cartesian plane
- Knowing motion concept and kinematics variables

The research methodology adopted is *Theory of Didactic Situations* by Brousseau (Brousseau, 1997). The laboratorial lesson was preceded and followed by the administration of a test, with the aim of evaluating the a priori and a posteriori students' behaviours. We made the qualitative analysis of the didactical activities analyzing the teaching/learning process through the analysis of the involved semiotic register. It refers to *APC space and Semiotic Bundles* by Arzarello (Arzarello & Robutti 2008). We made also a quantitative analysis of tests through *Statistical Implicative Analysis* by Gras (Gras et all, 2008). Cause of limited space, in this paper we show only the main results of our analysis.

Statistical Implicative Analysis

It is a non-parametric statistic, so it uses small samples and it is appropriate for this kind of research. We use the method of *implication* that establish the implication intensity between variables and the method of *similarities*, that classifies variables and groups them according to hierarchical levels (similarities) (Gras et all, 2008). Data were analyzed by using C.H.I.C.[3] software that visualizes *implication graphs and similarity tree,* working on Excel tables. We studied the implication of the *students' behaviours variables* by tables like this:

	Behaviour 1	…	Behaviour *n*
Student 1			
…			
Student *m*			

The values of this table are 0 or 1, depending if a student doesn't follow or follows the behaviour that corresponds in the table respectively. We analyzed the similarity of the *students' variables* using the *supplementary variables* method (Spagnolo 2005), (Fazio & Spagnolo, 2008). Here we use the supplementary variables as models of

[1] Lessons was conducted by the teacher-researcher M. L. Lo Cicero in her curricular classes.

[2] 1. December 2007, 4th class of Classical Liceo (17 years), (*Liceo Classico "Scaduto", Bagheria (PA), Italy*)
2. April 2008, 2nd class of Commercial Technical Institute (15 years), (*"Jacopo del Duca", Cefalù (PA), Italy*)
3. May 2008, 4th class of Classical Liceo (17 years), (*Liceo Classico "Scaduto", Bagheria (PA), Italy*

[3] Classification Hiérarchique Implicative et Cohésitive. Information regarding the software can be found at the following site of the A.R.D.M. (Association de Recherche en Didactique des Mathématiques): http://www.ardm.asso.fr/CHIC.html

student's behaviour, so the outcomes of our research depend from the similarities of the *students* respect to the *correct models of students' behaviour*. The correct models of students' behaviour are selected by combination of the correct behaviours. To obtain the similarity trees we used tables like this, with binary values:

	Student 1	…	Student *m*	model of student's behaviour 1	…	model of student's behaviour *p*
Behaviour 1						
NOT behaviour 1						
…						
Behaviour *n*						
NOT behaviour *n*						

Phases of the didactical activity

The phases of the didactical activity were the following ones:

1. Prediction, reading and comprehension of the graphs of rectilinear student's motion of three types:
   a. Leaving motion from the sensor
   b. Approach motion to the sensor
   c. Still body with respect to the sensor
2. Prediction, reading and comprehension of various rectilinear student's motion, with leaving and approach with respect to the sensor.
3. Study of rectilinear uniform motions of a train on tracks.

During phase **1** the students made a reflection on the variables studied by the sensor. They observed and calculated space and time of departure and arrival, the length of space and the time spent. Not all the students immediately realized the relation between abscises and ordinates. After the study of the leaving motion the students correctly predicted the other types of graphs. In phase **2** the topics of the previous phase were consolidated for every piece of curve of leaving, approach or stilling. Also the maximum and minimum distance reached with respect to sensor was read. The students noted that the slope of every piece of curve depended on the corresponding velocity of the student. Then the students were asked to make a relationship between spatial intervals and temporal intervals about pieces of a curve and to make comparisons. Besides the students calculated the mean velocities and compared them and the observations about the slopes of the pieces of the curve with the graphs velocity-time. In the phase **3** they studied the analytical representation of a uniform rectilinear motion by the fit of the data. The students noted that this is a particular type of straight line equation.

After the laboratory activity, the students were involved in a metacognitive reflection about the development of the lesson. The students reconstructed the phases of the modelling process and reached the devolution of these processes (Brousseau, 1997). They realized that physics phenomena, belonging to everyday life, could be representable by mathematical representation. In particular, uniform rectilinear motion can be represented by algebraic equation, commonly studied in scholastic mathematics. So this modeling process was an occasion to realize that mathematics is a tool to read the existence of mathematics in our everyday life (Lingefjärd, 2006), (Kaiser & Schwarz, 2006). During the didactical activity it was noted that motion sensor induces curiosity and desire of learning in students. They were encouraged to experiment several typologies of motion to compare the graphics produced with their own predictions. It was noted that the process of prediction is important to acquire the skill of *forming hypothesis on the base of experimental data.*

Test

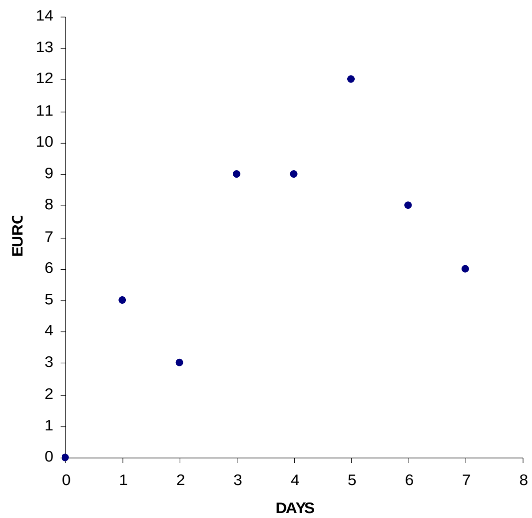
A test was administered before and after the laboratorial lesson. The students worked individually, they were not allowed consulting books or notes. They had sixty minutes to accomplish the task. The test contained items concerning reading and understanding of space-time graphs representing motion of bodies, contextualized in real life. So the students had to interpret models of kinematics phenomena. The students' improvements in kinematics graphical practises were remarkable, so they corroborated our research hypothesis. Besides the test contained the following exercise (*Sara's test)* concerning reading of not kinematics graph:

Sara's dad decided to reward his daughter every time she got a good grade at school by giving her five euro, which she could decide to spend or save as she pleased; but this would be her only source of income. The adjacent graph shows the money Sara possessed on each day of a week. Observe it and answer the following questions:

a. How many euros did Sara posses on the 4th day? (C1→COORD)

b. On which day/days did Sara surely get a good grade? (C5→R-MAX)

c. Knowing that on the second day Sara didn't get a good grade, how much money did she spend that day? (C2→INT)



d. Could she have gotten a good grade on the 6th day? (Justify your answer.) (C7→HP)

e. On which day/days did Sara possess the most money? (C4→A-MAX)

A priori analysis of students' behaviours of *Sara's test*

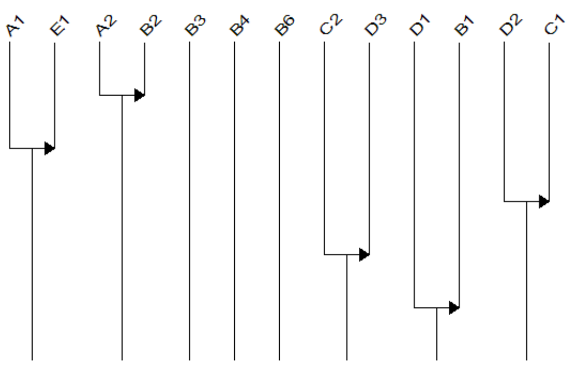
As it is indicated by *Theory of Didactic Situations*, we made an a priori analysis of students' behaviour in working out the test:

Q[4].	A.	BEHAVIOURS
a.	9	A1: Correct reading of the value of the ordinate in correspondence with abscissa
	26	A2: Sum of the euro that Sara possessed in the first four days. Interpretation of the graph like earned money.
b.	1,3,5	B1: Correct identification of the days corresponding to the relative maxima
	5	B2: Confusion between the concept of relative maximums and of absolute maxima
	1,3,4,5	B3: Writing, besides of the days which correspond to the relative maxima, also, of the 4th day, in which the euro remained constant
	3,4,5	B4: Writing of the highest values
	3	B5: Writing of the day corresponding to the major growth
	1,3,4,5,6,7	B6: Writing of all the days except the absolute minimum
	3,5	B7: Writing of the highest relative maxima
c.	2	C1: Correct identification of the width of the interval
	3	C2: Confusion between the concept of interval and of value of the coordinate. Wrongly interpretation of the graph like spent money
d.	Yes, …	D1: Affirmative answer to the question *d*, justifying with the affirmation "she could have spent the earned money": forming correct hypotheses on the base of experimental data
	No, …	D2: Negative answer to the question *d*, justifying with the affirmations "she spent 4 euro" or "her budget would have become 13 euro": not forming correct hypotheses on the base of experimental data
	Yes, …	D3: Affirmative answer to the question *d*, justifying with the affirmation "because she earned 8 euro": not forming correct hypotheses on the base of experimental data and wrongly interpretation of the graph like earned money
e.	5	E1: Correct identification of the absolute maximum
	3,4,5	E2: Writing of the highest values

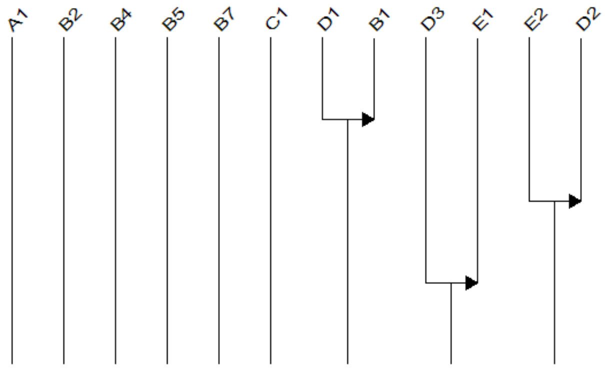
EXPERIMENTAL RESULTS AND CONCLUSIONS

We classified the behaviours of the students in tables. Using *Chic* software we obtained the following implicative graphs of the student's behaviours:

PRE-TEST



POST-TEST



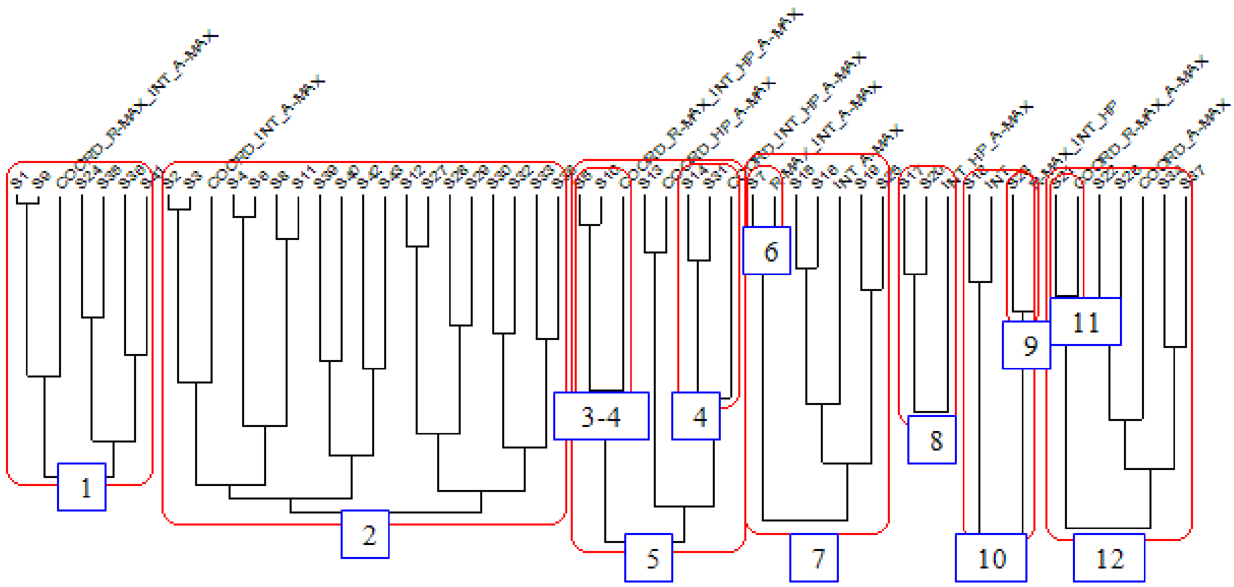
[4] Q=Questions. A= Students' Answers

In the implicative graph of the pre-test there is a strong implication of A2 towards B2: all the students that follow the behaviour A2 follow the behaviour B2 too. They represent two mistakes in reading of graph (reading of coordinates and relative maxima respectively). The implication A1→E1 inverts the expected implication between the reading of the coordinates and of the absolute maximum. It is due to the wrong interpretation of the graph like earned money in the answer *a*. D2 implicates C1 because the behaviour D2 includes the competence of reading of the width of intervals. The implication C2→D3 points out a wrong interpretation of the graph like spent money and earned money respectively. So the same students gave two wrong opposite interpretations of the graph. Since D1→B1, the students that form correct hypotheses on the base of experimental data are able to read relative maxima.

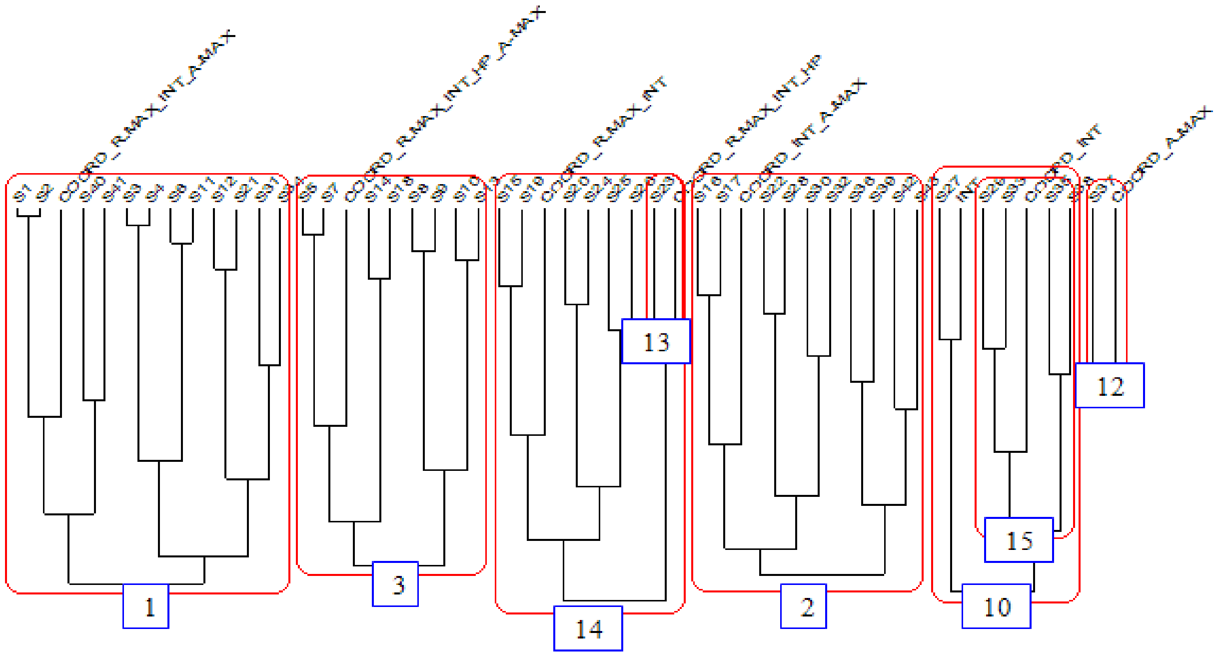
In the graph of the post test the implication D1→B1 is stronger than in the pre-test. Given that E2→D2, if the students don't read correctly the absolute maximum then they don't form correct hypotheses on the base of experimental data. Finally, since D3→E1, the students that don't form correct hypotheses on the base of experimental data and interpret the graph like earned money are however able to read absolute maximum.

We analysed the similarity of the variables s*tudent* respect to the variables *models of students' behaviour*. Below we report the graphs obtained by C.H.I.C.:

PRE-TEST



POST-TEST



In these graph we can observe the improvements of the competences of each students. The general improvements for each competence are:

	COORD	R-MAX	INT	HP	A-MAX
N° correct answers, pre-test	34	11	37	8	41
N° correct answers, post-test	43	27	42	9	31

Below we report a table extrapolated by the similarity trees. It shows the numbers of the students that possessed 5 or 4 or 3 or 2 or 1 competences in the pre and post-test.

	5 comp.	4 comp.	3 comp.	2 comp.	1 comp
N° stud, pre-test	2 (group 3)	8 (groups 1,4)	24 (groups 2,5,6,8,9)	8 (groups 7,12)	1 (group 10)
N° stud, post-test	8 (group 3)	13 (groups 1,13)	16 (groups 14, 2)	5 (groups 12,15)	1 (group 10)

In particular, in the similarity trees we note that the group n. 3, representing the students that possessed all the competences, is increased by 6 students in the post-test. The group n. 1, representing the students that possessed all the competences except the forming hypotheses, is increased by 6 students in the post-test.

Conclusions

The experimental results show that a laboratory activity with the use of motion sensor develops the competences of the students in reading, understanding and predicting of kinematics graph. This tool allows studying the steps of the modelling process of the phenomena *rectilinear motion* and to make metacognition reflection of their own learning. Modelling activities aid for the understanding of Cartesian graphs because they are the bridge between the real phenomena and the mathematical representations. According to the theory of *Embodiment* the students construct their knowledge observing the real phenomena and connecting it with its graphical and tabular representations. Our mind conceptualizes a function as a point that is moving on the plane and the use of motion sensor induces this kind of conceptualization. So, using motion sensor, the students acquire competences in reading, understanding and predicting Cartesians graphs not representing only a kinematics phenomena. In particular, our research shows improvements of the students in reading of the correspondence between abscises and ordinates, of maxima and width of intervals of a statistical function.

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