TOWARDS UNDERSTANDING TEACHERS' BELIEFS AND AFFECTS ABOUT MATHEMATICAL MODELLING

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Work in progress on a framework aiming at capturing teachers' beliefs about mathematical models and modelling is presented. It is suggested that the belief structure of mathematical models and modelling as perceived by teachers fruitfully might be explored as partly constituted of the teachers' beliefs about the real world, the nature of mathematics, school mathematics, and applying and applications of mathematics. Some aspects of the suggested framework are explored using two case study interviews. It is found that the two teachers do not have any well formed beliefs about mathematical models and modelling, and that the interpreted beliefs structure of the teachers contain inconsistencies which are made explicit within the framework. The empiric findings also suggest some modifications of the framework.

INTRODUCTION

Since the mid 1960s gradually more emphasis has been put on mathematical modelling in the written curricula documents governing the content in Swedish upper secondary mathematics courses (Ärlebäck, in preparation). In the latest formulation from 2000, *using and working with mathematical models and modelling* is put forward as one of the four important aspects of the subject that, together with *problem solving*, *communication* and *the history of mathematical ideas*, should permeate all teaching (Skolverket, 2000). Indeed, it is stressed that "[a]n important part of solving problems is designing and using mathematical models" and that one of the goals to aim for is to "develop their [the students'] ability to design, fine-tune and use mathematical models, as well as critically assess the conditions, opportunities and limitations of different models" (Skolverket, 2000). However, as noted by Lingefjärd (2006), "it seems that the more mathematical modeling is pointed out as an important competence to obtain for each student in the Swedish school system, the vaguer the label becomes" (p. 96). The question naturally arises what mathematical models and modelling are and mean for the different actors in the Swedish educational system. Ärlebäck (in preparation) concluded that the governing curricula documents, the *intended curriculum* (Robitaille et al., 1993), do not give a very precise description of the what a mathematical model or mathematical modelling is, but rather describe the concepts in an implicit manner as exemplified above. Therefore, focus is turned to teachers who interpret and realize the intended curriculum, and thereby have a big impact on which mathematical content and what view of mathematics students in classrooms are exposed to. One way to try to understand part of the process of what ends up in the classroom, the *(potentially) implemented curriculum* (ibid.), is provided by studying teachers' beliefs.

The question of how teachers' knowledge, beliefs and affects towards the learning and teaching of mathematics influence and relate to their practice is a highly active field of research (Philipp, 2007). Thompson, acknowledging the dialectic nature between beliefs and practice, argues that "[t]here is support in the literature for the claim that beliefs influence classroom practice; teachers' beliefs appear to act as filters through which teachers interpret and ascribe meanings to their experience as they interact with children and the subject matter" (Thompson, 1992, p. 138-139). Indeed, the six authors of the chapters on teachers' beliefs in the book edited by Leder, Pehkonen and Törner (Leder, Pehkonen, & Törner, 2002) all infer a strong link between teachers' belief and their practice, working from a premise that could be expressed by "to understand teaching from teachers' perspectives we have to understand the beliefs with which they define their work" (Nespor, cited in Thompson, 1992, p.129). In particular in connection with mathematical modelling, while discussing four different categories of mathematical beliefs, Kaiser (2006) concluded that depending on the mathematical beliefs held by a teacher, it is more or less likely that they build up obstacles for introducing applications and modelling in their mathematics teaching. Furthermore, Kaiser and Maaß (2007) looking at "what are the mathematical beliefs of teachers towards applications and modelling tasks?" (p. 104), found that for the group they studied, applications and modelling did not play a significant role in their beliefs about mathematics and mathematics teaching. The investigated teachers rather created/modified and adapted application-oriented beliefs in line with their existing mathematical beliefs.

In a research project aiming to design, implement and evaluate sequences of lessons exposing students to mathematical modelling in line with the present governing curricula carried out in collaboration with two upper secondary teachers, initial individual interviews was held with the participating teachers. The purpose being first to provide information about the teachers' background and their views and beliefs on the nature of mathematics, about their teaching, views on problem solving and mathematical modelling, as well as their opinion for the reasons and aims for mathematical education. Secondly, the interviews also intended to end up in a common understanding and agreement of key concepts among the researcher and the two teachers, laying the foundation for the collaboration project. The aim of this paper is partly theoretical in that we seek to develop a framework trying to capture and conceptualize beliefs about mathematical models and modelling and relate these to other types of beliefs studied in the literature. Nevertheless, it also aims to provide background about the two teachers participating in the research mentioned above and hence to feed in to the bigger analysis of that project.

BELIEFS, BELIEF STRUCTURES AND BELIEF SYSTEMS

Reviews on research on different aspects of beliefs in connection to mathematics knowing, teaching and learning often conclude that there is a great degree of variation of the involved concepts and their meaning used by different scholars (Leder et al., 2002; Pajares, 1992; Philipp, 2007; Thompson, 1992). The motive with the following small theoretical exposé is to establish the vocabulary used in the paper and to relate some of different concepts used in the literature.

As a point for theoretical departure we start from the work, and use the vocabulary, of Goldin (2002), who defines beliefs as one out of four "subdomains of affective representation[s]" (p. 61), distinguishing between *emotions*, *attitudes*, *beliefs*, and *values*, *ethics and morals*. More specifically, beliefs are "multiply-encoded cognitive/affective configurations, usually including (but not limited to) prepositional encoding, to which the holder attributes some kind of *truth value*" (p. 64, emphasis in original). For an individual, a collection of mutually reinforcing or supporting non-contradictory beliefs taken together with the individual's justifications for this constitutes a *belief structure*. Törner (2002) argues that beliefs generally are about something and introduces the notion of this something as a *belief object*, to which a set of beliefs, the *content set* is associated, which can be seen as the analogue of Goldins' beliefs structures. Other scholars often refer to similar constructs as *belief systems* or *cluster of beliefs*, but in Goldins' framework, a *belief system* is an "elaborated or extensive belief structure that is socially or culturally shared" (Goldin, 2002, p. 64). This terminology makes it easy to talk about and distinguish between beliefs held by an individual contra shared beliefs within a community, as well as the dialectic and tension between these types of beliefs.

Many authors deepen their discussion on beliefs drawing on Rokeach (1968) or Green (1971), or a combination of the two, introducing different *dimensions* of beliefs. Rokeach talks about a *dimension of centrality* for the individual, where a *central belief* is a belief which is non-contradicting within a persons' belief structure, whereas beliefs with some disagreeing features are less central for the individual. Green on the other hand introduces the construct of *psychological centrality* and uses *peripheral* and *central* to describe beliefs that the individual holds more or less strongly. Both Rokeach and Green argue that the more central a belief is, the harder it is to change it. Green also talks about *quasi-logicalness*, which captures the fact that some beliefs only are in consensus within a belief structure provided that a non-standard and personal logical explanation is provided. In connection to quasi-logicalness Green also proposed to differentiate *primary* beliefs from *derivative* believes. Returning to Goldins' framework of beliefs, part of the dimensions above are captured by the notion of *weakly*- or *strongly-held* beliefs. The two factors determining to what strength a belief is held are *importance for the individual of the belief being true* and the *degree of certainty the truth-value of the belief is attributed*.

MATHEMATICAL MODELLING

The literature on the aims, use and results of different approaches to incorporate and use mathematical modelling in the teaching of mathematics has steadily been growing since the beginning of the 1980s. The theoretical perspectives invoked display a great variety (Kaiser & Sriraman, 2006) as does the research methods used to explore this vast field of research; see for examples the recent 14th ICMI study (Blum, Galbraith, Henn, & Niss, 2007) and the published proceedings from ICTMA 12 (Haines, Galbraith, Blum, & Khan, 2007).

Mathematical modelling is often perceived as a multistep or cyclic problem solving process using mathematics to deal with real world phenomena. The student or modeller is supposed to use his mathematical modelling skills or modelling competencies (Maaß, 2006) to work through the steps, stages, phases or activities of the process. In this paper mathematical modelling refers to the complex and cyclic-in-nature problem solving process described for instance by Blum, Galbraith & Niss (2007), here illustrated in figure 1.

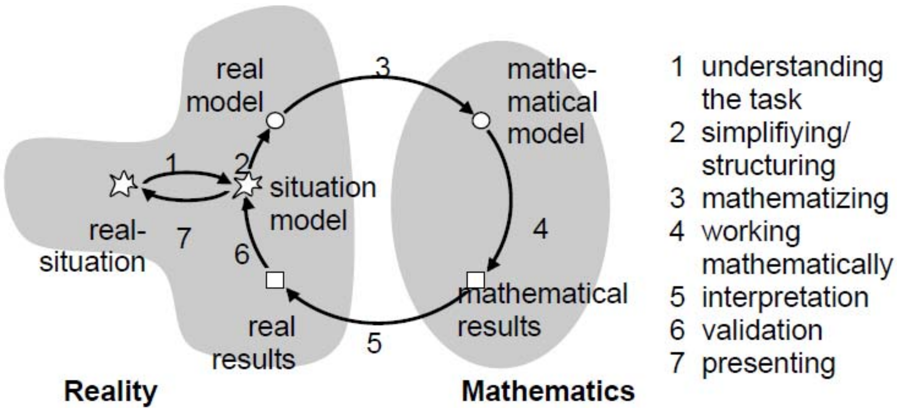

Figure 1. The modelling cycle from Borromeo Ferri (2006, p. 87)

It should be noted that this is only a schematic, idealised and simplified picture of the modelling process. For instance, in an authentic modelling situation the modeller normally jumps between the different stages/activities in a more non-cyclic, but rather unsystematic, manner (Ärlebäck & Bergsten, 2007).

A SUGGESTED BELIEF STRUCTURE OF SOME ASPECTS OF MATEMATICAL MODELLING

In setting out to investigate teachers' beliefs about mathematical models and modelling it is important to be explicit and specific about what object the beliefs should be about. Using the terminology of Törner (2002), the *belief object* under study in this paper is defined to be *mathematical models and modelling as perceived by upper secondary mathematics teachers*. For clarification we stress that the focus at this stage in the research process is not on the teachers' beliefs of the teaching and learning of mathematical models and modelling.

The literature review suggests the importance and influence on teachers' practice of their beliefs about mathematics and its teaching and learning. Hence, the validity of the framework suggested here steams both from analyzing the view taken on mathematical modelling in this paper and from research on mathematical beliefs of various sorts. A teachers' belief structure of mathematical models and modelling is suggested to be constituted of the beliefs of the following *(sub-)belief objects*:

Beliefs about **the nature of mathematics**. This is without question the most general of the constituting sub-belief objects, assumed to serve as a primary and central belief in the belief structure of modelling. The perspective taken on the nature of mathematics might radically change the interpretation and meaningfulness of fig. 1.

Beliefs about the **real world (reality)**. In our view, it is important that the problems used in connection with modelling to the greatest extent possible be from real problem situations in the real world. Different views, both philosophical and pragmatic, potentially influence the way one might think about mathematical modelling and models. In addition, how reality is perceived, especially in contrast to the nature of mathematics, can make a difference when it comes to the interpretation and validation of ones' modelling work. In fig. 1, beliefs about the real world might especially influence the phases 1, 2, 5, 6 and 7.

Beliefs about **problem solving**. In principle, depending on perspective, modelling is about problem solving or problem solving is about modelling (see Lesh & Zawojewski, 2007 for an overview). Regardless of which view adopted, the meaning of and role played by problem solving as a mathematical activity, seen as part of one's practise of one's mathematical knowledge and skill/competence might have important implications for how mathematical modelling and models are perceived. In connection to fig. 1, (mathematical) problem solving beliefs are important for the phases 3, 4 and 5.

Beliefs about **school mathematics**. Thompson (1992) concluded that the consistency between teachers' beliefs about the nature of mathematics and beliefs about the subject mathematics taught at schools are of varying magnitudes. Therefore, school mathematics beliefs are incorporated in the bigger belief structure to capture the potential influences they might have on other beliefs of the teachers.

Beliefs about **applying, and applications of, mathematics**. The application of mathematics is sometime synonymous with different views taken on modelling, and hence it is important to include beliefs about applying and applications of mathematics in the belief structure of mathematical models and modelling. Depending on point of view, beliefs about applications of mathematics are significant for phases 3 and/or 5 in fig. 1.

The five categories of beliefs above are suggested to constitute a way of describing the belief structure of mathematical models and modelling. This framework is initially based on the indicated links to the modelling cycle and will need empirical investigations to be further developed and validated.

This framework does not set up isolated beliefs but, by the discussion above, these beliefs are rather overlapping belief structures in themselves. Hence, an indication of the validity of the framework would be that the substructures display inner coherence, that is, display an inner quasi-logical structure. However, it is possible that taken all together as constituting the belief structure of mathematical models and modelling, incoherencies appear and then the question is which beliefs are more central, primary, and in line with official guidelines.

SOME EMPIRICAL FINDINGS

Although the empirical data used here was not collected primarily with the testing of the above framework in mind, due to its focus on teachers' views on mathematical modelling, we see it as relevant for discussing the viability and usefulness of the framework. As a result, it may also point out directions for how to develop it further.

Method

The interviews with the two teachers (here called Lisa and Sven) in the projects briefly described in the introduction were partly structured around five mathematical problems to serve as a basis for the discussion and reflection. Three of these were standard text problems from a widely used textbook in Sweden, one the so called *Fermi Problem* studied in (Ärlebäck & Bergsten, 2007), and one was *The Volleyball Problem*, a so called *modelling-eliciting activity*, described in (Lesh & Doerr, 2003). The interviews were recorded, transcribed and analysed using what may be called a contextual sensitive categorization scheme based on the five sub-beliefs object in mind. Due to the nature of the data, beliefs about the real world and applications and applying mathematics surfaced only sporadically and can therefore not be fully accounted for here. To economize with respect to writing space, the accounts of the teachers' beliefs are here given mostly in narrative form.

Lisa

Lisa, 36 years old, has been an upper secondary teacher in mathematics and physics for 13 years and is now working in her second school going on her 5th year. She teaches on a 70% basis and the other 30% she spend on administration, marketing and teacher education networking. She became a mathematics teacher because it seemed to make a lot of fun and as far back she can remember she always enjoyed doing and thinking about mathematics.

Beliefs about **the nature of mathematics**: Lisa talks about mathematics as a *tool* and something that *develops and strengthens ones' thinking* (logic). She connects mathematics to *structuring* and *organizing*, and a number of times talks about *geometrical pattern, forms and shapes in nature* and *mathematics as an art form*.

Beliefs about the **real world**: Lisa's comments in the interview seem to imply that the most prominent consequences of working on real problems are that the numbers occurring in the calculation are messy and that the calculations should be preformed and answered using better accuracy (more decimals).

Beliefs about **problem solving**: For Lisa problem solving is about *solving puzzles* and she associates feelings of satisfaction and happiness with the success of solving a hard problem. Problem solving is for Lisa something that preferably takes place in a technological environment with free access to every source of information possible. She also stresses the importance for the problem context to be familiar to the students.

Beliefs about **school mathematics**: Lisa repeatedly states the importance for school mathematics to be experienced as an entity, *a well defined course*, but also comments on the written governing curricula documents as theoretically formulated and hard to understand both for students and teachers. Lisa regretfully confess that some areas of mathematics (such as ordinary differential equations) only are taught as a set of procedures and recipes although the areas really have a great potential for making the subject more interesting and intriguing.

Lisa's direct talk about **mathematical modelling**: When asked about mathematical models and modelling, Lisa first seems to have a clear conception of what this means; without any time for consideration she says: *"Well, it might be a whole lot of things… a mathematical model… it might be that you describe a course of events or situation, or really just to make an assumption is a mathematical model, although a very simple one"*. Then she retreats and only considers a made connection/relation to constitute the model, not an equation or an algebraic representation of the relationship, but changes her opinion on this and clarifies that a mathematical model does not have to be expressed in mathematical terms. Rather, it should be the need of the situation that decides which degree of mathematization to use. The goal however, she continues, should always be a formulation of the model using mathematical symbols and ways of writing. Lisa also draws parallels between modelling and generalizing, and gives numerous of examples of what she considers to be different types of models when discussing the problems. She considers all five problems except The Volleyball Problem to be about, and include different aspects of, modelling.

Sven

Sven, 58 years old, has been teaching mathematics and physics (and computer science and chemistry) at the upper secondary level for 33 years and has been working at four different schools and last changed workplace in 1981. He teaches on a 60% basis and plans/manages the school schedules the rest of his working hours. It was mere coincidence that Sven became a teacher, following his personal fascination of mathematics, which led to physics and later also to teacher education.

Beliefs about **the nature of mathematics**: Sven describes mathematics as a *pure, exact and axiomatic science*, enabling to *part right from wrong*. It is about *logic*, the *relations between different quantities*, and it has a central *aesthetic component*. He emphasises that *"knowledge of the tools open up for the realization of the beauty"*.

Beliefs about **problem solving**: Sven talks about mathematical problem solving as an *exercise for the intellect*, as something decoupled form other subjects and contexts. When discussing the problems he carefully places them in a syllabus context; where, when, and how the topics touches in the problems are treated within the course.

Beliefs about **school mathematics**: When talking about school mathematics Sven expresses the importance to *learn to think logically* and to *prepare for learning in other subjects as well for higher education*. He thinks the aesthetic side of mathematics is something only a few students can appreciate and hence it plays only a minor role in the classroom.

Beliefs about **applying, and applications of, mathematics**: For Sven, application of mathematics is *"a tool used in other sciences; physics, chemistry and economics"*.

Sven's direct talk about **mathematical modelling**: When asked to describe mathematical models and modelling Sven answers, *"Yes, well… no, I don't know…"*

turning to the five problems and try to use them helping him to form and formulate his perception of mathematical models and modelling. To begin with Sven talks about a model as something to use solving problem, a tool, but elaborates his thinking further: *"I think it [a mathematical model] is something you create… in a more or less obvious manner…and there can be more than one model to use to solve a given problem."* Sven then describes different ways of working with a model; creating, using, and exploring it. He also strongly connects making assumptions and modelling, and considers all five problems used in the interviews as related to modelling. Sven also mentions that it is important for the students to learn to use and apply mathematics.

Discussion and conclusion

Although Lisa initially seemed to have a clear conception of mathematical models and modelling, it became clear throughout the interview that this was not the case. She rather, like Sven, had to make up and formulate her views as the interview went on. One explanation why neither of them had a clear conception of modelling might be the vague formulations found in the curriculum documents that provide no support and only circumstantial guidance. However, since they volunteered to participate in a research project about mathematical modelling, one could suspect that they had been doing some thinking about the project, and thus had some firm ideas about the central concepts. If they had, this was nothing that surfaced during the interviews. However, when talking about mathematical modelling, directly or indirectly during the interviews, the different categories of beliefs in the framework are touched on, as described above.

No flaws in the quasi-logic holding together the different sub-beliefs structure where detected in neither teacher's sub-beliefs structures. Sven for instance expressed the school mathematical belief that it is important for the students to learn to use and apply mathematics, and professed a similar belief about the application of mathematics. Lisa, when discussing The Volleyball Problem, on the other hand, strongly rejected it as a modelling problem since *"it is more about comparing advantages and disadvantages, structuring and organizing [than modelling]"*. This is in conflict with her beliefs about the nature of mathematics and a direct contradiction to what she said previously in the interview. One possible way to interpret this is that Lisa strongly held conflicting primary beliefs about the nature of mathematics on one hand, and mathematical modelling on the other.

Although the data was not initially collected for the testing of the suggested framework, the analysis indicates that it may be useful for exploiting beliefs about mathematical models and modelling, other professed beliefs, and relations between them. However, a thing to consider is to follow up the point made by Thompson (1992, p. 130-131), who lists a number of studies in mathematics education indicating the important impact teachers' beliefs about mathematics on the one hand, and about teaching of mathematics on the other, have on their practice. Including the teachers' beliefs on the learning and teaching of mathematics in general, and mathematical models and modelling in particular, seems to be the next logical step. A perhaps as urgent dimension to add to the framework is to include more actively affective considerations, which Goldins' (2002) framework make possible.

If indeed beliefs can be seen as filters influencing the teachers' practice, it is important to try to get a better understanding of beliefs about mathematical models and modelling if we want teachers to integrate it more in their mathematics teaching. Kaiser (2006) concluded that "beliefs concerning mathematics must be regarded as essential reasons for the low realisation of application and modelling in mathematics teaching" (p. 399), and we believe, like (Törner, 2002, p. 80), that higher consciousness about one's beliefs lead to a higher degree of integration of the beliefs in ones' practice. A question that we feel needs priority is how beliefs are formed.

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