MODELLING IN MATHEMATICS' TEACHERS' PROFESSIONAL DEVELOPMENT

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One of the chapters of the new Dutch handbook of didactics of mathematics, which is currently being written by a team of didacticians, concerns mathematical modelling. This handbook aims at (further) professional development of mathematics teachers in upper secondary education. In this paper we report about the issues we included: dispositions about modelling, goals, designing aspects, testing, the role of domain knowledge, and computer modelling. We also reflect on the relationship between mathematics, teaching of mathematics and modelling, and on the role of modelling in the Dutch mathematics curriculum.

INTRODUCTION

In this paper we describe how the subject of mathematical modelling is treated in the new Dutch handbook of didactics of mathematics, which is to appear within the next few years. The intended audience of the handbook consists of students in teachers' colleges as well as mathematics' teachers in upper secondary education who want to learn about teaching modelling as part of their professional development. We try to bridge the gap between educational research and teaching practice by bringing together results, scattered about the literature, thus making them accessible to (future) teachers. We highlight those topics which our post graduate courses for teachers have shown to be most urgent for their practical needs.

Many maths teachers are not familiar with modelling or do not want to spend time on modelling in math' class. Therefore we first address the question what modelling is (not) about and why it should be included in the mathematics curriculum. Next, we cover briefly some essential issues concerning the teaching of modelling.

We focus on the non-mathematical aspects of mathematical modelling, since the didactics of the necessary mathematics is dealt with in other chapters of the handbook. Furthermore, experience with professional development courses for teachers shows that these non-mathematical aspects of modelling deserve very careful consideration as they are often ignored.

We restrict ourselves to the question of how to apply known mathematics to non-mathematical problems. In particular, we do not discuss modelling as a tool to learn mathematics.

MODELLING, MATHEMATICS AND TEACHING OF MATHEMATICS

Goal of modelling

As mentioned above, we restrict ourselves to the application of mathematics using mathematical models to non-mathematical problems. Looking through the eyes of a scientist, it is our goal to understand the relations between the variables of our context. Mathematics is an important tool to achieve this goal. Scientists use mathematical models to experiment with variables and possible relations between them and answer specific questions, such as: Which percentage of Rhine water ends up in the ecologically important and sensitive Waddenzee? Of course, everyday life can also be a source of interesting problems, for example: Does it pay out to drive across the border to fill up the car? Students tend to that think models are copies of reality (Sins, 2006). It is important that they learn that models are made to answer specific questions and that the same context can lead to completely different models, depending on the question. A steal ball can be modelled as a point mass or a sphere or a conductor or a lattice or a free electron gas, depending on the question to be answered.

Modelling cycle

We describe the modelling process by a simple version of the modelling cycle. We start with a problem, which is to be solved using tools from mathematics. In the first stage the problem is described in terms of relevant non-mathematical concepts. During this stage one typically has to make some choices about (simplifying) assumptions. The result of this stage is a *conceptual model.* This conceptual model is then translated into a *mathematical model,* which can be analyzed mathematically. The actual translation of the conceptual model and the original question into mathematics may also be subject to certain choices. Next, the mathematical solution is translated back into the context and language of the original problem. We call this *interpretation*. Finally, one *validates* the solution. If necessary, one starts the modelling cycle all over again, adapting one or more of the steps.

Role of mathematics in modelling

The role of mathematics in modelling can vary considerably. It can be elementary or advanced. Sometimes computers are needed to aid mathematical analysis. The mathematics may involve calculus, algebra, geometry, combinatorics or some other field. The modelling problem can be well-defined with clear-cut data, a specific question, a standard mathematical model and ditto solution. In such problems mathematics and context science merge into a very potent mixture. The interplay between mathematics and context is then especially fruitful with techniques like dimensional analysis, where mathematical algebra is applied to physical units. The famous theoretical physicist Wigner was quite right when he spoke about "the unreasonable effectiveness of mathematics"! Conversely, a physical concept like velocity can be helpful to learn a mathematical concept like the derivative.

There may be several possible models and it is not always clear *a priori* which one serves our purpose best. If one doesn't have a complete theory describing the relevant phenomena, one usually fills the gaps by posing simple (e.g. linear) relations. For such models validation is a main point of concern. Most models are not built up from scratch anyway, but emerge as refinements and combinations of existing models.

Applications of mathematics in maths education

Mathematics started as an applied science, dealing with practical problems in trading, measurement, navigation, etcetera. The separation of theoretical mathematics from the empirical sciences is a relatively recent phenomenon, brought about by the development of non-euclidean geometry around 1800. In the middle of the nineteenth century mathematical education followed this trend and its focus shifted from applications to logical reasoning. Since then, the emphasis has swung back and forth between pure and applied mathematics (Niss, Blum & Galbraith, 2007).

Mathematics' education should pay attention to both sides of mathematics. However, many students consider mathematics as a theoretical, abstract subject, which hasn't much to do with reality (Greer, Verschaffel & Mukhopadhyay 2007). They have a blind spot for applied mathematics and the role of mathematics in the sciences or daily life. If students never learn how to apply mathematics, then their mathematical knowledge is indeed useless. Furthermore, it is counterproductive if common sense, intuition and reality are not used to aid mathematical understanding.

Modelling and the Dutch mathematics teaching programs

Non-mathematical contexts have played an important role in parts of Dutch mathematics education since 1985. Since 1998 all mathematics programs for secondary education involve modelling. The experiment which preceded the introduction of the new program indicated that assessment of open modelling tasks was a major problem and was avoided by many teachers. The modelling tasks in the national exams, too, paid little attention to conceptualization, interpretation and validation (De Lange, 1995). To counteract this deficit, the Freudenthal Institute in Utrecht started organizing modelling competitions for schools where these aspects *do* play an essential role.

All these efforts have partially paid off: PISA shows that Dutch students perform well on modelling related tasks. On the other hand, Wijers & Hoogland (1995) and De Haan & Wijers (2000) mention in their evaluation reports of the above mentioned modelling competitions that many students' papers lack in mathematical substance. Students tend to neglect relevant concepts and work by trial and error. Sins (2006) also laments the lack of conceptual thinking and understanding of the purpose of modelling. Future maths education should address these weaknesses more effectively.

PROFESSIONAL DEVELOPMENT

Ongoing professional development is obligatory by Dutch law since 2006. Since many maths teachers in upper secondary education have only scant knowledge of applications of mathematics, post graduate courses for teachers should fill this gap.

We use Schoenfeld's description of complex tasks like modeling (Schoenfeld, 2008, based on his work on problem solving 1985 and 1992). The essence of this framework is as follows. Anyone who takes up a complex task like mathematical modelling starts with certain *knowledge* (not only mathematical knowledge like facts, algorithms, skills, heuristics, but also domain knowledge), *aims* and *attitudes* (opinions, prejudices, preferences). Parts of these are activated, one makes *decisions* (consciously or not, depending on one's familiarity with the problem), one adjusts aims and designs a plan. During the execution of the plan one *monitors* the progress on several levels, going back and forth between the stages of the modelling cycle. Metacognition thus plays an important role in modelling.

We address the issues of aims and attitudes in the sections *Goals, Authenticity, Dispositions* and *Epistemological understanding*. Knowledge aspects are dealt with in the sections *Domain knowledge*, *Authenticity*, and *Computer modeling*. We conclude with a discussion of decisions and monitoring in *Monitoring* and *Assessment*.

Goals of teaching modelling

Modelling isn't easy. It takes a lot of time and is difficult to assess (Galbraith, 2007a) and (Vos, 2007). So why should we take up modelling in mathematics education? First, students have to learn how to apply mathematics, to prepare them for their further education and their jobs, as well as for everyday life. (It might improve their understanding of mathematics as well.) Modelling can help to achieve this (Niss, Blum & Galbraith, 2007). Second, modelling shows that mathematics is useful to scientists as well as practical problems solvers. Third, modelling is useful for students to make their picture of mathematics more complete: it is not a set of ancient, irrelevant algorithms, but an interesting, important, creative, still developing part of science, society and culture (Blum & Niss, 1991). Finally, modelling may help to counteract naïve conceptions like the illusion of linearity (De Bock, Verschaffel & Janssens, 1999; Greer & Verschaffel, 2007).

Authenticity

According to Galbraith (2007b): "Goals and authenticity are in practice inseparable, as the degree to which a task or problem meets the purposes for which it is designed is a measure of its validity from that perspective." Palm (2007) also emphasizes the importance of authenticity. He describes an experiment where two different tasks are distributed randomly among 160 Swedish school children. Mathematically, the tasks are identical: to determine how many busses are needed if 360 students have to be transported and each bus can hold 48 students. One version consisted of just this question, the other was much wordier, paying attention to other aspects of the school trip as well. The second, more authentic version was solved correctly by 95% of the students, whereas the first version was solved correctly by only 75% of the students! Greer & Verschaffel (2007) and Bonotto (2007) also describe how lack of authenticity can hamper students to use common sense in maths class. Authenticity is also beneficial for motivating students. Lingefjärd (2006) found that students are interested in problems concerning health, sports, environment and climate. Van Rens (2005) showed that mimicking scientific research practice in the class room, including writing papers and peer review, enhances motivation and improves the quality of the students' work.

Dispositions about modelling

Abstraction and generalization belong to the core business of mathematicians. Model building, on the other hand, depends critically on the characteristics of the context and the specific research question. This tension (Bonotto, 2007) between mathematics and modelling makes many maths teachers and students feel uncomfortable (Kaiser & Maass, 2007). In their opinion there is no place for modelling in the mathematics curriculum, which should be devoted to "proper" mathematics. We know, however, that even students with solid mathematical knowledge are not necessarily able to use this knowledge outside mathematics (Niss, Blum & Galbraith, 2007). In the minds of many students and teachers there is no connection between the subjects taught during maths class and the topics taught next door by the physics or economics teacher. We are not just talking about superficial problems like different notations, conventions or terminology, but also about deeply rooted opinions about mathematics and reality.

Greer, Verschaffel & Mukhopadhyay (2007) argue that students are trained to expect that problems in maths class are always solvable, that solutions are unique and that reality can be ignored. Students even think that using non-mathematical knowledge is forbidden (Bonotto, 2007). As Schwarzkopf (2007, 209-210) put it:

> The students do not follow the logic of problem solving, but they follow the logic of classroom culture.

This obviously impedes successful modelling in teaching of mathematics.

Understanding what modelling is about is strongly related to dispositions about modelling (Sins, 2006). He distinguishes between three levels. At the lowest level a model is considered a copy of reality. Students at the intermediate level understand that models are simplified representations of reality constructed with a specific goal. Different goals may lead to different models. At the highest level attention shifts towards theory building: Models are constructed to develop and test ideas. Sins experiments show that a higher level of epistemological understanding leads to better models. Students at the highest level use their domain knowledge to analyze the relevant variables and the relations between them. Most students, however, are at the middle level. They try to reproduce measurement data by varying the parameters one by one. They ignore domain knowledge, reason superficially and consequently produce poor models.

Epistemological understanding

Sins (2006) investigated the influence of epistemological understanding of modelling on the quality of models made by students. He advizes to make the goals of a modelling task explicit: what do we want to *understand* or which problem do we want to solve? He proposes that the teacher presents reasonable models to his students who have to analyze and improve them. This way students learn about the tentative nature of models: They are not perfect copies of reality, since they often depend on choices, approximations and incomplete information. Furthermore, this adjusting of existing models and iteration of the modelling cycle gives a fairer picture of the modelling process as performed by experts, who of course have lots of standard models at their disposal and rarely start from scratch.

It is not sufficient to just talk about modelling with students. Indeed, students who model themselves perform significantly better on modelling skills such as using various data, recognizing the limits of applicability of a model and adjusting models (Legé, 2007). However, even if students have a sound epistemological understanding of modelling, in very open modelling tasks they still do not always understand what is given, what is asked and how to attack the problem.

Domain knowledge

Modelling typically concerns extra-mathematical contexts. As a consequence, the maths teacher may find himself in an awkward position, since he cannot be an expert in all possible modelling domains, such as the natural sciences, computer science, economics, arts, sports or other specific (not necessarily scientific) contexts.

The same holds for students. We know, however, that lack of domain knowledge leads to poor models (Sins, 2006). So it is essential to choose a modelling context where students' lack of domain knowledge is not an issue. Furthermore, the teacher has to encourage the students to actually use their domain knowledge. Finally, the teacher has to be familiar with the modelling problem himself. In particular, he has to be aware that a problem can lead to several different models.

Computer modelling

Computers can be useful to in modelling, especially when the mathematics gets complicated. Using a graphic modelling tool it is easy to modify a model, run simulations and display the results graphically. The representation of a model in such a tool reminds one of a concept map in the sense that it indicates the relevant variables and the relations between them.

In Löhner (2005), who summarized claims and results from the literature on computer modelling, we find that computer simulations make validation and adaption of models very natural. It facilitates exploring the limits of validity of a model.

Unfortunately, it also facilitates the superficial ad hoc modifications and data fitting behaviour Sins (2006) warns against. Löhner (2005) finds that students who work with computer models over a longer period of time tend to start working in a *top down* fashion and develop a more mature, qualitative attitude towards modelling, although one shouldn't expect too much in this direction. Simulation results may lead students to new research questions. Computer modelling is challenging and motivating for students, as long as the models are not too complicated and the software is easy to use. It also helps to turn abstract, theoretical models into something more concrete, which makes it easier to discuss these models. Finally, experimenting using computer modelling helps students to understand and remember the phenomena and associated theory.

Monitoring

Monitoring the modelling process of a group of students can be very difficult. Different students make different and often implicit assumptions and simplifications, have different goals and use different data and notations. This makes monitoring the modelling process of a group of students very difficult if not virtually impossible (Doerr, 2007). It is thus very important to force students to make all of the above explicit. The teacher can make life easier by inserting go-or-no-go-moments at certain points of the modelling cycle. However, even if everything is written down neatly, it can still be difficult for teachers and students to compare different modelling results. Are the differences due to different conceptualization or to mathematical errors? This problem can be moderated by discussing and comparing the various conceptual models with the whole group. Monitoring becomes much simpler if consensus is reached about the data, the goal and notations. This also facilitates understanding and comparing the different results, which in turn improves motivation and understanding (Van Rens, 2005; Bonotto, 2007).

If modelling is new to students it is advisable to have them record their modelling process in a pre-structured log. In this log they have to describe all data, assumptions, etcetera. The log can also be very useful for assessment.

Assessment of modelling

One of the main obstacles when teaching modelling is evaluation. The goals of modelling can not be assessed as objectively as is customary in education of mathematics (De Lange, 1987). Maths teachers who take the non-mathematical aspects of modelling seriously have to come to terms with this lack of objectivity. To reduce the subjectivity one can use a team of assessors **(**Antonius, 2007; Vos 2002; Vos 2007**)** and weighted lists of evaluation criteria. One can search for *rubrics* on the internet and adapt them to the assessment at hand. One can use the modelling cycle to generate evaluation criteria: conceptualization (analysis of the original problem, data, relevant concepts, data, variables, relations, simplifications, modelling goal), mathematization, mathematical analysis (completeness, correctness), interpretation, validation, conclusions, adaptions. Other criteria which are mentioned by experienced assessors of modelling are general impression, readability, representation and originality. A common pitfall is to overestimate appearance, so it remains necessary to study and evaluate *thoroughly* the technical contents of students' work (De Haan & Wijers, 2000).

De Lange (1987) argued that traditional written tests are not suited very well to test higher skills like modelling. He mentions several alternatives, which may be more appropriate, like group work, home work, essays or oral examinations. Vos (2007) argues, however, that alternative tests like observation, interviews and portfolio's are often too time consuming and too subjective. She investigated experimentally how teamwork can indeed reduce subjectivity. Furthermore, she shows how alternative, laboratory like tests using manipulative materials can lead to valid assessment of modelling skills. These results are confirmed by Antonius (2007), who adds, however, that this kind of assessment levels out the differences between strong and weak students.

Above we emphasized the importance for teachers of taming excessive divergence for monitoring the modelling process. Similarly, assessment is facilitated by posing authentic "convergent" modelling tasks (Niss, 2001):

> Mathematical modelling involves the posing of genuine, non-rhetorical questions to which clear and specific answers are to be sought.

CONCLUSIONS

To prepare teachers for mathematical modelling teachers' colleges have to take into account (apart from the necessary mathematics and their didactics) the lessons learned from literature about the role and goals of modelling in science and mathematics education, the modelling cycle, dispositions, authenticity, epistemological understanding, domain knowledge, computer modelling, monitoring and assessment. Unfortunately, empirical research on modelling education is mostly restricted to short term teaching experiments. To design effective modelling education it is necessary to gain more experience and to systematically carry out *longitudinal* research into the effects of teaching modelling.

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