# CERME 6 – WORKING GROUP 10
## MATHEMATICAL CURRICULUM AND PRACTICE

## TABLE OF CONTENTS

# INTRODUCTION

# FROM A STUDY OF TEACHING PRACTICES TO ISSUES IN TEACHER EDUCATION

Leonor Santos (Portugal)

José Carrillo (Spain)

Alena Hospesova (Czech Republic)

Maha Abboud-Blanchard (France)

Group 10 is particularly interested in theoretical, methodological, empirical or developmental papers on issues concerning teachers' practices, professional knowledge and teacher education. Several themes are possible to be discussed, such as teachers' beliefs, teachers' activity, the role of the teacher in the classroom, professional knowledge, professional development, strategies for teacher education, and links between theory and practice, research and teaching, and teacher education and collaborative research.

This group received 57 proposals (48 for papers and 7 for posters). Each proposal was reviewed by the leader of the group and two authors, in general including one of the others co-leaders. Some proposals were immediately accepted (8 papers, 3 posters), others were asked some revisions (31 papers, 4 posters) and 9 proposals for papers were recommended to be transformed into posters. Fifty five authors from 19 nationalities participated in the sessions of the working group during the conference, through the presentation of 35 papers and 5 posters, all of them accepted to be included in the proceedings.

All the papers and posters have been grouped in different topics that constituted five panels. Each panel began with short presentations (5 minutes each), where the authors presented their paper contributions to the topic and posed three questions (maximum) to be dealt with in the working groups and the further discussion. This first part ended with a comment related with all the presentations (10 minutes), made by a previous invited participant of the working group. Afterwards, a discussion part took place. In general, this discussion had a first moment in small groups and a second one with the whole group.

The organisation of the sessions was highly valued by the participants, as well as the atmosphere. Nevertheless, due to the high number of presentations, the time for discussion was sometimes less than desirable. The group leader presented a different way to organize the working group for the future (some panels may occur in parallel), if the participation maintains so high, informing in advance the distribution of the papers in the different panels. One participant suggested that each author would be in a different small group permitting that the work in that group focuses on that author's paper. It has been also proposed a possible change in respect of presentations: the participants would present other participant's paper. We didn't get to any final agreement on this last proposition.

## Panels

We present the emerging issues and ideas that rose during the different panels.

Panel I: **Mathematical curriculum and practice**

- Is it possible a renewal of the curriculum, which implies changes in the teacher's style of work into the class, without any external stimulus (working at school in group, consulting only textbooks, even with the help of some experienced teachers)? If yes, what conditions are necessary at schools, and more widely in the social context?
- How can one develop a new curriculum in a mode that integrates top-down and bottom-up approaches?
- There is a specific role for mathematics educator, but which one and when? And for research?
- How does curriculum management influence students' learning of mathematics?
- Is the study of teachers' efficacy meaningful without taking into account the teachers' views about mathematics?
- What is the incidence and availability of such research, at international level? Can we think about common research on any topic in Europe without taking into account cultural and social differences among the countries?

Panel II: **Professional knowledge**

There are uses of similar, but different terms, within the notion of professional knowledge: knowledge base for teaching; pedagogical content knowledge; competence: disciplinary, didactic, and relational; subject didactical competence; practical knowledge (beliefs and knowledge)

- How can one present mathematics for the teachers to contribute to the development of their pedagogical content knowledge?
- What tasks can we use to diagnose the (students) teachers' subject matter knowledge (its possible weakness)?
- How can one change teachers' conceptions on mathematical communication (as information trasmission) through a collaborative work (eg. centered on teachers' reflection on their own practice)?
- How can one promote lasting classroom culture among teachers, one of its focus being the discussion of students' (right or wrong) strategies?

Panel III: **Professional development**

As for primary teachers, also for secondary teachers, mathematical content knowledge and pedagogical content knowledge must be interrelated in teacher education (having a mathematics degree isn't enough to understand the mathematics to teach).

• Professional development is about becoming autonomous and critical at designing and conducting classroom teaching. How do teachers develop professionally? In particular, what is the role of:

- theory (listening to lectures, reading papers, discussing issues, …)?

- practice (appropriating ideas from the practice of others, transforming ideas from his/her own practice)?

- reflection (reflecting on what? how? with what purpose?...)?

• How is it possible that groups of teachers develop towards a real learning (inquiry) community? What kind of impulses do they need?

• Which role could/should researchers/teachers' educators play in such professional development (taking account of their experience in international projects, in research studies, in the use of supporting tools of analysis…)

• How is it possible to promote real changes in the beliefs and the teaching practices of in-service teachers?

- How can we measure the sustainability of this professional development?

- What is the impact (if any) of the changes on the mathematical experience and learning of pupils?

• Co-learning is a means to promote professional development. But how to combine the expertise of teachers and that of mathematics educators/researchers in a way that can be useful to the two partners?

Panel IV: **Approaching reflection and collaboration in mathematics teachers' professional development**

Collaborating is not just sitting or working together and reflecting is not just thinking about or thinking aloud. Content and depth of reflection are determinant. Reflection is a privileged way for professional enhancement. Collaboration is a mean for professional development and for research strategy.

• What strategies, settings and content can we design to promote reflection and collaboration amongst teachers and between teachers and researchers in order to achieve a real professional development?

- How can we categorise data, statements, and phenomena? And why?

- What data should be analysed to measure the improvement of teaching via (joint) reflection?

Panel V: **Models to analyse the practice**

The practice of teachers includes classroom teaching, as well as training and other professional development contexts, …There are different examples of models to analyse the practice, such as: focusing on teachers' cognitions; focusing on interactions in a collaborative environment (bottom-up); and focusing on teachers' use of curriculum materials, textbook in particular.

• Enquiring into teachers' beliefs about teaching and learning mathematics through *focus* groups:

- What other uses might the focus group interview have in teacher education/teaching development?

- What are the special techniques for managing a focus group interview?

• How can we manage to make research results and instruments useful for teachers as means in their professional development, and for educators in training contexts?

## PAPERS

### Panel I: **Mathematical curriculum and practice**

Hellmig, L. *Effective blended professional development for teacher of mathematics: Design and evaluation of the "UPOLA" Program*.

Isler, I. & Cakiroglu, E. *Teachers' efficacy beliefs and perceptions regarding the implementation of new primary mathematics curriculum*.

Nunes, C. & Ponte, J. *Curriculum management in the context of a mathematics subject group.*

### Panel II: **Professional knowledge**

Andrá, C. *Gestures and styles of communication: are they intertwined?*

Doritou, M. & Gray, E. *Teachers' subject knowledge: the number line representation*.

Guerreiro, A. & Serrazina, L. *Communication as social interaction. Primary School Teacher Practices*.

Hosson, C.; Vandebrouck, F. & Robert, A. *Experimental devices in mathematics and physics standards in lower and upper secondary school, and their consequences in teacher's practices*.

Joubert, M. *et al*. *Professional development for teachers of mathematics: opportunities and change*.

Kattou, M. *et al*. *Teachers' perception about infinity: a process or an object*.

Kontoyianni, K. *et al*. *Perceptions on teaching the mathematically gifted*.

Larguier, M. & Bronner, A. *Nature on the numbers in second: A professional problem.*

Malara, N. & Tortora, R. *A European project for professional development of teachers through a research based methodology: The questions arisen at the international level, the Italian contribution, the knot of the teacher-researcher identity*.

Moscucci, M. *Why is there not enough fuss about effects and meta-effects among mathematics teachers?*

Murphy, C. *The role of subject knowledge in Primary Student teachers' approaches to teaching the topic of area*.

Reinup, R. *Developing of mathematics teachers' community: five groups, five different conceptions*

Rowland, T. *Foundation knowledge for teaching: contrasting elementary and secondary mathematics.*

Schwarz, B. & Kaiser, G. *Results of comparative study of future teachers from Australia, Germany and Hong Kong with regard to competences in argumentation and proof.*

Turner, F. *Kate's conceptions of mathematics teaching: Influences in the first three years*.

Ubuz, B. *et al*. *Pre-service teacher-generated analogies for function concepts*.

## Panel III: **Professional development**

Abboud-Blanchard, M. *Technology and mathematics teaching practices*.

Alatorre, S. & Saíz, M. *Teachers and triangle*.

Mgombelo, J. & Buteau, C. *Mathematics teacher education research and practice: researching inside the MICA program*.

Soto-Andrade, J. *Cognitive transformation in professional development: some case studies*.

Stehlíková, S. *What do student teachers attend to?*

Stylianides, G. & Stylianides, A. *The mathematical preparation of teachers: A focus on tasks*.

Tichá, M & Hospesová, A. *Problem posing and development of pedagogical content knowledge in pre-service teacher training*.

Zehetmeier, S. *Sustainability of professional development*.

## Panel IV: **Approaching reflection and collaboration in mathematics teachers' professional development**

Martinho, M. H. & Ponte, J. *A collaborative project as a learning opportunity for teachers*.

Matins, C. & Santos, L. *Reflection on Practice: content and depth*.

Pesci, A. *Developing mathematics teachers' education through personal reflection and collaborative inquiry: which kinds of tasks?*

Witterholt, M. & Goedhart, M.*The learning of mathematics teachers working in peer group*.

## Panel V: **Models to analyse the practice**

Kleve, B. *Use of focus groups interviews in mathematics educational research*.

Muñoz-Catalán, M. C.; Carrillo, J. & Climent, N. *Analyses of interaction in a collaborative context of professional development*.

Petrou, M. *Adapting the knowledge quarter in the Cypriot mathematics classroom*.

Ribeiro, C., Monteiro, R. & Carrillo, J. *Professional knowledge in an improvisation episode: the importance of a cognitive model*.

# EFFECTIVE 'BLENDED' PROFESSIONAL DEVELOPMENT FOR TEACHERS OF MATHEMATICS:

# DESIGN AND EVALUATION OF THE "UPOLA"-PROGRAM

Lutz Hellmig

University of Rostock, Germany

*The paper describes the implementation and evaluation of UPOLA, a one-year-long blended learning professional development (PD) program for teachers of mathematics. The use of polyvalent tasks in classes as the main issue of UPOLA proved to be appropriate to support changes in classroom practice. Based on a short overview of the concept of polyvalent tasks, a description of the design of the blended professional program is given by considering multiple dimensions of 'blending'. The evaluation of the program shows a shift in participants' perception over the time from rather environmental variables towards the impact of UPOLA for teachers' acting and students' learning. Furthermore, some findings on the implementation of web-based communication and collaboration are presented.*

Keywords: Professional Development, Blended Learning, Co-Operation, Evaluation, Polyvalent Tasks

## INTRODUCTION

The current practice of teachers' PD in Germany is predominantly a set of single events of limited time, with little impact on teachers' classroom activity and students' learning. Given the current situation in the field of PD of practicing teachers, a lack of effective, job-embedded PD for teachers can be observed (Sowder, 2007). Limited-time events, rarely longer than a single day, are the current practice of teachers' further education in Germany. The impact of most of these lectures, meetings, or workshops is weak, since they do not affect teachers' behavior and students' learning. A detailed analysis of the present state is given by Jäger and Bodensohn (2007).

According to Loucks-Horsley (2003) and Guskey (2000) PD should be an ongoing, intended and systemic process. However, there is no clarity about attributes of effective PD. A comparative study by Guskey (2003) shows that "[…] most of the identified characteristics [are] inconsistent and often contradictory" (Guskey, 2003, p. 4). Overall, implementing peer-cooperation and collaborative activities are frequently named as key features to ensure changes in classroom practice (i.e. Garet, Porter, Desimone, Birman, & Yoon, 2001; McGraw, Arbaugh, Lynch, & Brown, 2003).

Following Jäger and Bodensohn (2007), a successful PD-program has to consider the specific needs of participating teachers. Inside-differentiation in heterogenic classes is one of the most evident general issues for PD of teachers of mathematics (Jäger & Bodensohn, 2007). In the German province of Mecklenburg-Western Pomerania, where heterogenic classes in grade 5 and 6 have been established since 2006 in opposite to the common trinomial school system, teachers identify a higher need for differentiation especially in their classes.

UPOLA, which means "Teaching by using Polyvalent Tasks" *(in German: “Unterrichten mit POLyvalenten Aufgaben”),* focuses both on offering an appropriate topic (polyvalent tasks) to meet the needs of teachers and on a holistic blended approach for the design of PD. To adjust the ongoing program and to identify its strengths and weaknesses, evaluation on multiple stages was an essential part of the program.

## POLYVALENT TASKS – AN ISSUE OF PROFESSIONAL DEVELOPMENT

According to the idea of "Open-Ended Approach" (Becker & Shimada, 1997), Sill and Hellmig (2008) defined the concept of "polyvalent math tasks". A mathematical task is polyvalent, related to a group of students, if (1) every student is probably able to find a solution, and (2) the task has a set of solutions on different levels according to the use of mathematical skills. These attributes distinguish a relative small set of polyvalent tasks from a broad range of general open tasks. Thus, polyvalent tasks are highly appropriate to meet the needs of differentiation.

Asserting the benefits of these tasks requires an apposite style of teaching, which is different from the general practice in Germany. Hellmig *et al*. (2007) suggested a time-ratio of about 50% to 50% for two phases of implementing polyvalent tasks in classroom: First the students are asked to find answers to the task individually, by cooperating in pairs or in small groups. During the second phase students present their solutions. The teacher encourages less successful students to show their ideas first; further other students are asked to present different solutions with a higher degree of complexity. The aim of this phase is to develop a culture of communication about mathematics in classes. The course material (Hellmig *et al*., 2007), provided to every participant in the program, described the characteristics of these tasks, their use in classes, and contained a collection of 70 tasks for grade 5 and 6 students.

The use of polyvalent tasks in classroom supports the idea of openness, communication and cooperation. To take the mentioned ideas into teachers' practice, the design of the program itself is dedicated to these characteristics.

## DESIGN OF THE PROJECT – A BLENDED APPROACH

### General considerations

"All learning is blended learning." (Oliver & Trigwell, 2005, p. 20) Designing PD is always a blend of different goals, contents, and methods. Inspired by Cross (2006) the author sees a complementary interaction on several dimensions of PD with the main dimensions (1) instruction/construction, (2) presence/ distance, (3) individual/collaborative learning, (4) content/experience focus, (5) "traditional" media/e-learning. Regarding these dimensions, the project UPOLA was blended through

- Leading the course by two moderators; one with theoretical background, the other with more practical background.
- Giving content-related input (during meetings) and constructing knowledge by the participants through activity, reflection and discussion.
- Combining individual learning by teaching and reflecting with collaborative learning. This included discussions on didactical issues and about lessons, which were taught by the participants, as well as joint planning of lessons.
- Using a guideline linked to the curriculum during the school year and self-directed teaching, reporting and discussing.
- Meetings "off the job" and phases of experience and reflection "on the job".
- Using traditional channels and web-based environments to communicate.

A factor for transferring the topics of PD into classrooms is engaging more than one teacher per school. Transfer is influenced by organizational support of principals and acceptance by staff members of a school (Guskey, 2000; Krainer, 2002; Loucks-Horsley, 2003; Gräsel, Fussangel, & Parchmann, 2006). Thus, every teacher in grade 5 of the participating schools has been invited to attend the program. We assumed that a vast amount of fruitful peer communication and co-operation during PD could affect the growth of the local professional communities of the participating schools.

**Implementation of UPOLA in 2007/2008**

After a pilot study in 2006/2007, "UPOLA" was put into practice in 2007/2008. We grouped 44 teachers of grade 5 classes of Mecklenburg-Western Pomerania and Berlin into five courses. These courses were integrated in "Mathematics Done Differently", an initiative for PD of teachers of mathematics. A key feature of the programs in "Mathematics Done Differently" was the moderation by a tandem of a school- and a university-teacher (Rösken & Törner, 2008).

We combined four meetings "off the job" between August 2007 and May 2008 with three phases of PD "on the job"; each segment lasted 8-12 weeks in duration. This combination of presence and distance learning supports co-operative and collaborative work, associated with social interaction and flexible time management, which is important for preventing high drop outs (Lynch & Dembo, 2004; Nash, 2005; Picciano, 2006). A valuable list of factors for blended PD-programs was given by Wideman, Owston, and Sinitskaya (2007). We used the learning-management-system (LMS) "moodle" for online communication.

**Meetings**

The meetings mostly took place at the participating schools, the workplace of the attendants. We ensured a suitable atmosphere for the meetings, offered refreshments and agreed on an informal style to communicate with each other, even between participants and facilitators. Typically, a meeting started with a structured group interview as a review on the recent period of work, which often turned into a spirited discussion. The review ended by writing a collective summary. Second, a facilitator linked selected theoretical topics to the issue of polyvalent tasks and encouraged a discussion. Finally, participants selected a concerted task for the next on-the-job-phase and outlined first thoughts on teaching with the chosen task. Each meeting closed with a short written feedback on two open questions. A substantial amount of time of the first two meetings was spent for introducing the LMS "moodle" and the characteristics of asynchronous communication.

### Phases of experience and asynchronous communication

During an "on-the-job-phase", the attendants planned and conducted a lesson about the chosen polyvalent task. They were asked (1) to report and reflect upon their own lesson, (2) to comment on the reports of their peers, and (3) to discuss different teaching approaches with polyvalent tasks by using moodle.

For setting up the LMS we had to consider the skills and the attitudes of the attendants towards information technology. A certain number of teachers felt uneasy and tried to avoid the use of computers; some of the participants had to struggle with technical issues and deficient skills along the entire course. Hence we designed the structure of the moodle-course to be as clear and simple as possible into a general block and three topic-blocks, each for one on-the-job-phase. The main activity of each topic block was a discussion board for reporting everyone's experience in teaching polyvalent tasks and to discuss about didactical issues. Beyond that, we provided additional material such as manuals (i.e. how to write a report) and files of course-related content.

## EVALUATION

Success of PD depends both on content and design. Hence, the evaluation followed two main questions: (1) Are polyvalent tasks appropriate to address a broad range of students with different skills and encourage communication about mathematics in class?, (2) How far is this kind of blended learning applicable for teachers' PD and what sort of items can increase the outcome of the program? In this paper, we put our attention to the second question.

### Methodology

Guskey (2000) describes a model of evaluating teachers' PD that comprises five stages. We utilized this model, and gathered data for (1) participants' reactions, (2) participants' learning, (3) organizational support and change, (4) participants' use of new knowledge and skills, and (5) student learning outcomes. The author subclassified the second stage into (2a) process, and (2b) results of participants' learning.

Determined by our blended view of professional development, we had to separate two points of view from each other. On the one hand, we examined five courses in their entirety with certain attributes to find general correlations. On the other hand, we had to regard the participants as individual learners and teachers by case studies.

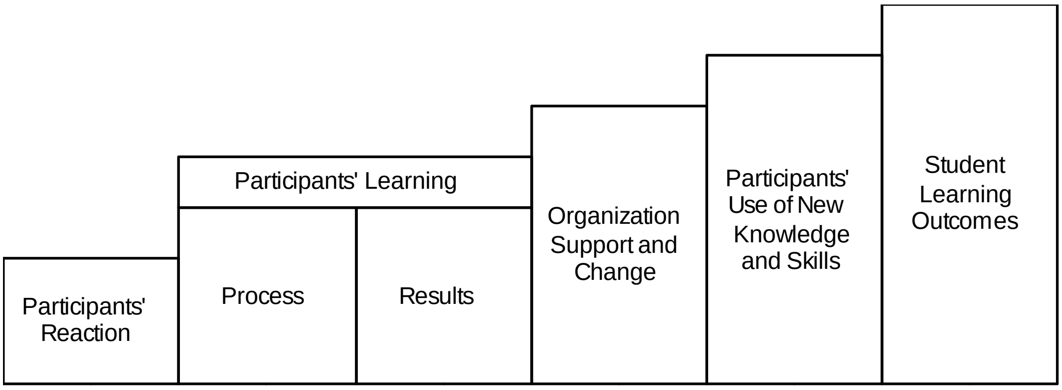



**Figure 1: 5 Stages of Evaluation adopted from Guskey (2000)**

Use of different means for evaluation was necessary to gain reliable data. The most important means were different questionnaires, interviews with teachers and principals, classroom observations, and monitoring discussion groups by quantitative and qualitative criteria. Finally, a modified method of the Repertory Grid interviewing technique (Collet & Bruder, 2006) was employed to capture the system of participants' personal constructs regarding math tasks before and after the course. Reflective reports and discussions during every face-to-face-session delivered very rich and useful "soft" data to get insights in participants' learning. The variety of tools for evaluation generated two separate sets of data: a set of personalized data, gathered by interviews, online- and face-to-face-discussions, and sampled classroom observations; and a set of anonymous data, collected by surveys and Repertory Grid. On the one hand, it was not possible to avoid getting some personalized data of the participants; on the other hand, protection of privacy is a precondition to get objective and reliable responses by participants. Three examination papers about the influence of polyvalent tasks on grade-5-students with different abilities were written.

Focusing on the use of the LMS, we analysed the number of insights in documents hosted on moodle, and quantitative and qualitative parameters of discussion threads. First, we simply counted the number of postings by every participant, differentiated by opening a thread and giving reactions to a posting. To rate the vitality of the discussion, we defined a scale for grading every thread. Beginning with the lowest degree we distinguished (1) posting by the moderator without a reaction, (2) posting by a participant without a reaction, (3) posting and one answer (one by the moderator) (4) posting and one answer without commitment of the moderator, (5) discussion (at least one posting regarding an answer) between a participant and the moderator, and (6) discussion without participation of the moderator. Furthermore, we viewed the dates of the postings to assess the continuity of participation. An analysis of qualitative variables (i.e. use of new terminology, deepness of reflection) complemented the observation of web-based communication. We compared these data with additional attributes, such as group-size, schedule of school-year activities and holidays.

Additionally, we could compare online activity of the participants with their contribution to the "off-the-job-meetings", and in some cases by observing classroom-activities concerned with the implementation of the subject.

UPOLA, as a part of "Mathematics Done Differently", was also evaluated externally by the Centre for Educational Research (zepf), University of Koblenz-Landau. Since that external evaluation was designed for one-day-events of PD, the usability of these data and the comparability with our self-evaluated data was limited.

## Findings

The description of the findings of the evaluation is grouped according to Guskey’s (2000) five stages of evaluation.

On stage 1, participants' reactions, participants appreciated the open and informal atmosphere of the meetings with possibilities to share experience with facilitators and colleagues. They reported about the importance of face-to-face-communication, many felt more comfortable to participate verbally rather than by online-written contributions. Participants attended the meetings regularly; we rated a small drop out (4 of 48) as an indicator of general satisfaction.

On stage 2, participants' learning, we observed that participants shared their individual approach to implement polyvalent tasks in profound discussions. We saw the quality of these discussions as a demonstration of increasing knowledge of participants. Frequently we heard that participants would rather communicate face-to-face than by using a discussion board.

In general, the use of the LMS for asynchronous communication felt short of our expectations. Although we defined a common and clear task for each experience phase, the number of postings by many participants did not match our demands. Most of the discussion-"threads" were only reports without a response by other participants. In some cases, participants received responses, but discussions developed rarely. We can confirm that the group size is an influential factor for the activity and intensity of discussion. Like Caspi, Gorski and Chajut (2003) and Wideman *et al*. (2007) we saw a better performance of courses with ten participants or more. The participants did not contribute postings continuously. First of all, the majority of the postings were written within the last two weeks before the meetings. This is critical regarding to the aim of developing discussions. Furthermore, we placed meetings into the last week before holidays. As a result, stimuli and motivation given during the meetings, faded out immediately due to the holidays.

To keep the attention of participants, daily alerts of ongoing activities had an influence on the activity of participants. Components of the LMS without delivering alerts (downloadable materials as well as some discussion groups) received measurably less attention or responses from participants. Since reading e-mails was not a daily routine for some participants, facilitators had to contact and motivate some teachers by using additional channels of communication, i.e. by making phone calls.

Participants started to reflect about their lessons just by giving an overview about different approaches of the students to solve polyvalent tasks. By continuing the program many of the attendants included thoughts concerned with planning or reflecting about their lessons.

Evaluating higher levels (stages 3-5 of Guskey’s model) of the impact of UPOLA has to regard the conditions of the attendants' workplace in addition to the program. Our research underlines the findings reported by Beaudoin (2002), who reported that a lack of online activity does not implicate a lack of adopting knowledge by participants. Observations of lessons of the UPOLA-project showed that in some cases teachers demonstrated sophisticated skills in teaching with polyvalent tasks, however, they gave no or very few reports to the discussion. Other participants admitted that they did benefit from ideas and experience of others, but hesitated to give themselves a reflection about their own work.

Finding relationships between teachers' PD and students' outcome is crucial, but challenging. Polyvalent tasks are usually not suitable for grading students by giving marks. Effects of polyvalent tasks were anticipated and observed in terms of motivating students, especially of students with lower skills, to think mathematically and to communicate about mathematics. Attendants reported that polyvalent tasks gave them the possibility to observe and assess their students in a broader variety of classroom settings. At this point, evaluation of the design of the program is closely linked to the evaluation of content.

Overall, an obvious change in teachers' perception of the PD program was observable. By classifying the comments of attendants on feedback-sheets (often so-called "happiness-sheets") it has become clear that teachers shifted their attention about the meetings from assessing the atmosphere or appreciating refreshment (after the first meeting) to higher-order categories such as content, quality of cooperation, or transferability. Although we encouraged teachers with the last feedback-sheet to report explicitly on their adapted 'knowledge', they focused more than before on their use of knowledge in classroom. In many cases, a possible impact on students' outcome was considered. Figure 2 shows the development of teachers' thinking towards students' learning over time, and indicates a solid impact of the program, according to Guskey's model of evaluation.

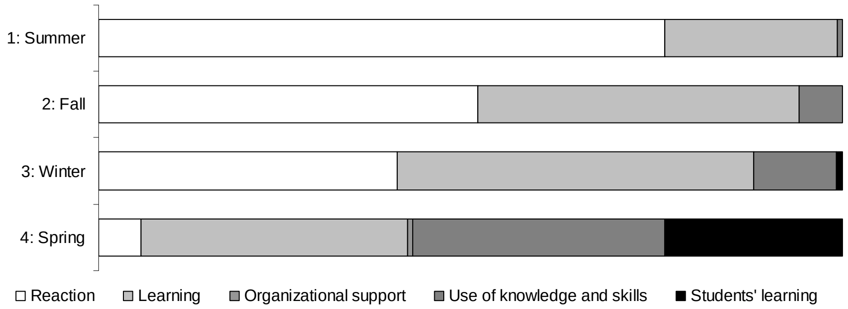



**Figure 2: Ratio of participants' responses after each meeting, on Guskey's (2000) five levels of evaluation**

In general, data of the internal evaluation was confirmed by the results of the external evaluation by the zepf, Landau.

## CONCLUSION

Constructing and developing lasting knowledge, skills and beliefs through teachers' PD must be seen as a process, which needs sufficient time and possibilities to gain experience situated at the workplace and to share ideas and experience in a collaborating group. Using a blended-learning setting – four face-to-face-meetings connected with three experience phases "on the job" – can be one way to meet the needs of participating teachers and to change classroom practice sustainably. We did not merely use a LMS-course to offer instructional and supporting material, but rather the teachers were asked to report and to discuss their lessons using discussion groups in the same moodle-course.

We identified a high acceptance of the topic and of the main structure of UPOLA. Teachers reported the importance of collaboration and discussion among teachers for their situated learning, and their own work. Still, the participants met our expectations about the use of a learning management system only partially.

Different types of weaknesses in terms of remote communication and co-operation have been observed. First of all, teachers were challenged by the faint culture of reflection and discussion about their own work, particularly in a written form. In some cases we identified a lack of motivation for continuous distance learning; teachers had not been aware of the benefits of informal, situated learning and ongoing cooperation. Insufficient technical skills and little experience and confidence, related to asynchronous communication with information technology, hindered the development of a vital and deep discussion. It was indicated that some attributes of the course-design, number of participants per group, dates of face-to-face-meetings, clear tasks for teachers' reports are key for the quality of web-based cooperation. Groups with a certain minimum of participants have to be built to ensure a vital discussion; however, exceeding a maximum of attendants could be a hindrance for developing social relationships.

Further suggestions for planning subsequent projects are to synchronize the course-structure with the schedule of teachers' workload during one school year, to avoid face-to-face-meetings that are immediately followed by holidays, and to design a plain and clear structure of the e-learning-platform, which requires no more than elementary technical skills. In addition, sufficient time and support has to be given to develop technical skills of every participant, including a prior phase for signing in and discovering the LMS through the participants themselves.

## REFERENCES

Beaudoin, M. F. (2002). Learning or lurking? Tracking the “invisible” online student. *Internet and Higher Education, 5(2*), 147–155.

Becker, J. & Shimada, S., Eds. (1997). *The Open-Ended Approach – a New proposal for Teaching Mathematics*. Reston: The National Council of Teachers of Mathematics.

Caspi, A., Gorski, P., & Chajut, E. (2003). The influence of group size on nonmandatory asynchronous instructional discussion groups. *Internet and Higher Education, 6(3*), 227–240.

Collet, C. & Bruder, R. (2006). Evaluation of a teaching concept for the development of problem solving competencies in connection with self-regulation. In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), Proceedings of the 30th conference of the International Group for the Psychology of Mathematics Education, (1), (pp. 345–352). Prague: Charles University.

Cross, J. (2006). Foreword. In Bonk, C. & Graham, C. (Eds.), *Handbook of Blended Learning*, pp. xvii–xxiii. Pfeiffer, San Francisco.

Garet, M., Porter, A., Desimone, L., Birman, B., & Yoon, K. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal, 38(4*), 915–945.

Gräsel, C., Fussangel, K., & Parchmann, I. (2006). Lerngemeinschaften in der Lehrerfortbildung – Kooperationserfahrungen und -überzeugungen von Lehrkräften. *Zeitschrift für Erziehungswissenschaft, 9(4*), 545 – 561.

Guskey, T. (2000). *Evaluating Professional Development*. Corwin Press, Thousand Oaks.

Guskey, T. (2003). Analyzing lists of the characteristics of effective professional development to promote visionary leadership. *NASSP Bulletin, 87(637*), 4–20.

Hellmig, L., Sill, H.-D., Hoffmann, S., Kleinschmidt, E., Kowaleczko, E., Kurtzmann, G., Leye, D., Lindstädt, M., Lorenz, P., & Roscher, M. (2007). *Gute Schule: Zum Arbeiten mit offenen Aufgaben im Mathematikunterricht der Klassen 5 und 6*. L.I.S.A. Mecklenburg-Vorpommern, Schwerin, 2nd Edition.

Jäger, R. & Bodensohn, R. (2007). *Die Situation der Lehrerfortbildung im Fach Mathematik aus Sicht der Lehrkräfte*. Deutsche Telekom Stiftung, Landau.

Krainer, K. (2002). Ausgangspunkt und Grundidee von IMST2. Reflexion und Vernetzung als Impulse zur Förderung von Innovationen. In Krainer, K. &

Kühnelt, H., eds., *Lernen im Aufbruch: Mathematik und Naturwissenschaften. Pilotprojekt IMST2*, Volume 1, Chapter I, pp. 21–58. StudienVerlag, Innsbruck.

Loucks-Horsley, S. (2003). *Designing professional development for teachers of science and mathematics*. Corwin Press: Thousand Oaks, 2nd Edition.

Lynch, R. & Dembo, M. (2004). The relationship between self-regulation and online learning in a blended learning context. *The International Review of Research in Open and Distance Learning*, 5(2). Retrieved September 28, 2008 from http://www.irrodl.org/index.php/irrodl/article/viewArticle/189/271.

McGraw, R., Arbaugh, F., Lynch, K., & Brown, C. (2003). Mathematics teacher professional development as the development of communities of practice. *Paper presented at the 27th Conference of the International Group for the PME held jointly with the 25th Conference of PME-NA, Honolulu*, Hawaii. Retrieved September 9, 2008 from http://math.arizona.edu/~rmcgraw/McGraw_et_al._PME27.pdf.

Nash, R. D. (2005). Course completion rates among distance learners: Identifying possible methods to improve retention. *Online Journal of Distance Learning Administration*, *8(4*). Retrieved September 29, 2008 from: http://www.westga.edu/~distance/ojdla/winter84/nash84.htm.

Oliver, M. & Trigwell, K. (2005). Can 'blended learning' be redeemed? *E-Learning*, *2(1*), 17–26.

Picciano, A. G. (2006). Blended learning: Implications for growth and access. *Journal of Asynchronous Learning Networks*, *10(3*), 95–102.

Rösken, B. & Törner, G. (2008). Mathematics done differently - An innovative approach to furthering the professional development of German teachers. *Paper presented at American Educational Research Association*, New York.

Sill, H.-D. & Hellmig, L. (2008). Konzept einer Lehrerfortbildung zu polyvalenten Aufgaben. In *Beiträge zum Mathematikunterricht*, Hildesheim. Franzbecker. (in press)

Sowder, J. (2007). The Mathematical Education and Development of Teachers. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics* (pp. 157-224). Charlotte, NC: Information Age Publishing.

Wideman, H. H., Owston, R. D., & Sinitskaya, N. (2007). Transforming teacher practice through blended professional development: Lessons learned from three initiatives. In Crawford *et al*. (Eds), Proceedings of society for information technology and teachereducation international conference. AACE, Chesapeake, VA, pp. 2148–2154. Retrieved September 28, 2008 from http://www.edu.yorku.ca/~rowston/SITE_projects.pdf.

# TEACHERS' EFFICACY BELIEFS AND PERCEPTIONS REGARDING THE IMPLEMENTATION OF NEW PRIMARY MATHEMATICS CURRICULUM

**Isil Isler and Erdinc Cakiroglu**

*Middle East Technical University, Turkey*

***Abstract***

*The purpose of this study was to investigate primary school and mathematics teachers' efficacy beliefs and perceptions in the context of the new primary mathematics curriculum in Turkey and identify differences, if any, in teachers' efficacy beliefs and perceptions based on their area of certification, gender, and experience. The sample consisted of 805 teachers, 696 of whom were primary and 105 of whom were mathematics teachers working in elementary schools located in 5 cities of Turkey. The questionnaire administered to participants was adapted by the researchers throughout the study. The results of the MANOVA analysis indicated that teachers' area of certification and experience had a significant role on the collective dependent variables, gender did not.*

*Keywords: Teacher Efficacy Beliefs, Teachers' Perceptions about the Curriculum, Mathematics Curriculum Implementation, Teachers' Practices, Primary and Mathematics Teachers*

## THEORETICAL FRAMEWORK

Mathematics curriculum change for elementary and middle grades was initiated in 2004 in Turkey. After a period of piloting, a new curriculum was started to be implemented in public and private schools throughout Turkey. Parallel with mathematics education reform movements in many countries, the new elementary and middle grades mathematics curriculum requires a significant shift in the teaching and learning of mathematics within the classroom. Compared to its precursor, the new Turkish curriculum includes a larger emphasis on learner-centered instruction, problem solving, open-ended explorations, modeling real-life situations, and the use of technology as a tool to support mathematics learning (MNE, 2005). Teachers are considered to have a critical role for the actualization of the ideas in the new curriculum. Hence, no matter what the curriculum suggests, it is the teacher who makes the ultimate decisions about what is going on in the classroom. Teachers' potential to learn and adapt to innovations can lead to students' learning and acquaintance with the innovations in classrooms. In that sense, teachers are seen as both the means and ends of curriculum reform movements (Cohen & Hill, 2001). Therefore, any curriculum change should pay attention to what teachers know and believe. The purpose of this study was to investigate teachers' efficacy beliefs about the implementation of the new national mathematics curriculum in Turkey. More specifically, it was aimed to investigate possible differences in teachers' efficacy beliefs based on their area of certification, gender, and experience.

Teacher efficacy has emerged as an important construct in teacher education over the past 25 years. It has been defined as "teachers' beliefs in their ability to actualize the desired outcomes" (Wheatley, 2005, p. 748). Teacher efficacy has been linked to teacher effectiveness and appears to influence students in their achievement, attitude and affective growth. Researchers have shown that teacher efficacy has positive effects on teacher effort and persistence in the face of difficulties (Soodak & Podell, 1993), professional commitment (Coladarci, 1992), student motivation (Midgley, Feldlaufer & Eccles, 1989), and openness to new methods in teaching and positive teacher behavior (Ghaith &Yaghi, 1997). In addition, teachers with a high sense of efficacy are more likely to use student-centered teaching strategies, while low-efficacious teachers tend to use teacher-directed strategies, such as didactic lectures and reading from textbooks (Czerniak, 1990). Thus, the importance of teacher efficacy is well established.

Teachers' sense of efficacy and reforms in curriculum has many common points (Smith, 1996). The changes teachers apply to their practices and adaptation to innovations require that they have a high sense of efficacy. Nevertheless, while both the implementation of reform in mathematics education and teacher efficacy beliefs have been studied in depth over the years, there have been very few research studies completed on the possible connection between the two.

The current study aimed to make a contribution to teacher efficacy research in the context of a major curriculum change initiated in Turkey. Furthermore, teachers' sense of efficacy has been described as "context and situation specific" (Bandura, 1986). Thus, many scales have been developed to serve different purposes, and some of them have been extensively used in different cultures. Therefore, for the specific purpose of the study, a questionnaire was adapted and utilized throughout the study to assess teachers' efficacy beliefs and perceptions regarding the implementation of the new curriculum.

## METHODOLOGY

In this study, a survey research design was employed. In the sampling method, schools rather than individuals were randomly selected. 57 schools selected for the study were public schools. The participants of this study included 696 primary teachers and 109 mathematics teachers who are teaching at upper primary level. Overall, there were 503 female and 302 male participants.

The data in this study were collected through a survey instrument, one part of which was adapted from another instrument called "Teachers Assessment Efficacy Scale (TAES)" (Wolfe, Viger, Jarvinen, & Linksman, 2007) and the other part constituted of " Teacher' Sense of Efficacy Scale (TTSES)" (Capa, Cakiroglu, & Sarikaya, 2005) which was originally developed in English by Tschannen-Moran and Hoy (2001).

## INSTRUMENTATION

Within the adaptation process, the TAES was translated in respect to the Turkish school culture. A conceptual translation method was employed. This method "uses terms or phrases in the target language that capture the implied associations, or connotative meaning, of the text used in the source language instrument" (Braverman & Slater, 1996, p. 94). Moreover, there were no negatively worded items in the original scale. However, Gable and Wolf (1993) suggest that both positive and negative items should be included in an instrument in order to control the response style. Therefore, some of the items were reworded to include a negative stem by maintaining the corresponded sub-dimension of the item. In addition, the confidence items were rephrased with "can" as Bandura (2006) suggested using "can" to refer to capability while developing efficacy scales because self-efficacy is a perceived capability. After the adaptation process of the instrument, various expert opinions were obtained for the content validation.

The final draft of the instrument consisted of four parts. The first part included 11 items measuring teachers' demographic characteristics such as gender, experience, educational level and area of certification. The second part included 22 items on a 5-point Likert type agreement scale (1-strongly disagree, 3-undecided, 5-strongly agree) related to the sub-dimensions of (1) efficacy beliefs in terms of the implementation of the new curriculum (e.g. I can prepare assessment tasks in accordance with the new curriculum) (2) beliefs about the impact of the new curriculum on classroom instruction (e.g. When based on the new curriculum, mathematics classes motivate the students to learn), and (3) perceptions about the utility or practicability of the new curriculum (e.g. The new curriculum can help me to identify the knowledge a students must master). The third part included 24 items on a 5 point Likert type frequency scale (1-never, 3-sometimes, and 5-always) about teachers' perceived utilization of the new curriculum (e.g. I use the new curriculum to plan problem-solving tasks for my students). Twelve items were added to the original sub-scale in order to assess teachers' utilization of special techniques such as cooperative group work and their use of manipulatives during instruction (e.g. I organize cooperative group work activities for my students). The fourth and the last part included the short form of Turkish teachers' sense of efficacy scale (TTSES) which consisted of 12 9-point scale items (1- inadequate, 5-moderately adequate to 9-extremely adequate) (e.g. How much can you do to control disruptive behavior in the classroom?).

In this study, common factor analysis was employed in order to discriminate the unique variance of each variable from common variance (Costello & Osborne, 2005). Factor analysis was conducted in two stages: factor extraction and factor rotation. Maximum Likelihood analysis with Direct Oblimin was used for each part of the questionnaire. Kaiser-Meyer-Olkin Measure of Sampling Adequacy (KMO) produced values higher than .9 for all parts of the questionnaire which means the sample size is appropriate for factor analysis (Field, 2005). Moreover, Bartlett's Test of Sphericity was significant evaluating the correlation matrix is not an identity matrix (Tabachnick & Fidell, 2007).

Results of exploratory factor analysis suggested six dimensions: Utility and Impact of the Curriculum, Impact of the Curriculum regarding Efficacy Beliefs, Efficacy Beliefs regarding the New Curriculum, Utilization of Curriculum, Utilization of Special Techniques, and Teachers' Sense of Efficacy. The reliability coefficients of the sub-scales produced high levels of reliability coefficients except the Efficacy beliefs regarding the new curriculum subscale.

Reliability of the subscales were satisfactory (Field, 2005) which were given in table 1.

Table 1. Reliability Statistics of the Sub-scales

| **Sub-scale** | **Cronbach's Alpha (α )** | **Number of Items** |
|---|---|---|
| Utility and Impact of the curriculum | .873 | 9 |
| Impact of the curriculum regarding Efficacy beliefs | .821 | 8 |
| Efficacy beliefs regarding the new curriculum | .670 | 5 |
| Utilization of Curriculum | .910 | 11 |
| Utilization of Special Techniques | .864 | 13 |
| Teachers' Sense of Efficacy | .912 | 12 |

## DATA ANALYSIS

For the inferential results, MANOVA was employed because of its advantage of controlling the risk of Type I error. Furthermore, MANOVA also provides univariate ANOVAs in the output to observe the separate effects of independent variables on each dependent variable (Field, 2005); however the significance of the follow-up tests should be evaluated by using Bonferroni method by dividing the alpha by the number of dependent variables in the analysis. In this study, three independent variables were chosen for investigations which were: teachers' area of certification, gender, and experience. Therefore, the alpha level was adjusted first dividing by three (0.05÷6) and then by the number of dependent variables (0.02÷6). The assumption the homogeneity of population covariance matrix for dependent variables of MANOVA was checked by inspecting Box's M Test of Equality of Covariance Matrices and Levene's test.

## RESULTS

The results of the MANOVA indicated that teachers' area of certification and experience had a significant role on the collective dependent variables, while gender did not (Table 2).

Table 2. MANOVA Results for Area of Certification, Gender and Experience

| **Effect** | **Wilks' Lambda** | ***F*** | **Hypothesis df** | **Error df** | ***P*** | **Partial $\eta^2$** | **Observed Power** |
|---|---|---|---|---|---|---|---|
| Area of certification | .976 | 2.800 | 6.000 | 697.000 | .011 | .024 | .884 |
| Gender | .966 | 4.124 | 6.000 | 697.000 | .000 | .034 | .977 |
| Experience | .929 | 4.124 | 24.000 | 2401.335 | .001 | .018 | .993 |

Further follow up analyses revealed that primary teachers (*M* = 3.76, *SD* = .538) had significantly stronger efficacy beliefs about the new curriculum than mathematics teachers (*M* = 3.57, *SD*=.545).

Moreover, teachers with 11 to 15 years and 21 and more years of experience were significantly found to perceive a higher utilization of special techniques than teachers with 10 years or less experience. In a similar sense, teachers with 16-20 years of experience were found to have a significant higher perceived utilization of special techniques than teachers with 5 years or less experience.

Table 3. Utilization of Special Techniques according to Teaching Experience

| **Teaching Experience** | **M** | **SD** |
|---|---|---|
| 5 years or less | 3.61[a] | .485 |
| 6-10 | 3.68[a] | .484 |
| 11-15 | 3.90[a] | .473 |
| 16-20 | 3.86[a] | .458 |
| 21 or more years | 3.88[a] | .521 |

[a] The possible highest score is 5; the possible lowest score is 1.

## DISCUSSIONS

Results indicated that primary teachers had significantly stronger efficacy beliefs about the new curriculum than mathematics teachers. This result is interesting in the sense that primary teachers who teach all subjects possessed higher efficacy beliefs in the implementation of the curriculum than mathematics subject-matter teachers. One of the reasons may be that primary teachers teach younger students than mathematics teachers. For example, Ross (1994) noted that declines occur in teacher efficacy when the grade levels taught are increased. Also, Capa (2005) found that elementary school teachers were more efficacious about student engagement than secondary school teachers in their first-year of teaching. Another possible reason for the lower sense of efficacy in the mathematics teachers may be because the new mathematics curriculum has been implemented since 2005 and it was first conducted in primary grades (1-5), then in the upper primary grades (6-8). Therefore, primary school teachers have been implementing the new curriculum for a longer time than mathematics teachers; thus, primary school teachers may be more acquainted with the new curriculum. Furthermore, primary teachers may have more congruent practices with the new curriculum such as developing and using hands-on activities with their students in the primary levels. Therefore, they may have felt more efficacious than mathematics teachers in the implementation of the new curriculum. A study was conducted by Wilson and Cooney (2002) including mathematics and primary teachers. The results showed that while the mathematics teachers focused on content knowledge; elementary teachers focused on different views of instructional strategies that claimed to have more "constructivist-oriented" views (p.143). Another claim for this result may be, in the grades between 6 through 8, middle grades, there are national examinations held at the end of each year for the purpose of placement of students to high schools after the 8$^{th}$ grade. Therefore, mathematics teachers may focus more on the scope of these examinations during their instructions rather than the requirements of the new curriculum, so that they may feel less efficacious about the new curriculum than primary teachers.

Results indicated that teachers with 11 to 15 years and 21 and more years of experience had significantly higher perceived utilization of special techniques than teachers possessing 10 or less years of experience. Moreover, teachers with 16-20 years of experience possessed significantly higher perceived utilization of special techniques than teachers with 5 or less years of experience. The first five years of teaching profession is a period where teachers are in the beginning of experiencing the learning to teach and developing ideas about themselves as a teacher. This may be a reason of why less experienced teachers perceive themselves to utilize the specific techniques suggested in the new curriculum less frequently. Ghaith and Shaaban (1999), founding their measurement on Veenman's (1984) list of teaching problems pointed out that teachers' concerns about teaching decrease after 15 years of experience. Therefore, more experienced teachers were expected to integrate special techniques more frequently than their beginning or less experienced counterparts since they may have less concerns about other issues such as maintaining classroom management and discipline. Veenman (1984) also called the first-year experience of teachers as a "reality shock" because of the gap between the theory they learned and the practice they are engaged in.

The study also revealed that, although found to be insignificant, teachers' efficacy beliefs about the new curriculum increased when teaching experience increased (Table 4).

Table 4. Efficacy Beliefs regarding the New Curriculum according to Teaching Experience

| **Teaching Experience** | **M** | **SD** |
|---|---|---|
| 5 years or less | 3.64[a] | .521 |
| 6-10 | 3.70[a] | .523 |
| 11-15 | 3.77[a] | .510 |
| 16-20 | 3.71[a] | .512 |
| 21 or more years | 3.75[a] | .581 |

[a] The possible highest score is 5; the possible lowest score is 1.

The findings of other studies in this issue is somewhat varying. Wenner (2001), for instance, indicated in his study with pre-service and in-service teachers that experience leads to greater perceived efficacy of teachers. De Mesquita and Drake (1994), on the other hand, investigated primary school teachers' attitudes and efficacy beliefs towards a nongraded state mandated educational reform and found that teachers possessed a lower-sense of efficacy when their experience increased. However, in the current study teachers' sense of efficacy beliefs, was found to increase when teaching experience increased although this increase was not statistically significant.

Moreover, gender did not reveal a significant difference in this study. However, descriptive results revealed that the sense of efficacy beliefs of male teachers was higher than females; despite not being statistically significant. On the contrary, Evans and Tribble (1986) found that females had higher teaching efficacy than males and Cheung (2006) found that female teachers had significantly higher general efficacy beliefs than male teachers by employing TSES. However, there have been some studies which indicate no relationship between gender and teacher efficacy (Hoy & Woolfolk, 1993; Ghaith & Shaaban, 1999).

It should be noted that change is a process rather than an event. Therefore, the teachers' adaptation process should not be underestimated. In-service trainings may aim to develop new sources for teachers' efficacy beliefs compatible with the reform efforts especially for mathematics teachers. For the design of the in-service training sessions, collaboration between schools and universities may provide educational opportunity for teachers. Furthermore, the in-service training should be parallel to the approach of what is expected from teachers as conductors of the curriculum, so that the teachers may gain mastery experiences which may provide them more efficacious about the new approaches of the innovation. In order to achieve the intended changes through implementation of the new curriculum, teachers' practices and beliefs in the adaptation process should continue to be analyzed well. Moreover, qualitative studies may be conducted to support teachers' self-report measures such as classroom observations and interviews in order to gain in-depth data about teachers' efficacy beliefs regarding the new curriculum and their adaptation processes to the new curriculum.

***References***


Bandura, A. (1986). Social foundations of thought and action: A social cognitive theory. Englewood Cliffs, N. J.: Prentice-Hall.

Bandura, A. (2006). Guide for constructing self-efficacy scales. F. Pajares & T. Urdan (Eds.). Self-efficacy beliefs of adolescents, 5, 307-337. Greenwich, CT: Information Age Publishing.

Braverman, M. T., & Slater, J. K. (1996). *Advances in survey research*. San Francisco: Jossey-Bass.

Capa, Y. (2005). *Factors influencing first-year teachers' sense of efficacy*. Unpublished doctoral dissertation, The Ohio State University.

Capa, Y., Cakiroglu, J., & Sarikaya, H. (2005). Ögretmenlik öz-yeterlik ölçeginin Türkçe uyarlamasının geçerlik ve güvenirlik çalısması. *Egitim ve Bilim, 30* (137), 74–81.

Cheung, H.Y. (2006). The measurement of teacher efficacy: Hong Kong primary inservice teachers. *Journal of Education for Teaching, 32* (4), 435–451.

Cohen, D. K. & Hill, H. C. (2001). *Learning policy*. London: Yale University Press.

Coladarci, T. (1992). Teachers' sense of efficacy and commitment to teaching. *Journal of Experimental Education, 60*, 323–337.

Costello, A. B., & Osborne, J. W. (2005). Best practices in exploratory factor analysis: four recommendations for getting the most from your analysis. *Practical Assessment, Research & Evaluation, 10*(7).

Czerniak, C.M. (1990). A study of self-efficacy, anxiety, and science knowledge in pre-service elementary teachers. Paper presented at the annual meeting of the National Association of Research in Science Teaching, in Atlanta, GA, April 8–11.

De Mesquita, P. B., & Drake J. C. (1994). Educational reform and the self-efficacy beliefs of teachers implementing nongraded primary school programs. *Teaching and Teacher Education, 10*(3), 291–302.

Evans, E. D., & Tribble, M. (1986). Perceived teaching problems, self-efficacy, and commitment to teaching among pre-service teachers. *Journal of Educational Research, 80*(2), 81-85.

Field, A. (2005). *Discovering statistics: Using SPSS for windows*. London: SAGE Publications.

Fraenkel, J. R. & Wallen, N. E. (2006). How to design and evaluate research in education, New York: McGraw-Hill Inc.

Gable, R. K., & Wolf, M. B. (1993). *Instrument development in the affective domain* (2nd ed.). Boston: Kluwer Academic Publishers.

Ghaith, G., & Yaghi, H. (1997). Relationships among experience, teacher efficacy, and attitudes toward the implementation of instructional innovation. *Teaching and Teacher Education, 13*, 451–458.

Ghaith, G., & Shaaban, K. (1999). The relationship between perceptions of teaching concerns, teacher efficacy, and selected teacher characteristics. *Teaching and Teacher Education*, *15*, 487-496.

Hoy, W. K., & Woolfolk, A. E. (1993). Teachers' sense of efficacy and the organizational health of schools. *Elementary School Journal, 93,* 335–372.

Midgley, C., Feldlaufer H., & Eccles J.S. (1989). Change in teacher efficacy and student self and task-related beliefs in mathematics during the transition to junior high school. *Journal of Educational Psychology, 81*, 247–58.

Ministry of National Education. (2005). *İlköğretim Matematik Dersi 6-8. Sınıflar Öğretim Programı [Elementary Mathematics Curriculum: Grades 6–8]*. Ankara: Devlet Kitapları Müdürlüğü Basımevi.

Ross, J. A. (1994). *Beliefs that make a difference: the origins and impacts of teacher efficacy*. Paper presented at the Annual Meeting of the Canadian Association for Curriculum Studies, June.

Smith, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal of Research in Mathematics Education, 27*(4), 587-616.

Soodak, L., & Podell, D.  (1993). Teacher efficacy and student problems as factors in special education referral. *Journal of Special Education, 27*, 66–81.

Tabachnick, B. G., & Fidell, L. S. (2007). *Using multivariate statistics* (5th ed.).Boston : Allyn and Bacon.

Tschannen-Moran, M., & Woolfolk Hoy, A. (2001). Teacher efficacy: capturing and elusive construct. *Teaching and Teacher Education, 17,* 783–805.

Veenman, S. (1984). Perceived problems of beginning teachers. *Review of Educational Research*, 54, 143–178.

Wenner, G. (2001). Science and mathematics efficacy beliefs held by practicing and prospective teachers: a five-year perspective. *Journal of Science Education and Technology, 10*, 181–187.

Wheatley, K. F. (2005). The case for reconceptualizing teacher efficacy research. *Teaching and Teacher Education, 21,* 747-766.

Wilson, M. C., & Cooney, T. (2002). Mathematics teacher change and development. In G. C. Leder, E. Pehkonen, &Törner, G. (Eds.), *Beliefs: a hidden variable in mathematics education?* (pp. 127-147). Dordrecht: Kluwer.

Wolfe, E. W., Viger, S. G., Jarvinen, D. W., & Linksman, J. (2007). Validation of scores from a measure of teachers' efficacy toward standards-aligned lassroom assessment. *Educational and Psychological Measurement, 67*(3), 460-474.

# CURRICULUM MANAGEMENT IN THE CONTEXT OF A MATHEMATICS SUBJECT GROUP

Cláudia Canha Nunes, EB 2/3 Olaias School, Lisbon

João Pedro da Ponte, *Faculty of Sciences of the University of Lisbon*

*This paper analyses how Simon, a mathematics teacher, manages the curriculum, uses the textbook to plan his practice and conducts students' assessment. It also seeks to understand the relationship between such curriculum management and the collaborative work undertaken by the mathematics teachers' subject group. This is a qualitative and interpretative case study, with data collection through participant observation, interviews and documents. The results show that the teacher manages the curriculum adjusting the expectations of different educational players (colleagues, students and parents) and his own expectations. They also show that curriculum management supported by the collaborative context generates tensions when a teacher makes decisions that diverge from those assumed collectively.*

Key-words: Curriculum management, mathematics, mathematics subject group.

A key aspect of professional practice is the way the teacher manages the official curriculum in order to meet the stated objectives, taking into account the students' characteristics and the conditions and resources of the school. In Portugal, curriculum management is particularly complex, giving the social tensions concerning mathematics teaching, largely fuelled by the performance of the students in mathematics in national (GAVE, 2002) and international assessments (OCDE, 2004). Innovative teaching practices are increasingly challenged in many forums, particularly in the public media. This paper aims to describe and analyze how a teacher manages the official curriculum, including the strategies and resources that he uses and how he assesses his students' learning.

## CURRICULUM MANAGEMENT IN MATHEMATICS

Different levels of the curriculum may be distinguished. There is the prescribed (or formal) curriculum of official documents, the available curriculum mediated by school textbooks, the planned (or shaped) curriculum by the teacher, the curriculum in action put in place by the teacher in the classroom, the curriculum learned by the students, and the curriculum evaluated, for example, through national examinations (Gimeno, 1989; Stein, Remillard & Smith, 2007).

Curriculum management refers to the actions of the teacher that contribute to the construction of the curriculum in the classroom (Gimeno, 1989; Ponte, 2005). The focus of the management process is students' learning, and it is according to such learning (at least in theory) that decisions are taken. Curriculum management has to do, essentially, with the way the teacher interprets and shapes the curriculum, on two levels: a *macro* level, concerning the overall planning of teaching for an extended period, and a *micro* level, corresponding to the teaching process in the classroom. The teacher makes decisions selecting tasks, strategies, and materials appropriate to the objectives and purposes of mathematics teaching, taking into account his/her students and working conditions. The teacher adjusts the curriculum as he/she evaluates and periodically reflects on his/her professional practices.

As a curriculum manager, the teacher faces new challenges. The cultural diversity of the student population requires the implementation and management of a dynamic curriculum that seeks to meet the demands of modern society. At the same time, the role of the teacher is changing from a "deliverer" of knowledge, to that of a facilitator of learning (Brooks & Suydam, 1993; Ponte, 2005). When planning his/her teaching, the teacher selects the tasks to propose to the students. These may be all similar (usually, exercises) or diversified (including, for example, problems, investigations, projects, and modelling tasks, as well as exercises) (Ponte, 2005). Tasks may be framed in mathematical contexts or refer to other contexts. According to current curriculum documents (ME-DGIDC, 2007; NCTM, 2000), the tasks should help the student to develop a comprehensive view of the mathematics activity, increase their understanding of mathematical processes, and help them to develop their mathematical reasoning.

School textbooks are important resources for curriculum management. Their use changes according to different perspectives on their role in different contexts (Ponte, 2005). In Portugal, the *Relatório Matemática 2001* (APM, 1998) indicates that textbooks are the teaching material most used by teachers from grades 5 to 12 (82% of the teachers use them always or in most classes). Textbooks have a large tradition in the field of education and occupy a central role in the classroom, influencing the work of teachers, and helping in delimitating the knowledge students are supposed to learn (APM, 1998). In general, teachers use textbooks to organize their classroom activity and to select tasks to propose to students to do in the classroom or at home. In this way, textbooks are key mediators between the different dimensions of the curriculum, particularly the curriculum taught and prescribed by the central government and the curriculum learned by students (Pires, 2005; Ponte, 2005).

Students' assessment is closely linked to curriculum management, playing a regulatory role in the teaching and learning process. Santos (2002), for example, suggests that assessment should be diversified and occur in formal and informal situations, with the active participation of students, contributing to their development and to the success of learning. The negotiation and establishment of an appropriate contract for assessment are important issues that can determine the success of students' learning (Nunes, 2004).

These challenges require teachers to work collaboratively, in order to frame and solve the many problems that arise in developing and adjusting the curriculum. It also requires the ability to reflect on teaching practice and students' learning, creating dynamics that promote their professional development and the school culture (Hargreaves, 1998; Nunes & Ponte, 2008). For schools to make a significant development in curriculum management and teaching practices, teachers' active involvement in innovative projects, carried out collaboratively, is an essential condition (GTI, 2008).

## METHODOLOGY

This study follows a qualitative approach (Erickson, 1986), with a case study design (Stake, 1994; Yin, 1989). The study involves a group of 14 mathematics teachers of a secondary school with 12-18 years old students. The mathematics subject group has an extensive experience of working collaboratively and in recent years has developed various projects at the school. Most of these projects emerged from the need felt by the teachers to improve their practice and to help students to overcome their difficulties. During the school year 2007/08 the subject group developed the project "Investigations, proof and problem solving tasks in textbooks and in curriculum management", involving all classes from grades 7 to 12. This project aims to diversify tasks in the mathematics classroom, to encourage the students' in learning mathematics.

This study focuses on the group of teachers of the project and within that group, on three teachers: Ana, the coordinator of the subject group, Matilde, a new arrival to school and to the group, and Simon a teacher at the school for 28 years. These cases provide several contrasts that may enable understanding to the relationships between professional knowledge and curriculum management, as well as with collaboration and leadership at the school. In this article, we present the case of Simon and give special attention to his curriculum management, because of his professional experience and role in the group.

Collection of data was done during the school year 2007/08 and includes participant observation (Jorgensen, 1989) of the group working sessions and two classes, with record of field notes in a research journal, two interviews with each of the three teachers selected for case studies, and collection of documents (Adler & Adler, 1994; Patton, 2002; Yin, 1989). According to the research plan, data analysis began simultaneously with data collection, to identify the need for further collection of data. The second level of data analysis involves the development of categories focused on professional knowledge, curriculum management, collaboration and leadership that may provide an interpretation of the data. The third level of analysis seeks to explain the meaning of the data, to provide contributions to the understanding of the phenomenon under study (Merriam, 1988).

## SIMON: MANAGING THE CURRICULUM

Simon is a teacher with 28 years of experience teaching mathematics classes from grades 7 to 12. Throughout his career he played several roles in his school such as deputy head teacher, in-service teacher education coordinator, department coordinator, and project coordinator (of mathematics projects and of other school projects). He is an in-service teacher educator in professional development courses and belongs to several working groups in and outside his school. Because of his professional experience and the initiatives he promotes in the group, Simon is recognized by his colleagues as the leader of the group. This academic year he has only grade 12 classes.

*Planning*. At the beginning of the school year, Simon makes the annual plan together with his colleagues who are teaching the same grades. This planning begins with the group of teachers browsing the school textbook and, together, making changes in the annual planning of the previous year. When questions arise, particularly about the number of lessons to assign to each unit, the group uses the mathematics curriculum and its “roadmap” with the methodological guidelines for planning. Once the overall plan is made, he and his colleagues direct their attention for the planning of the first unit [Group Working Session (GWS), 11/Sept/07]. At this stage, from inside his textbook, Simon hands several sheets, handwritten in pencil. In a table with just two columns he registered an analysis of all the tasks of the textbook, in a uniform way:

> This is my “curriculum management.” These sheets are worth gold! I have done this for all the textbooks that I use. (…) The first approach is always the textbook. I solve all exercises (…) This symbol [a ring], here around the number indicates that the task is very important and I note those tasks that are more difficult [marked with an arrow] and those that do not interest, because they are poorly structured or have errors [marked with a cross]. They [the teachers from the group] always ask me for my sheets. [GWS, 11/Sept/07]

> They [the students] know that everything I have decided to do I have solved before. I also see other textbooks, especially when I am introducing new units. [Interview, 16/Oct/07]

Simon seeks to be well prepared for his teaching. Therefore, he knows well the textbook that he uses, reading the sections on the subject that he is going to teach and solving all the exercises. His individual working plan is based on his vision for teaching mathematics. For him, the most important thing is that his students enjoy what they are doing and develop capacities that allow them to be autonomous and mathematically competent:

> To learn, students have to like what they are doing, then what I like most is that they solve their own problems. First, I would like them to be able to read a problem and not turn their arms down, not discouraging, therefore grasp the problem. (…) Achieving that with my classes is to get weapons to grasp and solve the problems which arise. [Interview, 16/Oct/07]

To achieve these goals, Simon diversifies both the tasks that he proposes to his students and the strategies he uses to solve them. However, he begins by assuming that it is not always possible to manage the mathematics curriculum diversifying the situations proposed to students. The major obstacles are time, or lack of it, coupled with the need to meet the official curriculum, taking into account the external evaluation of students at the end of grade 12:

> What I have more in mind, but I do not do always, is diversity, both of tasks and resources. I think it makes the lessons more attractive. Difficult things, easy things, open [tasks], closed [tasks], some [done] in groups, other individual (…) [I use] several resources: calculators, computers, manipulatives... I think some-

> times I have to do more! Until grade 12 I do. In grade 12 I do too little, just the calculator with great strength. [Interview, 16/Oct/08]

*Tasks*. In addition to the tasks suggested in the textbook, Simon selects other tasks to offer his students a variety of experiences to foster the different aspects of their mathematical competency. However, in grade 12, this is not always the case. It is perceptible that, at this grade level, he assigns an important role to problems that require using the calculator and to tasks that promote the development of written communication in mathematics. In such work, he highly values the textbook:

> First the textbook, then the other things. (…) We have a grade 12 textbook that has so many proposals that we have difficulty in selecting things. (…) We have to give everything and then we have no time for anything else! (…) Unfortunately, the textbook doesn't have much open tasks, but (…) problem solving, it has a lot. And it also suggests the use of technology, a little bit the computer, the calculator a lot. (…) The worksheets we have done [Law of Laplace, Slope, Lighthouse] were things related to communication, a bit following last year's project [project communication in mathematics]. [Interview, 8/Apr/2008]

Simon believes that the selection of tasks is not an easy job, and through the discussion that he develops with his colleagues who teach grade 12, he attempts to address their difficulties: "The collaborative work between colleagues can be a great help to feel more secure and confident on what we do and we developed in our classes and the materials we propose to our students" [Final reflection, 14/Jul/08].

*Curriculum materials*. The textbook is the curriculum material most often used by Simon when he is planning the work and assigns it a central role in the classroom. Therefore, he considers vital to choose a good textbook, highlighting as key elements in a textbook the nature and the diversity of the tasks. For other curriculum materials, he likes to diversify its use, but he acknowledges that in grade 12, because of the national examination, he just uses the calculator. However, he states that this is not always so: "I do not use the computer in grade 12 and I always use it in other grades" [Interview, 8/Apr/08].

*Classroom work and assessment*. Simon argues that the classroom work must be focused on the student. So, he seeks to promote since early the students' autonomy:

> Another thing I do is also autonomy, and as the years go they [the students] are increasingly autonomous. (…) I guide them! I say: "Look, I think that you should do this or that!" After, each one follows his/her path! There are some that do everything, others who do very little and I am not concerned to control it. The other day in a classroom, (…) they had questions in some exercises but they were all in different exercises and it could not be a lesson for all at the same time, so they made a request: "Look, do this and this and this," and I did it! [Interview, 8/Apr/08]

When performing tasks constructed by the mathematics group, Simon uses different strategies in the classroom, according to its purpose. Usually, he demands that students work in the tasks in pairs or in small groups. In assessment tasks, students work individually and in two phases.

Decisions about assessment provide an interesting episode concerning the relationship of Simon and the group. In fact, the other grade 12 teachers felt that the students should do assessment tasks just in one phase. That is what Ana and Diogo indicate:

> *Ana* – I think that if the task is to assess the students' learning then it has to be done individually. (...) I do not agree to give a second chance, because there are students with private tutoring and already know the task and many of them can provide ready-made answers.
>
> *Simon* - I think that they perform much better in a second stage. And I do not agree with you [Ana] that the reason is that they have external help and they already know the task.
>
> *Diogo* - I agree with Ana. In addition, if it counts for assessment, we have to do all in the same way, so that some [students] benefit and others do not. [GWS, 20/Nov/07]

However, Simon decided to use a different strategy. He chose to give a second chance to his students to improve their first response to the task, once corrected and commented. He did so because he strongly believes that this helps students to improve their learning. As he mentions, "students learn from the mistakes they do and a second chance allows them to improve their performance" [Interview, 8/Apr/08].

That decision was discussed in the following working session, as Simon announced his decision and suggested the group to analyze and reflect on the performance of his students in both phases. There were some negative reactions, especially from Ana and Diogo who have disagreed with Simons' decision [GWS, 4/Dec/08]. The issue was taken up later at meetings in which the group built tasks and discussed how to implement them in the classroom [GWS, 15/Jan/08; 19/Feb/08; 8/Apr/08, 6/May/08]. As a result, some other members of the group began to use Simon's strategy. In particular, at the end of the study Diogo admitted that this strategy can help students improve their learning, as he has verified with his own classes [GWS and Final reflection, 14/Jul/08].

The assessment of the students is one of the tasks that Simon acknowledges be the toughest for him. A major problem is the classification of the open tasks and its visibility in the students' final grade. With the collaboration of the subject group, he tried to overcome the difficulties, investing more in the construction and assessment of diversified tasks and testing different criteria for classification, starting from the criteria used in the national examination. Also the review of the assessment criteria established by the department of mathematics and the construction of a self-assessment grid helps to minimize this issue:

> I add under the formula, the four tests we had done so far, the three compositions [from open tasks], participation in the classroom in the first and second

> school period… In terms of knowledge and attitudes, and I gave a number. [Interview 2, 8/Apr/08]

Simon believes that to make decisions concerning curriculum management and to adjust his practices, the information concerning the work that he develops with his students in class is more useful than the one he collects from the tests:

> The assessment that I do all the classes is much more useful. Because everyone thinks they know [what I'm talking about], but when I come to the conclusion that they do not know I have to come back to do it in a different way. [Interview 2, 8/Apr/08]

However, the external assessment has a crucial role in the teaching strategies of Simon. That is visible in how the students do independent work in the classroom, in the tasks that he proposes, the curriculum materials and assessment instruments he most often uses (textbook, calculator, and tests). He is very concerned with the quality of his students' learning and their success, particularly in the mathematics' national exam and access to higher education. He also notes that,

> We [the math teachers] are always together, to speak of what happened [in class], and what we are going to do. (...) The assessment instruments are always made [together] and they are always the same. There are no complaints from our group, from anyone: the school community and parents. (...) The school realizes that we [subject group] work very, very in group. [Interview, 16/Oct/07]

Simon's words suggest that he seeks to take into account the expectations of students and parents. In this sense he also builds with his colleagues the assessment tools that he uses in order to harmonize them with the views of the other teachers and to support the decisions about his students' assessment.

*Work with the mathematics group*. Simon says that the discussions that the group has done in the project working sessions have been very "interesting" for him. In particular, he stresses the construction of open tasks, the definition of criteria to assess and to reflect on the results of students:

> The construction of tasks with a group of proofs, problems and explorations and investigations and their implementation in the classroom, the discussions we had in the sessions, has always been very enriching, and the exchange of ideas and clarification of points were a highlight of this project. (...) Discussions on the grading of the students' work on their achievements and to give them feedback were undoubtedly very important aspects for my learning. The contributions of all colleagues made me to reflect on my practice in these aspects, questioning what we did and discovering ideas and suggestions perfectly workable in practice in the future. [Final reflection, 14/Jul/08]

The collaborative work developed in the group played an important role in the individual work of Simon. His activity has also a major influence in the way the mathematics group works, with a culture of collaboration that has been strengthened over the years with the development of various school projects. This culture of collaboration seems to have been fostered by the way almost all teachers of the group have been involved in the project by joining and participating with enthusiasm. They appear to think that these initiatives are essential to their growth as teachers. These initiatives seem to be the key to the way they work as a group and have contributed to their working culture, where exchange of ideas, experiences and materials are welcome.

## DISCUSSION AND CONCLUSION

The curriculum management carried out by Simon at the macro level contains a collective and an individual side. The collective side involves the annual planning and the construction of units and tasks. In this process, we can see that he is an important element, particularly in its preparation, solving all the tasks of the textbook and feeding in this way the discussions of the group. Simon's curriculum management at micro level is markedly individual. He seeks to promote his students autonomy in mathematics learning, encourages them to take responsibility in their own actions and to be independent thinkers. This is much in line with the innovative teaching described by Boaler (1998). That is, mathematics education carried out in line with current curriculum orientations is possible at school level, both in Portugal and England.

His decisions have as a starting point, first, the school textbook. He seeks to understand the proposals presented and selects tasks in order to diversify the learning situations (planed curriculum). In addition to the tasks of the textbook, he offers other tasks to his students constructed together with his colleagues, and uses them for assessment. The information that he gets from his daily practice with his students helps him to regulate the teaching-learning process. The test is the instrument that he uses most. However, the formal assessment of students at the end of each term takes into account the information from students' work in the open tasks and involves the students' active participation. Simon manages the curriculum on the context of the mathematics teachers' group, but there is an individual mark that differs from the group. For example, the classroom strategies that he uses to perform the tasks in two phases differ from those initially supported by his colleagues. Also, we see that he tries to conduct the curriculum management dealing with the tension between different expectations in teaching and assessment of pupils, parents and colleagues and his own personal views. On the one hand, he proposes tasks from the textbook and, on the other hand, he gives his students more open and contextualized tasks which require the use of technology. Simon manages the curriculum taking into account its various dimensions. His practice (curriculum in action) goes beyond teaching from the textbook (mediated curriculum), exploring open tasks that involves students in significant mathematics activity (Boaler, 1998).

Second, his formal assessment practices essentially use the results of the students in tests and open tasks. However, to regulate his teaching practices he uses the information that it collects from his daily work with students (Nunes, 2004; Santos, 2002). Simon accepts the challenge of keeping diversifying his assessing practices, despite considering this to be one of the most difficult tasks of his work as a teacher. The experience of Simon, the various projects in which he participates and the collaborative work that he develops within the mathematics group of his school are key elements to help him to manage the curriculum in order to promote his students' learning (Hargreaves, 1998). Equally essential, seems to be his ability to address and solve issues of professional practice, reflecting in action, and about action (Schön, 1983).

Finally, the various initiatives of the group, in particular, its projects, are a key to the sustainability of the culture of collaborative work (Nunes & Ponte, 2008). This dynamic and working context seem to motivate the involvement of the teachers in teaching and learning. In particular, such dynamic appears to support the professional development of Simon and his capacity to accept new challenges. There are situations that generate conflicts in the group, especially when most participants favour some decision and some individual practices diverges from that. One important conclusion that we draw from this analysis is that Simon, the natural leader of the group, nurtures his relationship with his colleagues using curriculum management as a focal activity. The professional practice of these teachers, supported by this working environment, shows that current curriculum orientations may be implemented not just at an individual or small group level, but by a whole school mathematics subject group. From this study new issues emerge for future research, namely: How teacher's practices and curriculum management influence students' learning of mathematics? What conditions are necessary at schools, and more widely in the social context, so that this kind of collective curriculum management takes place, very much in line with current curriculum orientations?

## REFERENCES


Adler, P., & Adler, P. (1994). Observational techniques. In N. Denzin & Y. Lincoln (Eds.), *Handbook of qualitative research* (pp. 377-391). Thousand Oaks, CA: Sage.

APM (1998). *Matemática 2001: Recomendações para o ensino e aprendizagem da Matemática.* Lisboa: APM.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education, 29 (1),* 41-62.

Erickson, F. (1986). Qualitative methods in research on teaching. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (pp. 119-161). New York, NY: Macmillan.

GAVE (2002). *Conceitos fundamentais em jogo na avaliação de literacia matemática e competências dos alunos portugueses.* Lisboa: ME-GAVE (available at http://www.gave.pt/pisa/conceitos_literacia_matematica.pdf in 19.10.2004).

Gimeno, J. (1989). *El curriculum: Una reflexión sobre la práctica*. Madrid: Morata.

GTI (2008). *O professor de Matemática e os projectos de escola*. Lisboa: APM.

Hargreaves, A. (1998). *Os professores em tempos de mudança: O trabalho e a cultura dos professores na idade pós-moderna*. Lisboa: McGraw Hill.

Jorgensen, D. (1989). *Participant observation*. Newbury Park, CA: Sage.

Merriam, S. B. (1988). *Case study research in education: A qualitative approach*. S. Francisco, CA: Jossey Bass.

ME-DGIDC (2007). *Plano da Matemática* (available at http://www.min-edu.pt/outerFrame.jsp?link=http%3A//www.dgidc.min-edu.pt/, in 18.09.2007).

NCTM (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Nunes, C. C. (2004). *A avaliação como regulação do processo de ensino-aprendizagem da Matemática: Um estudo com alunos do 3.º ciclo do ensino básico* (Tese de Mestrado, Universidade de Lisboa). Lisboa: APM.

Nunes, C. C., & Ponte, J. P. (2008). Os projectos de escola e a sua liderança. In GTI (Ed.) *O professor de Matemática e os projectos de escola* (pp. 11-37). Lisboa: APM.

OCDE (2004). *Learning for tomorrow's world: First results from PISA 2003*. Paris: OCDE.

Patton, M. (2002). *Qualitative research & evaluation methods*. London: Sage.

Pires, M. V. (2005). *Os materiais curriculares na construção do conhecimento profissional do professor de Matemática: Três estudos de caso* (Tese de doutoramento, Universidade de Santiago de Compostela).

Ponte, J. P. (2005). Gestão curricular em Matemática. In GTI (Ed.) *O professor e o desenvolvimento curricular* (pp. 11-34). Lisboa: APM.

Santos, L. (2002). Auto-avaliação regulada: Porquê, o quê e como? In *Avaliação das aprendizagens: Das concepções às práticas* (pp. 77-84). Lisboa: Ministério da Educação, Departamento de Educação Básica.

Schön, D. A. (1983). *The reflective practitioner: How professionals think in action*. Aldershot Hants, NY: Avebury.

Stake, R. E. (1994). Case studies. In N. Denzin & Y. Lincoln.(Eds.), *Handbook of qualitative research* (pp. 236-247). Thousand Oaks, CA: Sage.

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester (Ed.) *Second handbook of research on mathematics teaching and learning* (vol. I, pp. 319-369). Charlotte, NC: Information Age.

Yin, R. K. (1989). *Case study research: Design and methods*. Newbury Park, CA: Sage.

# GESTURES AND STYLES OF COMMUNICATION: ARE THEY INTERTWINED?

Chiara Andrà

Department of mathematics – Turin University

*The resources used by mathematics teachers include gestures, drawings and extra-linguistic modes of expressions, which can be analysed through a semiotic frame. Teacher's words may go with his gestures, his written signs on the blackboard or slides projection on a screen. Depending on the emphasis given to one among these three possibilities, the styles of communication could be classified into three main trends, where the body of the speaker, the speech and the blackboard play different roles with respect to each tendency. Gestures and styles of communication seem to be intertwined, since giving importance to the body or the written signs leads to different communicative styles; conversely, the style of communication influences the type, the frequency and the role of gestures/written signs accompanying the speech.*

Key-words: teacher, gesture, communication, multimodality, semiotic bundle.

## INTRODUCTION

This paper focuses on teacher's use of gestures, drawings and extra-linguistic forms of expression when talking about mathematical subjects. It investigates whether it is possible to define a relation between teacher's modes of using gestures and his style of communication. An answer is given trough a case study. Moreover, in the same case study possible effects on students' learning process are shown.

Different resources, spreading from words to gestures to ICT instruments, are employed by teachers in the class. Sometimes they become communicative tools, supporting students in their comprehension and learning process. A semiotic approach to teaching-learning processes in mathematics is useful to understand the personal appropriation of signs by persons within their social contexts (Arzarello, Paola, Robutti & Sabena, in print).

At a more or less deep conscious level, any teacher formulates his communication strategy. An analysis of communication strategies chosen by teachers is useful to understand the way mathematical concepts are told to the students. Specifically, it can be interesting to focus on the *objectives* of the message (in the case of mathematical lessons they mainly concern giving information and knowledge), on the *target* to which the lesson is managed; and on the *definition* of messages.

It can be fascinating to combine both semiotic and communication approaches, when examining the acquisition of knowledge by students. In this paper teachers' way of communicating mathematical concepts is considered. How they use gestures, what gestures they make, and which tools support their lesson, is taken into account.

This paper is divided into five main parts, this *Introduction* and a conclusion. *Section 1* focuses on the semiotic bundle, introduced by Arzarello (2006), who adopts a Vygotskian approach and presents an enlarged notion of semiotic system, which reveals particularly helpful for framing all the semiotic resources found in the learning processes in mathematics. *Section 2* is centred on communication strategies (Di Raco, 2000) adopted by teachers. Considering a mathematical lesson, common features and a classification based on styles of communication is presented. *Section 3* presents the methodology used in the case study. In *Section 4* the analysis of some videos is sketched and the main traits of different styles of communication are modelled on both bases of semiotic bundle and of communication strategies. *Section 5* reports some considerations about the relation between teacher's communicative choice and its impact on students' feelings. The *Conclusion* closes the paper.

## THE SEMIOTIC CONTEXT OF SIGNS

In a semiotic approach to mathematical teaching, the role of signs and the way they are adopted by individuals within their social context is central (Arzarello, Ferrara, Paola & Robutti, 2005). According to Peirce, a sign is anything that "stands to somebody for something in some respect or capacity" (Peirce, 1931-1958). Within this wide perspective, Arzarello (2006) has introduced the semiotic bundle, which allows studying gestures – and teaching-learning processes – in a multimodal approach. Recent discoveries in neuropsychology (Gallese & Lakoff, 2005) underline the embodied aspects of cognition and show that the brain's sensory-motor system is multimodal rather than modular. Multimodality consists in interactions among the different registers within a unique integrated system, composed by different modalities: gestures, oral and written language, symbols, and so on (Arzarello & Edwards, 2005 and Robutti, 2005).

An important example of semiotic bundle is given by the unity speech-gesture. McNeill claimed that gesture and spoken utterance should be regarded as different sides of a single underlying mental process (McNeill, 1992). Gesture and language constitute a semiotic bundle, made of two deeply intertwined semiotic sets. Researches on gestures have discovered some important relationships between the two, for example match and mismatch has been studied (Goldin-Meadow 2003).

The term "gesture" includes a variety of behaviours that do not form a single category. According to McNeill, the term designates any spontaneous movement of the hands and harms that people perform when talking. Gestures are characterized by the following features (McNeill, 1992): they begin from a position of rest (the *preparatory phase*), move away from this position (the *peak*), and then return to rest (the *recovery phase*).

McNeill (1992) identifies two types of gestures: the *propositional gestures*, which have a main pictorial component, and the *non-propositional gestures*, which are discourse gestures. The propositional gestures could be *iconic gestures*, if they bear a relation of resemblance to the semantic content of discourse; *metaphoric gestures*, similar to iconic ones, but with the pictorial content presenting an abstract idea that has no physical form; *deictic gestures*, if they indicate objects, events or locations in the concrete world. Among the non-propositional gestures, McNeill distinguishes the *beats* (e.g. the hands move along with the rhythmical pulsation of speech, lending a temporal or emphatic structure to communication), and the *cohesive gestures*, that tie together thematically related but temporally separated parts of the discourse.

Since recent findings in psychology show that gestures can contribute to creating ideas (Goldin-Meadow, 2003), investigating how gestures are used by the teacher can be useful. In fact, it has been shown that – when gestures accompany the discourse – the listener retains more information with respect to a situation in which no gestures are performed (Cutica & Bucciarelli, 2003).

The types, the frequency and the use of gestures vary not only from teacher to teacher, but also depend on the choice of supporting tools like the blackboard or the slide projector, during the lesson (Andrà, in print).

## STRATEGIES OF COMMUNICATION

Semiotic activities are classically defined as communicative actions utilizing signs. This involves both sign reception and comprehension via listening and reading, and sign production via speaking and writing. In researches of the Turin group (Robutti, 2006), it has been investigated both the role of gestures and written signs in the mathematical discourses of students, and the role of teachers' gestures with respect to the learning processes of students: how they are shared among students and how they influence their conceptualisation processes (Furinghetti & Paola, 2003).

In order to analyze the phases that a teacher follows to prepare a lecture, the classification used by Di Raco (2000) is adopted. The first phase is the phase of *knowing*, which consists of defining theoretical objectives, choosing communication policy and investigating about expectancies and needs of the target to which he refers; in this phase, the teacher get conscious of the teaching-learning situation in which he is involved.

The phase of *designing* consists in modifying theoretical objectives and adapting them to the target, creating events and communicative situations, selecting communication channels and identifying tools that can help the teacher to talk as more clearly as possible. In this phase the teacher chooses tools that can support him while teaching (the blackboard or the slide projector).

The phase of *planning* consists in defining lengths of time, resources, structure and style of the communicative activity.

The phase of *implementing*: it is the only part that the researcher can analyse when watching videos (as it is the case of this paper), and by this examination it is possible to know something about the previous phases.

## METHODOLOGY

The case study focuses on teacher's use of gestures, drawings and extra-linguistic forms of expression when talking about mathematical subjects. Defining a relation between teacher's modes of using gestures and his style of communication is the purpose. Only university lectures have been chosen for the analysis, in order to avoid any noise given by lack of discipline from students.

In a first step, seven videos have been analysed: they concern university lessons on mathematical subjects and each one lasts about 30 minutes. They have been examined from both the semiotic context and the communicative strategies perspectives. Contributes from communication strategy researches supply a background for the semiotic analysis that is the core of this paper. The results of the analysis in the first step are reported in the next section.

In a second step, six new lectures (speakers are labelled respectively F, G, H, I, L, M) had been analysed, following the classification defined in the first step. At the end of each lesson, a questionnaire was given to students, in order to have an immediate feedback on their feelings. The questionnaire was structured in four parts: the first one contains a series of couples of opposite adjectives describing the teacher's attitude (the students and the teacher were asked to agree at a certain level to one between the two adjectives of each couple); in the second part an opinion about the rhythm of the lesson was requested; the third part was focused on students' perception of understanding: how they take notes, whether or not they remember previous lessons and what was the subject of the lecture. In the last part, an opinion about teacher's gestures was asked. A similar questionnaire was given to each speaker, in order to have the possibility of comparing the teacher's intentions whit the student's receptions. The number of students involved in answering the questionnaire is 178: 35 students in lecture F, 18 in G, 70 in H, 26 in I, 24 in L and 5 in M.

## GESTURES AND COMMUNICATION STYLES

From a semiotic perspective, it is possible to distinguish four phases in each lecture. In fact, the semiotic unity speech-gesture evolves in time. Each phase corresponds to a particular relation between words and use of signs, gestures, drawings and so on.

The *"zero" phase* consists of the first few minutes: the speaker ties with his audience. In this phase, either the speaker does not gesticulate, or his gestures have few relevance. The *introductory phase* is characterized by a great number of gestures: during this phase the teacher introduces the language that becomes shared between him and his audience. The strong relation between speech and gestures is evident. The *main phase* is more extended temporally than the previous one, but is characterized by a decreasing number of signs. In fact, the teacher has already introduced the main concepts he needs and the words he uses evoke themselves the ones – combined with signs – he has utilized in the previous phase. Some signs, utilized in the introductory period, are utilized again. The *concluding phase* varies from teacher to teacher, but a common feature is that an increasing frequency of signs is observed. A possible explanation could be that in this phase there is the need of fixing the concepts firstly introduced and then explained in the previous phases.

On the side of communication strategies, all videos have in common some main features. In fact, the objectives are mostly cognitive and didactical ones (transmitting knowledge is at the core of the activity); the professor speaks neither to equals nor to a generic public: the target is a group of professionals with a lower level of knowledge; messages he communicates are mathematical contents; and channels of communication consist always in front lessons.

There are some differences, from speaker to speaker, in communication policies and in tools accompanying talks (slides projection, blackboard...). Focusing on the semiotic bundle speech-gesture leads to consider also such supports the teacher may use. The role of such instruments is crucial. The choice of the communication policy influences not only the quantity and the quality of signs but also the preference for certain tools accompanying talk, instead of other ones.

Referring to these choices, in analysed videos it is possible to distinguish three distinct trends. When the communication takes place mainly through *the body* of the speaker, iconic and metaphoric gestures are predominant, because it is the same body of the teacher that talks with the audience. In the speech-gesture unity, the second component has a central role. The use of the blackboard or slide projection is limited or it is absent. Among non-propositional gestures, beats are numerous. In the "zero" phase the teacher does not make signs nor gestures. The introductory phase is characterized by a great number of iconic and metaphoric gestures, and some signs are pictured on the blackboard. The strong relation between words and gestures is clear and it reveals its potential power. Gestures used in this phase are repeated in the subsequent phase. The speaker is introducing the lecture and the concepts he is talking about will return during his speech in the next phase. He will broaden these concepts, and gestures utilized at this time would be repeated, going with words as an inseparable unity. During the main phase the creation of iconic and metaphoric gestures falls off, while the number of beats holds steady. Some iconic and metaphoric gestures of the previous phase are utilized. At times cohesive signs are used, for example to connect what the teacher is telling to what had been written on the blackboard. Signs written on the blackboard are not erased and accompany the whole speech. Written signs enrich the semiotic bundle made of words and gestures. In the last phase gestures utilized during the introductory one get back.

In the second trend observed in those videos, the communication takes place mainly through *the blackboard*, i.e. trough written signs that are contemporary of speech. The unity speech-written sign is central in the semiotic bundle, and gestures serve to enrich it. Deictic and cohesive gestures are dominant. In the "zero" phase the blackboard is already at the centre of attention, because the speaker is writing on it or because he just points it (e.g. no sign has already been made, but the speaker indicates, while he is introducing concepts, the point where he will start to write few minutes later). The introductory phase is characterized by the use of the blackboard. Cohesive and deictic gestures as well as beats are frequent. At the beginning of the central phase the blackboard is erased. It is continuously utilized and it is erased many times. In the final phase the blackboard is employed in a manner that is, in some way, symmetric with respect to the introductory phase.

In the last tendency identified, the communication happens substantially trough the projection of slides. In this case the signs produced by the speaker are very limited in number. Iconic and metaphoric gestures are absent. Beats are slightly incisive. It is hard to distinguish the phases shown for the previous trends. The semiotic bundle is made mainly of words and of signs projected on the screen.

The reader is referred to Andrà (in print) for an exhaustive analysis of those seven videos.

## IMPACT ON STUDENTS

It has been shown that it is possible to piece together theoretical aspects belonging to the semiotic context and to strategies of communication. The result of this mix is a framework in which one can analyse a didactical activity such as a lecture from a more complex point of view. Four different phases in the teacher's speech have been distinguished. These phases are characterized by aspects referring to both gesture studies and to communication techniques. Different styles of communication involve different uses of signs, in quality and in quantity. And how a speaker uses his body rather than other didactical tools such as the blackboard determines different strategies for the communication of mathematical concepts.

The question of interest is now about the effect of each strategy on students' feeling. Till now, the semiotic analysis of gestures has focused only on the teacher. The teacher, however, communicates to students. Students are listening to him, they are learning the concepts he teaches. Following Vygotsky (1986), how do the choices he has made influence the way students internalize what he has said?

According with the analysis from the six new lectures and the questionnaire, two professors (F and G) followed the first communication strategy: their body plays a central role when they speak. I, L and M followed the second communication strategy: the blackboard was the main tool to teach. Speaker H used slide projections in conducting her lesson. In tables 1, 2 and 3 the main trends in students' answers are reported. When the proportion of students choosing a certain response is lower than ¼, it is not reported, since it has revealed as little significant.

In table 1 the six couples of opposite adjectives describing the teacher's attitude are shown. For each couple, the major trend is indicated for each teacher's style (the students' proportion of the main trend is given). Looking at table 1, when in the unity speech-gesture the second component (i.e. the body) prevails, students' perception is mainly in involvement. Students feel them near to the teacher's world. If the blackboard plays a central role, this involvement is a little lost and it is not perceived when the blackboard is replaced by the slide projections. In this last case, students' perception of conciseness and of a schematic presentation increases with respect to the other two cases.

| | **F (body)** | **G (body)** | **H (slides)** | **I (blackb.)** | **L (blackb.)** | **M (blackb.)** |
|---|---|---|---|---|---|---|
| **Interesting Boring** | 80% appealing | | 60% quite boring | 60% appealing | | |
| **Involving Detaching** | 70% involving | | 60% detaching | 50% involving | | |
| **Concise Lengthy** | > 50% lengthy | | 60% concise | 50% quite lengthy | | |
| **Schematic Convoluted** | >50% quite convoluted | | 80% schematic | 50% quite convoluted | | |
| **Clear Confused** | 60% sufficiently clear | | 50% clear | 60% in the middle | | |
| **Passionate Cool** | 80% passionate | | 70% quite cool | 50% passionate | | |

**Table 1: Main trends (percentages) in judging teachers' attitude are compared**

The opinion on the rhythm of the lesson varies from one strategy to another. How students perceive the speed of the lesson may reveal how quickly they interiorize concepts explained. If the rhythm is suitable or slow for a student, probably he finds little difficulty in understanding what the teacher is saying.

| | **F (body)** | **G (body)** | **H (slides)** | **I (blackb.)** | **L (blackb.)** | **M (blackb.)** |
|---|---|---|---|---|---|---|
| **Teacher's rhythm** | 45% suitable<br>45% quite fast | | 25% slow<br>25% suitable<br>35% fast | 30% slow<br>30% suitable<br>30% fast | | |

**Table 2: Main trends (percentages) in judging teachers' rhythm are compared**

Table 3 reports the main trends in students' perception of understanding. The body-style had lead to a broaden spread of key-concepts perception. In the slide case, on the contrary, the key-concept is definitively perceived by a larger percentage of students. A possible interpretation is that grasping mathematical knowledge seems to be easier when slide projections are employed, rather than when the teacher speaks with no support like this.

| | F (body) | G (body) | H (slides) | I (blackb.) | L (blackb.) | M (blackb.) |
|---|---|---|---|---|---|---|
| **Notes?** | 100% often | | 50% only displayed | 60% often | | |
| **Previous lessons?** | 70% remember | | 30% don't remember<br>30% remember<br>30% know where in the program this lecture is | 50% remember<br>20% know at what point in the program this lecture is | | |
| **The subject of this lecture?** | 50% skin-deep concepts | | 70% quite good understanding | 30% quite deep<br>40% superficial | | |

**Table 3: Taking notes, remembering previous lessons and understanding the analysed lecture are shown by comparing the main trends**

Finally, an opinion on teacher's gestures was asked. Students had to indicate whether the teacher had made signs during his lesson and whether these gestures were bothersome. The purpose was of knowing students' perception of gestures and words as a unitary entity: if students did not notice teachers acts, movements or signs, one can hypothesize that gestures are felt as intertwined with the speech.

In the body-centred case, iconic and metaphoric gestures are heavily utilized, but a percentage of 20% of students had never noted them, an analogous percentage said that the teacher wrote on the blackboard mostly and only a half of students realized that the speaker made gestures, and they were not bothersome.

In the blackboard-centred case, only 5% of students said that the teacher wrote mostly on the blackboard, 40% said that he did not make signs or that it had never been noticed and 60% that the speaker gesticulated mainly.

In the slide-centred case, 45% of students said that the teacher gesticulated but it was not bothersome, 40% said that they had never noticed it and 15% that the speaker did not make signs.

It seems that the main tool chosen by the professor in communicating has not been noticed: students' attention is driven on the other supports (on the blackboard in the body-centred lessons, or the body in the blackboard-centred ones). One can suppose that the main tool (the body, the blackboard and the slides respectively) has been perceived by the students as an underlying entity, which forms a semiotic unit with the speech. Conversely, students noted that the teacher has been using different tools, those tools he did not concentrate on.

## CONCLUSION

Both semiotic standpoint and researches on communicative strategies can help to frame teacher's way to conduct his lesson. It has been shown that types, frequency and the use of gestures are closely related to the style of communication chosen by the speaker. The impact of each strategy on students learning process has been analysed from four distinct perspectives: how the teacher's attitude has been perceived by students, how the rhythm of the lesson has been felt, what level of perception of understanding students had and how teacher's gestures had been noticed.

Students seem to be mostly involved in the case the professor used mainly his body when speaking. When the blackboard plays a central role, a little lost of such involvement has been observed and, when the blackboard is replaced by the slide projections, it has not significantly perceived. In the slides case, conciseness and precision have been more perceived, rather than in the other two cases.

When the teacher used his body to communicate, students often take notes and are able to remember the previous lecture. When the slides were utilized, the notes taken are less, because they wrote only fundamental concepts, but a greater percentage of students was able to indicate in which part of the program the lesson was located.

If the blackboard is heavily used, further investigation is needed. It is not clear neither if students remember the subject of the previous lesson, nor how they take notes. Their level of understanding is not evident. A possible interpretation of this fact is that the use of the blackboard assumes all the students be able to capture the concepts at the same speed, namely the speed of the teacher's writing.

As a final consideration, it has to be pointed out that students reversed the rule between the main and the accessory tools chosen by the teacher. For example, they had said that teacher F mainly wrote on the blackboard while he had primarily used his body, but whit a regular pacing on the blackboard: in the introductory phase he wrote the concepts he recalled at the end of the lecture, without erasing them. The main tool is perceived as integrated with the speech. The rhythm of the lecture is beaten by the use of this tool (e.g. the body). Students noticed a change in the rhythm (associated to a change in the tool used, for example from gestures to the blackboard), rather than the smooth use of the main tool. Accessory tools became central in their perception, since they corresponded to a change in the rhythm of the lecture.

## REFERENCES

Andrà, C. (in print). Teaching mathematics: the role of gestures and styles of communication from a semiotic point of view.

Arzarello, F. (2006). Semiosis as a multimodal process. *Relime, numero especial*, 267-299

Arzarello, F. & Edwards, L. (2005). Gesture and the Construction of Mathematical Meaning (Research Forum 2). In H. L. Chick & J. L. Vincent (Eds.), *Proc. of the 29$^{th}$ Conference of the International Group for the PME, 1,* (pp. 122-145). Melbourne, AU: University of Melbourne

Arzarello, F., Ferrara, F., Paola, D. & Robutti, O. (2005). The genesis of signs by gestures. The case of Gustavo. In H. L. Chick & J. L. Vincent (Eds.), *Proc. of the 29th Conference of the International Group for the PME, 1,* (pp. 73-80). Melbourne, AU: University of Melbourne

Arzarello, F., Paola, D., Robutti, O. & Sabena, C. (in print). Gestures as semiotic resources in the mathematics classroom.

Cutica, I. & Bucciarelli, M. (2003). Gestures and the construction of models. *The 2nd International Conference on Reasoning and Decision Making, Reasoning and understanding: Mental models, relevance and limited rationality approaches.* Padua, IT

Di Raco, A. (2000). La comunicazione gestionale tra organizzazione e interazione simbolica. *Notebook on Sociology of the Turin University, 44(24)*, 15-24

Furinghetti, F., & Paola, D. (2003). To produce conjectures and to prove them within a dynamic geometry environment: a case study. In N. A. Pateman, B. J. Dougherty & J. T. Zilliox (Eds.), *Proceedings of the joint meeting PME 27 and PMENA, 2*, (pp. 397-404). Honolulu, HI.

Gallese, V. & Lakoff, G. (2005). The brain's concepts: the role of the sensory-motor system in conceptual knowledge. *Cognitive Neuropsychology*, *22,* 455-479.

Goldin-Meadow, S. (2003). *Hearing gestures: how our hands help us think.* Chicago: University Press.

McNeill, D. (1992). *Hand and mind: what gestures reveal about thought*. Chicago: University Press.

Peirce, C.S. (1931-1958). Collected papers (Vol. I-VII). In C. Hartshorne, P. Weiss & A. Burks (Eds.). *Collected papers*. Cambridge, MA: Harvard University Press.

Robutti, O. (2005). Hearing gestures in modeling activities with the use of technology. In F. Olivero & R. Sutherland (Eds.), *Proceedings of the 7th International Conference on Technology in Mathematics Teaching* (pp. 252-261). Bristol: University of Bristol.

Robutti, O. (2006). La matematica in laboratorio e l'interazione tra studenti. The 26th meeting UMI-CIIM, http://www.ciim26.unimore.it/abstract/abs_robutti.pdf Reggio Emilia, IT: UMI Editions

Vygotsky, L. S. (1986). *Thought and Language.* Cambridge, MA: MIT Press.

# TEACHERS' SUBJECT KNOWLEDGE: THE NUMBER LINE REPRESENTATION

Maria Doritou
University of Nicosia
Cyprus

Eddie Gray
University of Warwick
United Kingdom

*This paper considers the perceptions that trainee and experienced teachers have of the number line. Grounded within the theoretical perspective highlighted by Herbst (1997) the paper examines the interpretations that 'teachers' place on a core classroom representation advocated for teaching the number system in English schools (DfEE, 1999; 2006). The outcome suggests that primary school teachers have conceptions of the number line that do not portray conceptual understanding of its abstract nature as a representation of the number system. Descriptive characteristics of visual models, ambiguity and an emphasis on use overshadow the deeper understanding that would lead to the realisation of the potential as a valuable metaphor*.

Key-words: Number Line, Teachers, Conception, Interpretation, Ambiguity.

## INTRODUCTION

This paper brings to surface teachers' knowledge about the number line representation. A representation used extensively within English mathematics classrooms and that appears frequently within curriculum documentation – the National Numeracy Strategy (DfEE, 1999; 2006) and the National Curriculum for Mathematics (DES, 1991). Within these two documents, there is no explicit reference to the conceptual knowledge associated with the number line's form and use, despite the fact that this representation is identified as a "key classroom resource". The number line appears not only as an alternative version of the number track, but it also is frequently fragmented to emphasise particular features of the number system such as whole number and fraction. The difference between a number track and a number line lays both in the perceptual and the conceptual sense identified by Skemp:

> The number track is physical, though we may represent it by a diagram. The number line is conceptual – it is a mental object, though we often use diagrams to help us think about it. The number track is finite, whereas the number line is infinite. … On the number track, numbers are initially represented by the number of spaces filled, with one unit object to a space. … On the number line, numbers are represented by points, not spaces; … The concept of a unit interval thus replaces that of a unit object.
>
> (Skemp, 1989, pp. 139-141)

Using evidence drawn from the way in which practicing teachers and teacher trainees perceive and talk about the number line, this paper indicates that knowledge based on the perceptual characteristics of the number line together with an ambiguous use of the term number line with that of the number track, express an incomplete and compartmentalised understanding of the conceptions associated with a representation which is used on a frequent basis in their primary school practice.

## THEORETICAL FRAMEWORK

The association between number (real number) and line has been evident since Babylonian times (Wilder, 1968). The Greeks intuitively conceived real numbers as corresponding to linear magnitudes. The Greek idea of “magnitude”, which is substituting magnitude for number, implied that one may think of “numbers as measured off on a line” (Bourbaki, 1984, p. 121). The number line is, therefore, an abstraction of a representation strongly associated with the notion of a measure instrument since continuity underscores it. Starting from the Euclidean line, a “sense of continuity” can be created for and by the individual and the result be used as a number line to represent natural numbers.

Herbst (1997) concurs that the number line is a metaphor of the number system and in order to form a number line:

> one marks a point 0 and chooses a segment u as a unit. The segment is translated consecutively from 0. To each point of division one matches sequentially a natural number. (Herbst, 1997, p. 36)

All kinds of numbers can be represented on it. If a series of different number lines each introducing different numbers is built, then the number line could be in one-to-one correspondence between numerical statements and number-line figures. Growing sophistication with its formation supports representation of a number line containing natural numbers, followed by number lines illustrating the positive rationals, the integers, the negative rationals and finally one containing all numbers — the real numbers number line, which would include all numbers. It is these features that would appear to suggest the use of the number line as a pedagogical tool whilst the “dense” quality of the number line enabled Herbst to write about what he calls the “number line metaphor” and the “intuitive completeness” (Herbst, 1997; p.40) of the number line, evolving from plane geometry.

Such features are relevant in the context of teachers’ subject knowledge and awareness of conceptual issues associated with understanding the nature of the number line. Shulman (1986) defines subject matter (content) knowledge as “the amount and organisation of knowledge per se in the mind of the teacher” (p. 9) and distinguishes between the aspects of knowing “that” and knowing “why”. Aubrey (1994) suggests that every teacher has different subject knowledge and personal beliefs about teaching and learning, which are factors affecting their work in classroom; and in order for teaching to be effective, conceptual understanding of knowledge is essential. It is suggested, therefore, that in the context of the number line, teachers would be effective if they conceptualized the representation as a “metaphor” of the number system.

Ball (1990) argues that subject matter knowledge for teaching not only entails '*substantive knowledge of mathematics*' – specific concepts and procedures – but also '*knowledge about mathematics*' – mathematics as a field. Examining what teacher trainees understood about division with fractions as they entered formal teacher education, she focusing on what they have learned as students and what they need to know as teachers. She concluded that the students' had narrow understanding of division that was compartmentalized and based on rules. This was a view supported by Ball, Hill & Bass (2005) who, as the result of their attempt to measure teachers' mathematical content knowledge (an amalgam of common content knowledge and specialized knowledge) for teaching, concluded that teachers in general lack strong mathematical understanding and skill.

This paper aims to present one aspect of primary school teachers understanding of the number line as identified by their conceptions of what the number line is. The insight may provide some indication of their potential effectiveness.

## METHOD

The results presented in this paper form part of a broader study carried out during 2003 and 2004 (Doritou, 2006) that, given the explicit recommendations regarding the use of the number line within curriculum material, investigated the relationship between teacher's presentation and children's understanding of the number line. The study is a case study of an English primary school that follows guidance within the National Numeracy Strategy (DfEE, 1999). The issues addressed the primary school teachers' perception and understanding of the number line. This paper address one aspect of these issues but it draws its data from two samples that are considered to be related and complementary: (a) teacher trainees and (b) practicing teachers.

As part of an examination of their understanding and perception of the number line the full final year cohort of BA(Ed) students within the Education Department of a large Midland University were invited, through a questionnaire, to "Define a number line". The response to this question forms part of the focus of this paper. The 69 teacher trainees in the sample, had had the benefit of a four year course associated with the content and pedagogy of primary school (children aged 5-11) mathematics, were fully conversant with the contents of the National Numeracy Strategy, had experience teaching it within school and been provided core lectures associated with the number line. The respondents were followed a mixture of subjects, such as English, art, music and a third of them followed mathematics and science.

The full-time teachers' sample (also referred to as practicing teachers) contained teachers who taught mathematics within each of the year groups 1 to 6 (median ages 5.5 to 10.5). Through lesson observation and informal interviews on a one-to-one basis the teachers' perspective of the number line at a personal level and the way they presented it to the children as a pedagogical tool was investigated. Placing the trainees conceptions of the number line within a perspective associated with practising teachers, it is hypothesised a valuable insight may be gained into what primary school teachers think a number line is.

## RESULTS

### Teacher Trainees' Conceptions of the Number Line

When the participating Teacher Trainees (TT) were asked as part of a questionnaire to define a number line, only one student provided a definition that implied that the number line was infinite and contained all numbers:

> A line that contains all rationals and irrational numbers. It is an infinite line. (TT4)

One other suggested it was:

> A continuous line of all of the numbers within our number system. (TT1)

Two others provided definitions that evoked either the notion of infinity but with no further explanation, one indicated that the number line was limited to rational numbers, whilst one other defined a number line with a response that may be interpreted as an association with magnitude:

> A sequence of numbers arranged on a line which has an infinite number of divisions. (TT23)

> A line of numbers on which any number can be placed. (TT48)

> A line where you may place all the rationals at some point on the line. (TT32)

> Representation of value according to how far the number is along the line. (TT43)

None of the above students gave any explicit reference to the notion of a repeated unit, which could be partitioned, although partitioning may be implied from the statement of TT4. However, almost one quarter of the students (16/69) did make reference to some form of equal spacing associated with the line, although there was some evidence of little formality about the way they articulated this underlying feature:

> A line which is separated equally into different portions. (TT2)

> A straight line with equal distances marked. (TT7)

> A piece of apparatus with equal divisions marked. (TT10)

13 of these sixteen students associated the notions of equal spacing with numbers although in two instances the students referred to digits:

> A line with digits equally spaced along it. (TT47)

> A line with numbers attached at equal intervals. (TT66)

> A line which numbers are spaced evenly across it in a specified pattern. (TT17)

> An equally segmented line, each segment numbered in ascending order. (TT20)

Although it is not certain, TT20’s definition suggests that she is thinking about a number line that only has positive numbers. This type of definition was relatively common:

> Numbers placed at identical intervals marked on a line in ascending order. (TT15)

and indeed, no student made explicit reference to the notion that a number line could contain negative numbers.

TT17’s reference to pattern was, together with notions of order and sequence, a feature of the number line identified by 42% of the respondents:

> A string of numbers in a pattern. (TT27)

> Numbers in a correct order. (TT9)

> A sequence of numbers in a row. (TT22)

> A sequence of numbers ordered from left to right. (TT24)

> A line in which there is a number sequence reaching from lowest to highest number. (TT11)

> An ordered set of numbers in sequence, horizontal. (TT6)

Here again we see no explicit reference to negative numbers. The implications in two of these quotes (TT24 and TT11) suggest that the number line only contains whole numbers, an issue confirmed by the comments of some trainees:

> A line with number patterns on it — or from zero to a number. (TT12)

> Numbers that have been arranged in some form of sequence mainly from 0 to 10. (TT35)

> A horizontal line with a series of digits on it that have a pattern: one to ten; ten to one hundred. (TT42)

The above comments also give the sense that the number line is finite and none of these particular trainees made any reference to the notion of partitioning the intervals. However, one student did provide an indication that partitioning was associated with the line by using the word “divided”:

> A horizontal line divided into ten equal sections allowing it to be divided into fractions or quantities. (TT64)

Interestingly, in addition to these students who explicitly mentioned order, pattern or sequence, six others introduced the word “chronological” to define the number line:

> A chronological line of numbers. (TT37)

> A line with marked number intervals in chronological order. (TT56)

> A horizontal line where positive numbers ascend in some sort of chronological order. (TT61)

We can see from the definitions provided by the trainees identified through the above examples, that reference to the underlying qualities of Herbst's (1997) definition — the consecutive translation of a segment U as a unit from zero, the partitioning of U in an infinite number of ways — is extremely limited. We note that only three students referred to infinity, but only one of these implied that through partitioning all numbers could be represented. However, though there was no reference to the notion of "consecutive partition", almost 25% of the teacher trainees indicated that a number line possessed equal divisions but these definitions appear to be founded upon partitioning rather than the continued replication of a defined unit.

Herbst further indicated that a number line could be formed by choosing a unit, repeating it from zero and then attaching to the end of each repeated unit a natural number. Though just over 80% of the teacher trainees associated the notion of the number line with a number or numbers, the majority of the remainder focussed on defining the number line as a tool (see below) but, as TT6 (above) indicated, there was also some evidence that the reference to numbers was not linked to the notion of line.

The overall impression left from the trainees' definitions of the number line was that they did not define it, but instead indicated how it may be seen. The sense was that they were describing a specific number line but often this specificity was limited to the more obvious perceptual characteristics rather than conceptual aspects of the line. In doing this, essential features were often omitted. Only in the first six instances quoted above do we see the trainees' explanations rise above specificity to give more sophisticated responses.

An additional feature of the trainees' definition of the number line was its identification as a tool. Almost 10% of the trainees suggested that the defining feature of the number line was either its use in calculation or in solving mathematical problems:

> A continuous line in which numbers can be placed and used to aid calculations. (TT3)
>
> A piece of apparatus with equal divisions which children use to help them count. (TT10)
>
> A line with numbers on representing intervals, aid to solving mathematical problems. (TT34)

or associated it with the notions of counting:

> A device to aid learning, involving counting on and counting back. (TT39)
>
> A method used to count on or back horizontally. (TT62)
>
> To aid children when counting up or down. (TT65)

In one instance, the identified process was left open to interpretation:

> A way of roughly finding out any numbers between any two given extremes at each end.

(TT52)

Although the above responses emphasise the nature of the number line as a “helping tool” – used as a metaphor to support thinking – and although Herbst (1997) suggested that its dense nature meets such a requirement, there seems little indication from these particular responses that other qualities could be associated with the number line. Additionally, the responses suggest that those students who emphasise use are drawing upon experience, either as learner or as teacher and, it is hypothesised, were drawing upon episodes from within that experience.

## Practicing Teachers’ Perception of the Number Line

When the practicing teachers were each informally interviewed about their conceptions of the number line, one issue that was raised was whether or not they thought that the number line was a good representation of the number system.

3 of the 5 teachers identified the number line as a good representation of the number system because it carried the very ideas that 42% of the trainee teachers expressed with their definitions of the line. That is an emphasis on order and sequence:

> Yes! I suppose it is because it is natural order in a sequence, isn’t it? (Y2 Teacher)

> It’s a good representation for them to be actually able to see it! It has it (numbers) all in order and they can see it! (Y5 Teacher)

The fact that children could ‘see’ the number line was one of the reasons why a Year 4 teacher (teaching children with a median age of 8.5) thought the number line was a good representation of the number system

> Because it’s visual and children like visual things, and they can come up and interact with it. (Y4 Teacher)

Having something to see enabled some of the teachers to be quite specific in talking about the number line although there was evidence that this could lead to the sort of confusion identified by Skemp (1989), particularly if we recognise the hundred square as a segmented number track:

> I have got the number line, which is really useful, but because it’s so long, it is quite hard… It’s at least two metres (a number line on laminated card under the board). I do refer to it quite a lot, but I do use the number square as well. I do try and encourage the children that it’s the same. (Y2 Teacher)

This similarity between the hundred square and the number line was also volunteered by the Year 3 teacher. He indicated that the number line is a good representation of the number system when used to develop subtraction, but not so easy as the hundred square which is easier than sometimes using the number line. Really, they're sort of similar things, but this goes zero to one hundred, this goes from one to one hundred, so it's the same really… (Y3 Teacher)

Other evidence associated with seeing and with accessibility came from the Year 1 teacher, who when asked if there was a difference between a number line and a ruler, replied:

I just use the ruler, because it's a good individual tool and easily accessible. So if they want to use the number line it's immediately accessible. (Y1 Teacher)

Within her teaching of the classroom lessons, this teacher and the Year 2 teacher both drew an analogy between the number line and the ruler:

A ruler is a bit like a number line. (Y2 Teacher)

The number line here is like a ruler. Use a ruler[1] as a number line to help you. (Y1 Teacher)

However, the Year 2 teacher preferred to use the hundred square

I do use the hundred square as well in the classroom, coz that's easier to display to be honest. (Y2 Teacher)

One of the teachers explicitly thought the number line was a good representation of the number system, because of the arithmetic that could be done with it:

Yes! Very good! Use it to bridge through multiples of ten. Partition the numbers and then the tens and then the units, if they're doing addition. And if you're working out subtraction. (Y3 Teacher)

The teacher teaching Year 6 among other classes was the only teacher who gave a response that made any reference to the fact that the number line (although finite in her terms) was a representation of the number system:

… I think Year 6 children are quite good to see that the number line represents quarters, halves, numbers up to a thousand or even negative numbers.

This teacher's response to the question "Is the number line a good representation of the number system?" bore remarkable similarities to the trainee teachers' conceptions of the number line. Two of the five teachers referred to pattern, order and sequence. There was reference to the number line as tool, but only one reference to the variety of numbers that could be represented on it. However, whilst all of the teachers could talk about what it may look like or what it may be associated with, none provided a sense of its continuity and density. Those teachers who referred to the hundred square or to the ruler did not make a distinction between the abstract nature of the number

[1] The ruler the teacher referred to and given out to the children was one that represented a number track. It was a wooden 30cm stick divided in squares, with the first coloured yellow, the next green, the next yellow and so on and so forth. Within each box a natural number was written, starting from 1.

line representing continuity and the more concrete nature of the alternatives that represented the discrete nature of number.

## DISCUSSION

In their consideration of effective teachers Askew, Brown, Rhodes, Wiliam & Johnson (1997) suggest that effective teachers can be distinguished from less effective teachers in terms of increased fluency in discussing conceptual connections in the context of classroom practice whilst less effective teachers may express a more procedural rather than conceptual personal subject knowledge. The former, generally identified as “connectionist”, appeared to value both pupils’ methods and teaching strategies, in an attempt establish links with mathematical ideas. The latter, those associated with the notion of “transmission”, appeared to prioritise teaching over learning and considered mathematics to be a collection of routines and procedures.

The data presented in this paper would suggest that connectionist values associated with the number line seldom featured in the responses of either trainees or practicising teachers. Indeed, most of the English curriculum material presents the number line as a concrete model supporting actions with little if any reference to its strength as an abstract representation of the number system. Such a focus may be more strongly associated with, and possibly even instrumental in, promoting beliefs that are associated with transmission. Though the classroom teachers in this survey applauded the pedagogical benefit of the number line as a tool, neither they nor the trainee teachers provided little explicit or implied indication that this benefit had a formality based upon the repetition of a unit interval and the partition of this interval. Instead we see that perceptual features, frequently implicitly associated with episodes and with a particular “line”, dominated the definitions and additional comments obtained, though, particularly in the case of the teachers, these were frequently tempered by representational ambiguity and supported by counting episodes associated with moving backwards and forwards.

Gray and Doritou (2008) suggest that such conceptions lead to similar conceptions amongst children and though these do not appear to mitigate against the success of younger children in elementary arithmetic they eventually led to confusion amongst the older children. Specific interpretations of the features and use of a number line fail to provide children with a platform from which they may recognise its potential to contribute to the development of a global perspective on the number system. They also fail to contribute towards procedural efficiency as number size increased.

Bright, Behr, Post & Wachsmuth (1988) suggest that the number line is currently an extensively used model in the teaching of mathematics in elementary school, and whilst generally effective is also the source of difficulty both in instruction and its use by children. This paper provides one explanation for this difficulty – a very limited conceptual understanding of the representation by the teachers who use it. It was more general for the number line to be conceptualised as a series of discrete representations of particular elements of the number system. The notion that it evolved from a unit that could be repeated and partitioned was less important than the notion that actions could be carrying out with it. This emphasis essentially associated with transmission caused ambiguity in the way teachers referred to a number line and, it is hypothesised, a consequent limited understanding of a sophisticated representation by the children who are faced with it.

## REFERENCES


Askew, M., Brown, M., Rhodes, V., Wiliam, D., & Johnson, D. (1997). Effective teachers of numeracy in UK primary schools: Teachers' beliefs, practices and pupils' learning. In Pehkonen, E. (Ed.), *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2 (pp. 25-33). Lahti, Finland: Gummerus.

Aubrey, C. (1994) *The role of subject knowledge in the early years of schooling.* London: Falmer Press.

Ball, D. L. (1990). The Mathematical understandings that prospective teachers bring to teacher education. *Elementary School Journal, 90*, 449-466.

Ball, D. L., Hill, H.C, & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Mathematics Educator, Fall*, 14-46.

Bourbaki, N. (1984). *Elements of the history of mathematics*. Berlin, Heidelberg: Springer-Verlag. (Original work published 1984)

Bright, G., Behr, M., Post, T., & Wachsmuth, I. (1988). Identifying fractions on number lines. *Journal for Research in Mathematics Education*, *19* (3), 215-232.

Doritou, M. (2006). *Understanding the number line: Conception and practice*. Unpublished PhD., Mathematics Education Research Centre, University of Warwick.

DES (Department of Education and Science) (1991). *Mathematics in the National Curriculum*. Her Majesty's Stationery Office, London.

DfEE (Department for Education and Employment) (1999). *The National Numeracy Strategy: Framework for Teaching Mathematics from Reception to Y6*. London: DfEE.

DfES (Department for Education and Skills) (2006). *Primary National Strategy: Primary Framework for literacy and Mathematics*. Norwich: DfES Publications.

Gray, E. & Doritou, M. (2008). The number line: Ambiguity and interpretation. In O. Figueras, J. Cortina, S. Alatorre, T. Rojano & A. Sepúlveda (Eds.), *Proceedings of the 32nd Conference of the International Group for the Psychology of Mathematics Education*, Vol.3 (pp. 97-104). Morelia, Mexico.

Herbst, P. (1997). The number-line metaphor in the discourse of a textbook series. *For the Learning of Mathematics*, 17 (3), 36-45.

Shulman, L. S. (1986). Those who understand: knowledge growth in teaching, *Educational Researcher, February*, 4-14.

Skemp, R. R. (1989). *Mathematics in the Primary School*. London: Routledge.

# COMMUNICATION AS SOCIAL INTERACTION PRIMARY SCHOOL TEACHER PRACTICES

**António Guerreiro**

*School of Education, University of Algarve*

**Lurdes Serrazina**

*School of Education, Lisbon*

**Abstract.** *This article reports the reflections of a primary school teacher on her communication practices in the classroom and the interaction between the students. It is part of a large research which intends to study the evolution of collaborative work among three teachers and the first co-author of this article, with regard to the knowledge of and development of processes of mathematical communication and interaction in the primary school classroom.*

**Key-words**: mathematical communication; collaborative work; teaching practices; professional knowledge; teacher education.

Communication as an instrument of the relationship between teacher and students has been the target of widespread dispute in the field of education, given its relevance in the teaching and learning process. The greater value given to the role of dialogue and the sharing of information is opposed to a more traditional form of communication based on a one-way discourse, undertaken by the teacher (Brendefur & Frykholm, 2000).

From this point of view, the transference of information and codes (linguistic and others) is not approached nor studied in itself, but in its use, and communication is characterised as a process of social interaction. It is in this process of interaction that the subjects as well as society itself undergo their construction, through the negotiation of meaning between individuals (Yackel, 2000).

Founded on this desire to understand the role of communicational changes in the teaching and learning of mathematics, the first co-author of this article developed a collaborative research into mathematical communication with three primary school teachers, with the supervision of the second co-author.

This article proposes to explore the way in which the communicative practices of the teacher can raise the value of the communication among the students in the classroom. It results from the work undertaken with one of the teachers who participated in the study – Laura.

## COMMUNICATION AS A PROCESS OF SOCIAL INTERACTION

From the point of view of communication as interaction, learning by the subjects arises from interactions between the individual and the culture (Sierpinska, 1998), including the interactions between students and the teacher.

Communication is characterized as a process of social interaction, which permits the subject to identify himself/herself with the other, and at the same time, express and affirm his/her singularity (Belchior, 2003), and has the function of creating and maintaining understanding between individuals.

Thus, teaching is understood as an interactive and reflective process, with a teacher continually engaged in differentiated and updated activities for his/her students. With these activities, meanings are formed in the process of interaction between the subjects, and not only in the transmission of a codified knowledge which is given beforehand (Cruz & Martinón, 1998; Godino & Llinares, 2000; Yackel, 2000).

It is assumed that mathematics teachers' knowledge is a specialized knowledge of and about mathematical (Ball, 2003), practical and personal knowledge (Chapman, 2004; Elbaz, 1983) that teachers develop through the process of reflection. Thus the collaborative work between teachers and researcher is a privileged way for knowing the teachers' professional practices (Boavida & Ponte, 2002).

## METHODOLOGICAL OPTIONS

The background investigation for this article fits into a qualitative methodology (Bogdan & Biklen, 1994), which adopts the interpretative paradigm and follow the design of a case study (Stake, 1994; Yin, 1989). Three primary school teachers participated in this study, in a context of collaborative work with the researcher, regarding the reflection about their professional practices concerning mathematical communication.

The study has been conceived in two phases: the *characterization phase* in order to characterize the participants and interpret *the state of the art* (carried out during 2006/2007 academic year) and the *collaboration phase* in order to work together on mathematical communication in the process of teaching and learning (carried out during 2007/2008 academic year).

The data collection consisted of initial and final interviews (audio taped) with the teachers, description of the collaborative meetings (audio taped) between the researcher and the teachers (collectively and individually) and classroom reports (audio and/or video taped). The data were transcribed and reduced in expressive episodes.

In the *characterization phase* an interview was carried out with each teacher. The researcher attended two lessons of each one and carried out two meetings with them. In the *collaboration phase* there were two meetings with each teacher and five meetings of collaborative work. The researcher attended nine lessons of each teacher. The final interviews were carried out at the beginning of 2008/2009 school's year.

The collaborative work implicated theoretical framing discussion, elaboration of mathematical tasks for the classroom and the reflection based on the transcription of teachers' lessons and the video of teachers' and students' communication practices in the classroom.

The data analysis was organized in case studies. Each one with the characterization of the teacher and school context, namely the teacher's mathematical communication conceptions and practices. These were the reflections of the teachers about the facts and situations that gave added value to social interaction between students and mathematical learning.

## INTERACTION IN THE CLASSROOM

This section shows how the interaction in Laura's classroom evolved from the beginning of the study and throughout the collaborative work.

The initial reflections (in the characterization phase) of Laura about the interaction among the students in the classroom, in the class group, seem to reflect conceptions associated with the notion of communication as transmission of information:

> Normally explaining how they did things, the reasoning, the calculations, but also in relation to the problems. [Interview, December 2006]

This presentation of strategies and reasoning is conducted by the teacher, requiring sometimes the participation of the rest of the class. The students were presenting their productions of rectangular panels constructed with twelve paper squares. (Appendix 1):

> Teacher: Which was the first one that you made together?
> [The students in the group, up by the blackboard, point to one of their stuck-on designs]
>
> Teacher: That one. How did you make this one here? [Points to the first rectangle]
> Student: Four…
> Teacher: Four.
> Student: Four, four and four…
> Teacher: Was it like that?
> Another student: Four, three and three…
> Teacher: And the second one?
> Student: We made it two by two and four by four.
> Teacher: Not four.
> Student: One, two, three, four…five, six.
> Teacher: Ah, and the last one, how was that one? You just said: "We have to make three, three, three...", I said, "no, you already have three, three, three…", "ah, of course there is. So we have to make four, four, four…", "but you already made that here", "Ah, of course that's right. So we have to make two", "but you already have that here". What did you say to me then?
> Student: We can make it one by one. [First Year Class, June 2007]

The omnipresence of the teacher in the classroom, allied to the *monologue* of the students, appears to result in an understanding of communication as a way to put forward previously constructed ideas which have been validated by the teacher.

**Interaction and Exploration of Error.** The avoidance of error in the construction of mathematical knowledge seems to be one of the causes of this constant validation of the activities of the student by the teacher. As Laura tells us, her main worry in relation to the work of her students was the attempt to avoid error, "*always to get the thing right*" [Collaborative Work, October 2007], given that "*we really love it when they get it right straightaway*" [*Idem*].

The reflection, in the collaborative group, on the role of off-the-cuff validation and of error, implied that teachers involved in the study made an effort to try to avoid validation of the activities of their students when group work was taking place.

Laura tried to get the students to interact among themselves, in spite of her very much present mediation. As Laura says, despite trying not to interfere so much, the students constantly need her approval, "*Mine look at me and wait for me to say something*", while they are putting questions to each other [Collaborative Work, November 2007].

In the development of this strategy of communication among the students priority was given to presentation of the incomplete or wrong strategies of the students and consequently to the discussion of the mathematical aspects or other causes for the errors put forward.

In the problem of the River Crossing (Appendix 2), the teacher opted to begin the discussion with a solution that was incongruent with the conditions of the problem. The student Monica presented the solution of her group, writing:

> Little Johnny takes the rabbit in the boat. Little Johnny takes the cabbage in his lap and the dog on one side, and they go on their way

While the student was writing on the board, some students were waiting with their hands up, as a sign that they wanted to question their colleague.

| | |
|---|---|
| Teacher: | There are hands up. |

The teacher alerted Monica to the questions of her colleague and she ended her presentation and chose one of the other students to ask her a question. After an intervention directed towards the correct solution, one of the students who had identified the incongruency of the resolution with the statement of the problem explained:

| | |
|---|---|
| Gonçalo: | The group wrote "the cabbage in his lap and the dog on one side" but he can only take one animal. |
| Teacher: | Where? |
| Gonçalo: | In the boat. |
| Teacher: | One thing. But three things went. |
| Gonçalo: | Yes, but the cabbage can't go on Little Johnny's lap. There can only go the dog or the cabbage, only one thing. [Second Year Class, March 2008] |

The teacher valued the interaction among the students and passed this conclusion on to the group which was at the blackboard, highlighting the impossibility of more than two *passengers* in the boat. Faced with this *rejection*, one of the members of this same group – Tiago – presented a new proposal for the solution, writing:

> First goes the dog [the students become agitated because they consider what their colleagues wrote to be wrong]. Second goes the cabbage. And last goes the rabbit.

Gonçalo, observing the solution written by Tiago, says:

> Gonçalo: I know what's wrong.
> Teacher: So go up there Gonçalo. Go to the blackboard and say what's wrong.

Gonçalo went up to the blackboard and put his reasons to Tiago.

> Teacher: Tiago, stay there to defend yourself.
> Gonçalo: The dog can't go first, because if Little Johnny took the dog…. If Little Johnny crossed the river with the dog, then the rabbit would eat the cabbage [idem]

The comments of the teacher were intended to promote the interaction between the students – "*There are hands up*" – and to encourage the justification of student' reasons - "*stay there to defend yourself*". This attitude of this teacher promoted a greater interaction between the students in the classroom.

**Interaction and Teaching and Learning.** Laura recognizes and values the students change in attitude towards communication by the students, emphasizing that they have also changed their attitude in the other subject areas:

> I try to get them communicating among themselves, no matter what the subject is. [Meeting of the Teacher with the Researcher, April 2008]

This attitude of the students also appears to be related to a significant change of the teacher's attitude in the classroom, in particular with regard to her expectations about students:

> I bide my time, I wait, listening more carefully, because at times what they say is important, although sometimes it isn't. [*Idem*]

This seems to have contributed to a greater autonomy of the students in the learning and construction of knowledge:

> [The students] are more at ease, they have a different dynamism. They participate more. They are more attentive to what they are doing. [*Idem*]

The development of communication and interaction among the students has changed the way of working in the classroom. As Laura says, *"we are working at a deeper level because there's more discussion"*. [*Idem*]

## SOME FINAL CONSIDERATIONS

The teacher's practice in relation to the interaction among the students is initially associated with the valorisation of the attitude of exposition of their activities according to the role of the teacher in explaining mathematical concepts.

Teachers were involved in reflecting on their classroom practices in mathematics. With this reflection they began to give more importance to the role of error in mathematics learning, and to allowing students to interact with their peers. This led to increase the interaction among students, either mediated by teachers or not.

## REFERENCES

Ball, D. (2003). *What mathematical knowledge is needed for teaching mathematics?* Paper presented at the U.S. Department of Education, Secretary's Mathematics Summit, Washington, DC, February 6, 2003 (File PDF)

Belchior, F. (2003). Pedagogia, comunicação e existência. *Revista Portuguesa de Pedagogia*, ano 37, nº 3, 197-230.

Boavida, A. & Ponte, J. (2002). Investigação Colaborativa: Potencialidades e problemas. GTI (Orgs) *Reflectir e Investigar sobre a prática profissional* (pp. 43-55). Lisboa: Associação de Professores de Matemática.

Bogdan, R. & Biklen, S. (1994). *Investigação Qualitativa em Educação.* Porto: Porto Editora.

Brendefur, J. & Frykholm, J. (2000). Promoting Mathematical Communication in the Classroom: Two preservice teachers' conceptions and practices. *Journal of Mathematics Teacher Education*, 3, 125-153.

Chapman, O. (2004). *Facilitating peer interactions in learning mathematics: teachers' practical knowledge*. Proceedings of the 28th Conference of the International: Group for the Psychology of Mathematics Education, (File PDF)

Cruz, G. & Martinón, A. (1998). Interacción y construcción significativa del conocimiento: notas teóricas y una práctica educativa. *UNO. Revista de Didáctica de las Matemáticas*, nº 16, 85-100 (file PDF, 1-13).

Elbaz, F. (1983). Teacher thinking: a study of practical knowledge. New York, NY: Nichols Publishing Company

Godino, J., & Llinares, S. (2000). El Interaccionismo Simbólico en Educación Matemática. *Revista Educación Matemática,* Vol. 12, nº 1, 70-92.

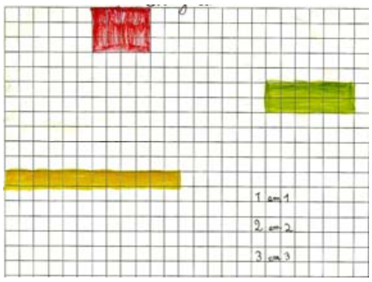
Sierpinska, A. (1998). Three epistemologies, three views of classroom communication: Constructivism, sociocultural approaches, interactionism. In H. Steinbring, M. G. B. Bussi, & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 30-62). Reston, VA: NCTM.

Stake, R. (1994). Case studies. In. Dezin, N. & Lincoln, Y. (Eds.) *Handbook of qualitative research.* Londres: Sage Publications.

Yackel, E. (2000). *Creating a Mathematics Classroom Environment that Fosters the Development of Mathematical Argumentation.* [On-line] Available in http://www.nku.edu/~sheffield/eyackel.html.

Yin, R. (1989). *Case study research: Design and methods*. Londres: Sage Publications.

**Appendix 1**



**Appendix 2**

River Crossing - The hunting dog, the rabbit and the cabbage

Little Johnny was crossing a dry, unshaded field on the way to his grandfather's house. He was taking with him a hunting dog to go with his grandfather on the hunt, a jack rabbit for his grandmother to put in her rabbit hutch with a pretty female rabbit and a nice cabbage for lunch.

All along the way, the dog wanted to eat the rabbit and the rabbit to eat the cabbage. Little Johnny had to be very careful as he walked along to avoid anything going wrong. After a while Johnny came to a river he had to cross.

In order to cross the river there was a small boat which he could use, but it was so small that he could only take with him one passenger at a time: the dog or the rabbit or the cabbage. He could never leave the dog alone with the rabbit, nor the rabbit alone with the cabbage, so how can he get all of them across without any problem? You are going to have to help to resolve this problem.

# EXPERIMENTAL DEVICES IN MATHEMATICS AND PHYSICS STANDARDS IN LOWER AND UPPER SECONDARY SCHOOL, AND THEIR CONSEQUENCES ON TEACHER'S PRACTICES

Fabrice Vandebrouck, Cecile de Hosson, Aline Robert

University Paris-Diderot, Laboratoire André Revuz

*The new French Standards for the teaching of science subjects in secondary school advantage the experimental dimension by a revival of words such as "experiment", "experimental" and by the introduction of quite new teaching concepts such as "inquiry-based teaching" and "practical experiment test". Our study deals with the introduction of a new teaching paradigm which includes a strong experimental dimension in both mathematics and physics instructions. The "double approach" frame, including both didactic and ergonomic approaches, constitutes the global frame for the analysis of the teachers' practices we wish to focus on. This allows us to go back over some variables that could be essential to take into account in order to choose appropriate educational devices.*

The new French Standards for the teaching of science subjects in lower and upper secondary school advantage the experimental dimension by a revival of words such as "experiment", "experimental" and by the introduction of quite new teaching concepts such as "inquiry-based teaching" and "practical experiment test". This novel approach is common to mathematics, physics, chemistry and biology instruction in lower secondary school. Conversely, in upper secondary school specificities appear depending on each scientific subject. In mathematics, this specific approach leans on more or less implicit references to the use of ICT.

Our study deals with the introduction of a new teaching paradigm which includes a strong experimental dimension in both mathematics and physics instructions. First, we will survey the meaning and the possible place of experiments in the physics and mathematics learning by examining the textbooks and standards. Then, we will focus on the practices of the teachers who intent to implement such experimental elements in their classroom. In that perspective, we use a common frame of analysis ("double approach didactic and ergonomic") in order to raise the predictable complexity of the recommended approach. Some examples are given which analysis leads to the conclusion that either the approach suggested by the teacher is too open and nothing happens or it is too restrictive or reductive, and students have no real access to what is required.

In the "double approach" frame, a didactic point of view and an ergonomic one are interwoven. It constitutes the global frame for the analysis of the teachers' practices we wish to focus on (Robert & Rogalski, 2005, Robert, 2008, Pariès, Robert, Rogalski, 2007). This frame allows us to describe both planed sequence and expected tasks proposed by teachers (in terms of available knowledge and adaptations), and to confront them to an analysis of the possible children's activity.

To conclude, we will go back over some variables that could be essential to take into account in order to choose appropriate educational devices, that is, concepts or situations that fit with a relevant experimental approach. At the same time, the efficiency of our methodological frame will be thus attested.

## THE PLACE OF THE EXPERIMENT IN THE MATHEMATICS AND PHYSICS CURRICULA

In an international context, a lot of researchers looked into experimental activities and enlightened their different objectives. Their results led the authors of curricula to take new directions for science education. It consists in showing a richer image of scientific processes, giving more autonomy to pupils and proposing more open tasks allowing them to develop higher level cognitive activities: the statement of scientific questions, the statement of hypotheses, the design of experimental protocols, the choice and treatment of data and the communication of the results. These different elements have been made explicit in several projects, such as Science for All Americans or in the recent report ordered by the European Commission. More particularly in France, this kind of process in the classroom at low secondary school, is a continuation of a pedagogical practice implemented at primary school since 2000. In France, it appeared in the curriculum in 2005, and was reasserted in 2007 under the name of "démarche d'investigation" in French, that has been translated here into "inquiry-based teaching" (IBT). This process concerns both mathematics and science teaching.

Despite this common educational text for both mathematics and physics instruction (grade 6 and 7), it seems difficult to implement and to analyze this type of approach in the classroom in the same way in mathematics and physics, insofar as the actual objectives are on both sides different. Indeed, this requires at least to question the very nature of the subject itself (in an epistemological point of view) and the different type of problems involved in a scientific process learning such as modeling the real world, complex operating of tools previously elaborated, etc.

In mathematics, the experimental test in upper secondary school (end of grade 12) includes a consistent and open problem. Students can be asked to model a part of this problem, but this is not systematic (BOEN HS n°7, 2000). From the perspective of potential acquisitions, the experimental test doesn't seek to introduce new knowledge but to make students' knowledge (assumed available) operate. This type of process includes rich, various and possibly new adaptations of this knowledge. Students often face a number of choices: choice of cases to deal with specific software, choice of the software itself, etc. It seems appropriate to *a priori* consider what we want to "win" in terms of students' knowledge (start-up knowledge, knowledge supposedly already there, and also the distance between the two). It is to estimate how students can stage and work with the "experimental" part itself, given the management developed by the teacher that determines the whole work in the classroom and also number of other constraints such as time, material organization, etc.

In the IBT context, physics teachers are now invited to elaborate problems that are favorable to the development of processes and construction of new knowledge by the pupils themselves (BOEN HS n°6, 2007). At the same time, pupils are given more responsibility and autonomy (the statement of hypothesis or conjectures, the elaboration of an experimental device in order to test these hypotheses). At last, teachers are expected to know pupils conceptions in various subjects and be able to exploit them in the elaboration of sequences that would aim at making these conceptions evolve by using a hypothetico-deductive process. The implementation of the IBT in the classroom requires profound changes in science teachers’ practices and experience. A focus on the spontaneous transition between IBT in the curriculum and teachers’ practices leads us to draw a picture of the way teachers appropriate the new instructions and allows us to identify the underlying difficulties.

## SOME COMMON ELEMENTS OF METHODOLOGY FOR ANALYSING TEACHERS’ PRACTICES IN THE CLASSROOM

The « double approach frame » (Robert & Rogalski, 2005) postulates that the analysis of teachers’ practices requires for the researcher to draw what tasks are chosen by the teacher for its pupils, and to derive the way its courses are organized. The corresponding analyzes lead to reorganize the activities the pupils could have performed. These analyzes are guided by the choices of the teachers, but they remain inadequate to understand teachers’ practices as a whole. Other analyzes, inspired by the ergonomic framework complete the former ones: they include the constraints and the resources associated with the profession of “teacher”: institutional constraints (connected with the curricula), social constraints and the constraints connected with the personal resources of the teacher, that is, his beliefs, knowledge and experience.

This theoretical framework is not a model; it is drawn from the Activity Theory (Leont’ev, 1984, Vygotsky, 1997, Vergnaud, 1990). The conversion of fundamental elements of this theory into specific theoretical elements adapted for mathematics or physics and for learning situation allows us to question teachers’ practices and to legitimate our research questions whether there are local or global. Thanks to this approach, our questions can be in kipping with a unique framework associated with specific methodologies.

These methodologies involve on the one hand the presentation of a large planed-teaching course that includes the analyzed sequence(s) (either because many sequences are involved or at least to clarify the place of the sequences into the whole course), and on the other hand, the statement of the possible activities of the students. The latter is done trough the confrontation of an *a priori* analysis (including the study of expositions or instructions and the examination of the data given by tools) with an analysis of the teaching processes.

**The *a priori* analysis** provides the tasks the students should perform and the corresponding knowledge **(**Horoks and Robert, 2007**)**. The second analysis (**the analysis of the teaching processes**) refines the *a priori* analysis by taking into account teachers' interventions. This concerns the organization of students' work (including the timing of the different phases) and this also covers their actual work (self-working, part of initiatives, students' involving, teachers' help to the making tasks, aid to overcome the action, reports). Starting from **the recovery of students' activities** we can question and understand the choices done by the teachers and think about alternatives strategies that take into account the standards, different constraints (e.g. time), the habits of the job, and individual characteristics.

## CASE-STUDIES

### In mathematics

We develop in this communication two examples of grade 12 teaching sequences ($12^{th}$ grade). The two sequences last one hour, with pupils working alone on a computer, and with the teacher helping them individually.

| |
|---|
| **The objective of this session is to discover a property of the slope of the exponential curve, then to prove this property.** |
| **EXPERIMENTAL PHASE** |
| To answer this question, you will use the software **Geogebra** |
| 1) Realization of the diagram |
| (…) |
| 2) Experimentation |
| Vary the point A on the curve. Observe simultaneously the X-coordinates of A and B |
| 3) Hypothesis |
| **What property seems to be true for all positions of the point A? Try to imagine a method to confirm this hypothesis with experimentation.** |
| **RESEARCH OF A PROOF** |
| 1) Let *a* be a real and A the point on the curve y=exp(x) which X-coordinate is *a*. Find the equation of the slope T of the curve on A. |
| 2) Can you use this equation to prove your hypothesis? |
| 3) Make the proof of the hypothesis. |

**Table 1: exposition given in the first example of mathematics teaching sequence**

**The a priori analysis** shows that the experimental activity potentially made by the pupils is banished. Indeed, the ICT tool to be used is given and the objective “discover a property of the slope of the exponential curve” is too hazy to allow an autonomous pupils’ activity. Then, the experimental construction is given by the exposition “realization of the figure” (question 1) and the activity described as experimental (question 2) is reduced to vary a point on the curve and to observe the conjecture as an evidence (question 3): “The X-coordinate of A is always the one of B plus 1”. There is no more one demonstration exercise fairly traditional with no experimental dimension anymore. Even if the introduction of the parameter and the calculation of the equation of T is explicitly asked in the exposition, some intermediary tools have to be introduced by pupils. So this traditional exercise is complex in comparison with the task.

The analysis of **the teaching process** confirms this complexity: the teacher says that “even the best student asks for an indication” and that she finishes the session by showing in a collective way how to do the proof. So, in this first example, there is no experimental activity of students but only several immediate applications of some explicit pieces of knowledge.

The exposition for the second studied sequence is the following:

Let $k$ be a real positive. **We are interested about the number of roots of the equation ln($x$)=$kx^2$ for $x$ positive..**

1. Open the software **Géogébra**.
2. In the entry windows, enter f($x$)=ln($x$) then validate. Enter $x$^2 then validate. Do the same with 0.5$x$^2, then 0.1*$x$^2 and then $-x$^2. Fill in the table :

| Value of $k$ | | | | |
|---|---|---|---|---|
| Nomber of roots according the graphical curves | | | | |

3. We want now to determine in a more precise way the number of roots. Click on “Fenêtre”, then “Nouvelle Fenêtre” and then let appear the curve of the function ln in this new frame.
4. Enter $k=1$ in the entry window then validate. This number appears in the algebra window. In the entry window, define now $g(x)=kx^2$.
5. Vary the number $k$, then click with the right button of the mousse on this number, then click on “Afficher l’objet”. A cursor appears. Click on  to define the mode “déplacer”, and then displace the cursor with the mousse.
6. **Conjecture following the values of $k$ the number of roots of the equation** $\ln(x)=kx^2$**.**

*Call the teacher to validate your answer.*

7. If $k>0$, graphically find a value of $k$ with two right digits after the decimal point for which the equation admits only one solution (you can right click on $k$ and then on “Propriétés”, “Curseur”, to reduce the increment inside the

| |
|---|
| interval)<br>*Call the teacher to validate your answer.*<br>Demonstrate on your sheet that for any negative value of k, the equation admits a unique solution. |

**Table 2: exposition given in the second example of mathematics teaching sequence**

This second example assumes a level of software's competencies which is lower than the first one but we don't want to enter in this problematic for this communication. However, **the a priori analysis** shows that the experimental construction is again given by the exposition (questions 3, 4, 5, 6). Moreover, the exposition initiates the activity of testing some particular cases of the whole open problem (question 2). The so called experimental part is again isolated from the one more strictly mathematical. This last one is cute in two sub tasks (questions 7 and 8) while a real experimental activity should lead to treat the whole task.

**The teaching process** shows that lot of pupils don't see the link between the curve they draw during question 2 and some particular cases of the problem. The teacher says that they didn't see how to fill in the table. This reinforces the idea that there is not at all experimental activity during this sequence. Moreover, the question 8, even if it is simplified by the exposition, remains very difficult for pupils.

With these two examples, we understand that the expositions, as in educational texts, are effectively open problems: "to discover a property of the slope of the exponential curve" and "we are interested in the number of roots of the equation $\ln(x) = kx^2$ for $x$ positive". But the field of activity is too large to allow an autonomous activity of students and the tasks are simplified by the expositions: "Realization of the figure", "Fill out the table". In other words, the experimental management is not in charge of the students but it is explained by the detailed expositions. So the hypotheses are evidenced at the end of the explained manipulations. There is no reason to question these hypotheses even if some questions can be awkward in this direction, as in the first example: "Try to imagine a method to confirm this hypothesis with experimentation".

Then, a classical proof ("research of a proof", question 8), isolated from the manipulation phase, is asked. Moreover this proof can be difficult for students because of complex uses of available knowledge and because manipulations don't help for this purpose at all. However, in general, we think that there could be an interaction between the two parts of the session. For instance, in the first example, the manipulation of software Géogébra requires the internalisation of some commands. More precisely, the command "curseur" of the software is deeply associated to the introduction of parameter to prove the hypothesis. So there could exist a though to help students to introduce parameters in their proofs by training them to associate parameters and "curseur" in ICT environment.

## In physics

An analysis of 26 teachers' worksheets available on pedagogical websites and supervised by the educational authorities was conducted a few months ago (Mathé & al. 2008). This analysis revealed important gaps between IBT in the curriculum and teachers' perceptions or appropriation. In particular, it has been shown that few of them make pupils' conceptions explicit in their worksheets and build their sequence in order to destabilize these conceptions. Moreover, while the curriculum comprises a phase of statement of hypotheses, only 11 worksheets ask pupils for stating hypotheses. Furthermore, only 9 protocols are entirely designed by the pupils. In the other worksheets, the teacher plays a more or less important part: whether he designs the protocol himself or he imposes the experimental equipment, or corrects the pupils' propositions (Mathé 2008, Mathé & al. 2008).

The sequence we take as an example concerns combustion processes. The new knowledge aimed by the sequence is exposed as following:

- the combustion of carbon requires oxygen and produces carbon dioxide;
- a fire naturally occurs when air, heat and fuel are combined.

These three elements form the "fire triangle". When one of these elements is missing, the fire stops. The problem to be solved –"How to extinguish a fire"– is connected to an everyday-life starting situation which is supposed to motivate the pupils. They are asked to go outside the classroom, to find all the anti-fire and fire protection devices of the school and to explain the way they operate. Doing so, the teacher expects the children to make hypotheses on combustion process such as "oxygen is necessary for the combustion process" or "combustion produces carbon dioxide". This hypothesis should be tested by appropriate experiments elaborated and performed by the children. The sequence is implemented with grade 7 children and last two hours. It is video-recorded and transcribed. We focus here on specific heading: children's conceptions, the statement of hypotheses, and the hypothetico-deductive process.

**The a priori analysis** shows that the tasks proposed to the pupils can't destabilize children's conception about fire such as "fire is an object endowed with material properties" widely studied by philosophers and science education researchers (Bachelard 1938, Méheut 1982), and we wonder to what extent it doesn't strengthen it. Indeed, attention to the anti-fire devices operation does not automatically leads to the idea that the air supply is necessary in the combustion process. Consequently, the problem to be solved can't lead to the statement of the expected hypotheses either. Thus, no spontaneous hypothetico-deductive process can be expected.

**The analysis of the teaching processes** confirms this difficulty. Children are easily involved in the preliminary activity which consists in describing the anti-fire and fire protection devices of the school. A difficulty appears when the teacher asks them to describe the way the devices operate. We observe a misunderstanding between the teacher's expectation which concerns the underlying chemical process and the pupils' answers that exclusively focus on the description of the way the device is used. This unexpected difficulty leads the teacher to formulate a more precise and guided question: “can you explain why these devices extinguish the fire?”. At that time, a second difficulty occurs which is directly connected to the way that “the fire” is considered in pupils’ mind. As an example, pupils think that fire-resisting doors close in order to prevent the fire to move forward. According to them when the doors are closed the fire “bounces” on them. None of the pupils spontaneously establish a link between the air (specifically the oxygen) and the existence of the fire. This difficulty is widely underestimated by the teacher during the effective sequence. Finally, after one hour of discussion, the expected hypotheses are given by the teacher himself: “oxygen is necessary for a fire to exist” and “a fire produces carbon dioxide”. Pupils are then invited to elaborate experiments in order to test the hypotheses. In this phase, they must isolate the different air contents to prove that only the oxygen plays a part in the combustion process. They also have to elaborate an experiment in order to evidence the carbon dioxide. In the next course, contrary to what was planned, the experiments are imposed and performed by the teacher. This is directly connected to management constraints.

According to the a priori analysis, we observe significant gaps between the teacher’s intentions and what really occurred during the effective sequence. Children’s ideas about the burning process and the fire are not destabilized by the inquiry-based activity itself. The teacher plays a determining part in the knowledge transmission and the starting situation doesn’t allow the implementation of a cognitive-conflict as expected in the IBT. Moreover, the teacher asks the pupils to design an experimental protocol but he finally imposes his own experiment.

## CONCLUSION

We assume that no generalities can be asserted as the analyses previously presented remain clinical. Nevertheless, some regularity seems to emerge that form tracks to explore.

What is specific to us is the need for teachers to make a quadruple prior analysis, lighter than the researcher’s one of course, in order to effectively implement this type of process in their classroom:

- an analysis of the aimed knowledge or the knowledge to be used (different from a subject to another);
- an analysis of the available knowledge to permit an autonomous activity of pupils;
- an analysis of the role played by the experimental process in the connection between the aimed knowledge (or knowledge to be used) and the available knowledge considering both content and teaching processes;
- an analysis of the way the teacher can manage this experimental activity.

Moreover, in physics, depending on the nature of the referred content, an inquiry-based teaching can be adapted or not. IBT in the classroom requires choosing relevant scientific content and problem that aim at destabilizing pupils' conceptions and that allow the implementation of a hypothetico-deductive process by the pupils implying more autonomy for the statement of hypotheses and the design of a protocol. However, it may be that students cannot develop hypotheses highlighting their misconceptions. In that perspective, the choice of the scientific subject remains fundamental.

In mathematics, we have seen that there is a problematic amalgam between an experimental approach of mathematical activity and an activity with ICT tools, these tools being able to lead pupils easily to emit correct conjectures for complex problems. The experimental constructions being given by the expositions, the experimental activity can only exist in a one to one correspondence between manipulations (not experimentations) and proofs. This activity, even if it is far from scientific one, can be interesting for using mathematical knowledge (activity with available knowledge or activity with adaptations of knowledge). But it is difficult for students who are not accustomed with these activities. It is also difficult for teachers who have to find adequate situations permitting these go and return between manipulations and proofs and who have to manage at the same time the learning of the new knowledge as well as the learning of software's competencies. This kind of studies has to be completed by some results on individual different students' attitudes when working on computers (Vandebrouck, 2008).

## REFERENCES


Bachelard, G. (1938). *The formation of the scientific mind*. Clinamen Press Ltd.

Horoks, J., Robert, A. (2007). Tasks Designed to Highlight Task-Activity relationships, *Journal of Mathematics Teacher Education*, vol 10, 279-787

Leontiev, A., (1984) Activité Conscience Personnalité. Moscou, Editions du Progrès

Mathé, S., Méheut, M., de Hosson, C. (2008). La démarche d'investigation au collège : quels enjeux. *Didaskalia*, vol. 32, 41-76.

Mathé, S. (2008). Inquiry-based teaching in low secondary school in France: which stakes? *Proceedings of the ESERA summer school.* York (England): 25-29 august 2008.

Méheut, M. (1982). *Combustions et réaction chimique dans un enseignement destiné à des élèves de sixième*. Thèse de troisième cycle. Université Paris 7.

Robert, A., Rogalski, J. (2005). A cross-analysis of the mathematics teacher's activity. An example in a French 10th-grade class. *Educational Studies in Mathematics*. Vol 59, 269-298.

Pariès, M., Robert, A., Rogalski, J. (2008). Analyses de séances en classe et stabilité des pratiques d'enseignants de mathématiques expérimentés du second degré, *Educational Studies in Mathematics*. Vol 68, 55-80.

Robert, A. (2008). La double approche didactique et ergonomique pour l'analyse des pratiques d'enseignants de mathématiques. In Vandebrouck (Ed.) *La classe de mathématiques : activités des élèves et pratiques des enseignants* (pp 59-68). Toulouse: Octarès

Vandebrouck, F. (Ed) (2008). La classe de mathématiques : activités des élèves et pratiques des enseignants, Toulouse: Octarès

Vergnaud, G., (1990) La théorie des champs conceptuels, *Recherches en Didactiques des Mathématiques,* Vol 10 (2/3), 133-170.

Vygotski, L. (1997) Pensée et Langage. Paris, Plon

# PROFESSIONAL DEVELOPMENT FOR TEACHERS OF MATHEMATICS: OPPORTUNITIES AND CHANGE

Marie Joubert[1], Jenni Back[2], Els De Geest[3], Christine Hirst[4] and Rosamund Sutherland[1]

*University of Bristol[1], Kings College London[2], Oxford University[3], University of Birmingham [4]*

*The RECME research was set up to develop understanding of 'effective' Continuing Professional Development (CPD) for teachers of mathematics by looking at a large number of initiatives adopting a variety of models, taking a non-interventionist, non-participatory approach. In addition to building a 'big picture', it also aims to develop an in-depth understanding of the individual initiatives by looking at the structure and organisation and at the responses of individual teachers to their CPD. The paper develops and uses an analytical framework to help us understand one particular initiative and the learning and teacher change of individual teachers participating in this initiative. We conclude with a discussion of the factors contributing to the effectiveness of the CPD.*

Keywords: Professional development, mathematics, teachers, CPD

## INTRODUCING RECME

In 2006 the National Centre for Excellence in the Teaching of Mathematics (NCETM) was set up in England in order to build a coherent infrastructure to support the continuing professional development (CPD) needs of teachers of mathematics. In 2007 the NCETM funded an eighteen month research project, Researching Effective CPD in Mathematics Education (RECME). The aims of the project include the characterisation of different types of CPD for teachers of mathematics and the investigation of the factors contributing to 'effective' CPD. In order to understand the range and scope of CPD opportunities existing in the UK, the project team researched a sample of thirty initiatives representing different models of CPD in mathematics education, run by a variety of providers, in different locations, and aimed at about 250 teachers of students in pre-primary, primary, secondary, further and adult education settings.

RECME is an ongoing project and has not yet produced comprehensive findings or recommendations. These are due by March 2009. However, most of the data for the project has been collected and this paper introduces a framework for the analysis of the data and uses it to analyse the data from one initiative.

## THEORETICAL FRAMING AND METHODOLOGICAL DECISIONS

We adopt a broad sociocultural perspective which suggests that all human activity, including the learning of teachers, is historically, socially, culturally and temporally situated. This suggests that the experiences and contexts of teachers will have a major influence on their learning and implies a need to pay attention not only to the situation, the opportunities and the context of sites of learning (in our case initiatives of professional development), but also to the individuals (teachers of mathematics) taking part in professional development.

## Data collected

For each initiative we asked the leader/ coordinator for data concerning the form and structure of the professional development. We also observed at least one professional development meeting and took observation notes. The data we collected included dates of meetings, structure of meetings, number of participants, duration of the CPD, what takes place in meetings, funding/costs, support and communication structures, recruitment procedures and leaders of the meetings. For some initiatives not all this data was applicable.

With the help of the leaders/co-ordinators, we identified two teachers from each initiative. We visited these teachers in their classrooms and observed them teaching mathematics in order to develop understanding of the context in which they work, and interviewed them after the observed lesson. The interview data included questions about professional background, perceptions of their professional identity, thoughts on the observed lesson, influence of the CPD on the way they teach, motivation to take part and remain involved in the CPD, their CPD histories and how they felt about the CPD.

## Analytical framework

An initiative of professional development can be described in terms of the content, context and processes in which participants engage (Harwell, 2003). There is a wide range of different models of CPD (see for example Kennedy, 2005) but most CPD aims to provide opportunities for teachers to become involved in processes of learning and change. We suggest that different teachers, influenced by the contexts in which they work and their personal motives, beliefs, theories and experience, will perceive different opportunities, and these perceptions may shift over time.

The professional development of the individual teachers inevitably relates to the opportunities provided by the CPD initiative (Muijs, 2008), and may lead to learning and changes in attitudes and beliefs (actual PD). Teachers may also change their classroom practice, but it is possible that changes in classroom practice could also be influenced by other formal and informal learning. Changes in practice could lead to changed student behaviours and possibly improved student learning (Guskey, 2002), although once again there are other factors which might influence any changes that do take place. In turn, changes in student behaviour and learning could influence the teacher learning (Cooney, 2001), their perceptions of the opportunities and experiences offered by the CPD, and the opportunities and experiences they decide to take up.

Finally, a sociocultural perspective suggests that we also need to take into account the influences of the school and national context on the design of the CPD initiative (Bishop & Denleg, 2006; Cobb, 2008) and of the motives, beliefs, theoretical understanding and experience of the designers of the CPD (Rogers et al., 2007), the feedback they receive from the ongoing CPD, as well as the specific aims of the initiative (Goodall, Day, Lindsay, Muijs, & Harris, 2005)

Figure 1, below, provides a diagrammatic representation of the interrelationships of all these factors.

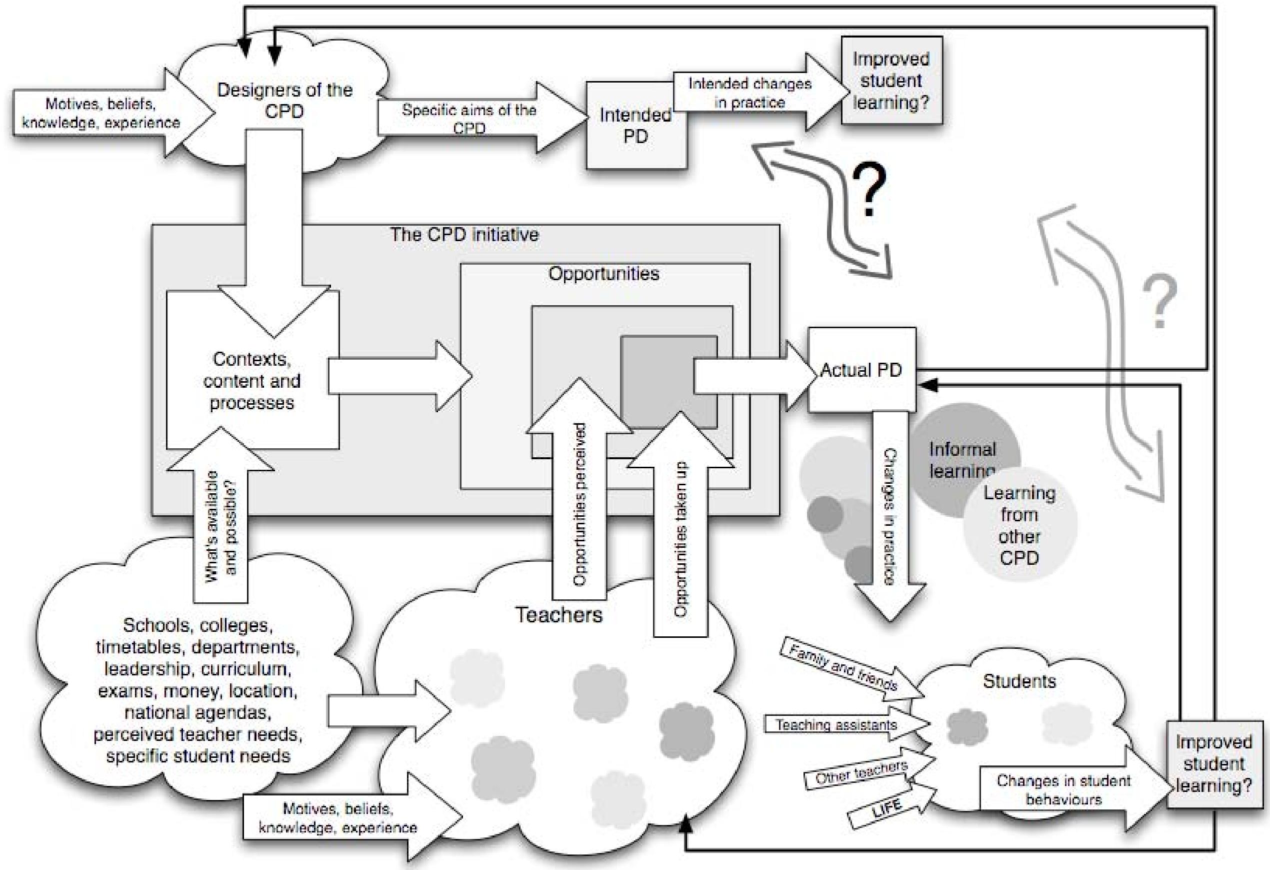



**Figure 1: Understanding a CPD initiative**

As with many analytical frameworks, this representation could be seen as 'too neat', yet the data is messy and complex. Further, it is a static diagram which cannot represent the ways in which the nature of the CPD may be dynamic and changing in response to feedback from teachers and their changing needs over time. However, we suggest that it provides a useful lens for understanding both the CPD initiative itself and the participation of individual teachers. In addition some of these asrrows could, in many cases, be two ways.

Further it explicitly attends to the teacher professional development intended by the organisers of the CPD and the intended changes in teachers' practice, and to learning and changes that do take place. This is important in our view, because both these can be seen to provide some 'measure' of the effectiveness of the CPD (Garet, Porter, Desimone, Birman, & Yoon, 2001; Goodall, Day, Lindsay, Muijs, & Harris, 2005; T. R. Guskey, 2000; Thomas R. Guskey, 2003) (although we do recognise that the ultimate aim of the CPD is usually improved student learning).

## CASE STUDY: ONE INITIATIVE AND TWO TEACHERS.

### Context, content and processes of the CPD initiative

This initiative is run by a local authority mathematics adviser and a university-based teacher educator. The initiative is now in its third year; two cohorts have already completed the programme. The participants are all secondary school mathematics teachers who attend five separate day-long meetings over the course of a year.

During the meetings the course leaders initiate discussion, frequently asking the participants to discuss issues (for example, how they feel about group work in the mathematics classroom) and then to report back to the group. Frequently one of the course leaders notes down the points made on a flip chart and, when each small group has reported back, draws out some of the key points. During the meetings they also introduce new resources to the teachers and discuss how they might be used and hand out research papers and give the teachers time to read them and then lead a discussion about them. Much of the material they hand out focuses on questioning techniques and much of the discussion concerns using open questions and tasks rather than closed questions and tasks.

In addition, they introduce various classroom mathematics activities and ask the teachers to work in small groups to complete them. For example, one of these activities uses small cards with equations, graphs and co-ordinates of points printed on them, although some are left blank. The task is to decide how to group them, but importantly there is no correct or incorrect answer, and consequently can be seen as providing rich learning. Further, when these activities are used in the classroom, they provide opportunities for teachers to assess their students' prior knowledge. The teachers are asked to experiment in their classrooms between the meetings by using either this activity (suitably adapted for their particular circumstances) or some other activity designed by themselves. The activity they choose to use is called a 'gap' activity (because it is to be carried out in the 'gap' between meetings). There is no prescribed type of gap activity; the key point about the gap activity is that it represents something new for the teacher to try out in the classroom. Teachers are asked to bring some of the students' work from these gap activities to the next day meeting to form the basis of discussion.

Teachers are also asked to keep a journal. At the last day meeting, they are asked to make a presentation to the group, outlining how their practice has developed through the project.

## Aims of the CPD

Although the course leaders state that ‘this project focuses on helping teachers to understand the underlying principles of assessment for learning and applying these to embedding effective practice in the classroom’ (www.nctem.org/recme), they told us that the actual content addressed in each of the days is, to some extent at least, informed and influenced by the work of the teachers both during the meetings and in the classroom, and by their concerns and questions. In order to be free to follow this flexible approach, the course leaders deliberately do not have any further documented specific aims.

However, they told us that their general aims are threefold and they see them as related and interdependent: to provide time for the teachers to reflect, to encourage teachers to put their learning into practice in the classroom and to engage the teachers with relevant research.

They also said that the course aims to create a community in which teachers meet, talk, share and learn from one another. The leaders have created a community web page where the teachers are able to share resources, thoughts and ideas, away from the face-to-face sessions.

## Intended professional development (teacher learning)

The course leaders told us that they hoped that by providing the opportunities described above, participating teachers would be inspired to think more critically about their own practice and revise it accordingly, to pay more attention to how pupils learn mathematics, and to develop the confidence to allow pupils to follow their own directions rather than scripting their lessons in detail.

## Intended changes in practice

The intention is that teachers will change their practice in the short term by experimenting with the gap tasks. In the longer term the course leaders said they hoped that teachers’ practice would change in three main ways:

- They would use more challenging and open tasks in the classroom, with less reliance on textbooks and closed questions, leading to more exciting and unpredictable lessons for the students
- They would reflect more on what happened in mathematics lessons, thinking more about what the learning had been rather than about how much material had been covered
- They would become more relaxed in their interactions with the students and develop more collaborative classroom practices.

## The teachers: Barbara Bircher and Peter Millward

This section discusses the CPD experiences of Barbara and Peter, the two teachers who were invited to take part in the in-depth part of the research. It reports on what they said when our researcher interviewed them and on the observation of their lessons, and uses the framework developed above to structure the discussion. It begins by describing the backgrounds of the teachers and the contexts in which they work.

Barbara has been teaching mathematics in secondary schools since 1976 and is now subject leader for mathematics in her school. Peter is in his third year of teaching at a large comprehensive 11 – 18 school where he has overall responsibility for the first three year groups in the school (known as Key Stage 3 and culminating in a standardised national test).

Barbara became involved in the current CPD because she had heard a lot about the course, which is now in its third year, and she liked what she heard: the approaches she heard they promote are similar to the ones she believes in. She thought it would be valuable for someone in the department to attend and decided to go herself (rather than sending someone else from the department), because then she could cascade her learning to the rest of the department. She saw this as an opportunity for her to develop herself in order to 'move the department forward'.

Peter said that he decided to take part in the CPD because a member of the senior leadership team asked him if he wanted to go. He said that much of the CPD he had previously experienced had taken place in school and 'seems to be more about technical jargon than new stuff but that he chose to attend this CPD because he was looking for something with more mathematics.

## Opportunities

In this section we report on those opportunities provided by the course that Barbara and Peter seemed to value. Both teachers mentioned the resources they had been introduced to, with Barbara saying that she valued having time to investigate them and Peter saying they were useful.

Barbara said that she values the time out of school to reflect and think and discuss, she enjoys having time to read. Peter also said he liked the fact that there was enough time for discussion and he seemed to value the opportunity to meet with other people in order to 'stock up' with ideas to try out in the classroom.

Peter did not mention the value of gap tasks, but he did say that, as a result of the course, he has to 'push' himself to try something out and this is the most useful thing about the course. Barbara told us that she had used most of the gap tasks with her classes and reported back on them. She said that knowing that she 'had to' report back on how she had found teaching these gap tasks meant that she had actually done them, and that otherwise she may not have. She said she enjoyed reporting back to the group after doing a gap task. She said that the course had given her the opportunity to do what she believes is good maths teaching.

To Peter, the course leaders are very important; ‘they prepare the stuff, they help us along’. He says that they provide a link between the theory and practice in both his own classroom and what other schools are doing. The local authority advisor has a good overview of what happens in his local authority, and he says this is useful for the teachers.

## Actual professional development

Barbara said that using the gap tasks had challenged her embedded practice of expecting the students to work in a predetermined direction and reawakened her awareness that ‘the obvious isn’t obvious’. She said that it has kept her interest in mathematics teaching and her desire to be a reflective practitioner continuing to improve. She said that the course had reminded her about what she really liked doing; teaching mathematics, adding that in recent years she has moved gradually away from her passionate interest in teaching, because of the pressures of school and management. Barbara said the course made her very excited and gave her the opportunity to do what she believes is good maths teaching. She finished the course wanting more. More specifically, she reported that she had learnt the value of sharing students’ work and of developing a classroom culture in which ‘it is ok to be wrong, as long as you are thinking about your learning’.

Attending the course had made her think about the direction she wanted to move in, in terms of her role in the school, and has provided her with clear ideas about the way she intends to develop the department.

Peter was much less forthcoming about telling us about his learning and changes in beliefs. However, he did report that the course ‘replenishes my enthusiasm’. He also remarked on a change in awareness:

> ‘I am more aware of what I am doing and thinking much more about what I am doing and why’.

## Changes in practice

Both teachers reported that they had implemented some new teaching tasks as a result of the CPD. Barbara had tried some of the gap tasks and is now incorporating more open and investigative tasks in her everyday teaching. For example, she gave the class coloured paper and scissors and provided the students with instructions on how to create the shapes she wanted them to work with. Over the course of several lessons, the students investigated angles and lengths in the shapes, as well as tessellation properties.

Peter, on the other hand, did not use a gap task but told us that he has tried to integrate some of the ideas from the CPD into his normal practice, rather than relying on the textbook too much. He has also used ideas for new tasks which came from another teacher in the group. For example, he asked a year 9 class to write a test and devise a mark scheme and he was very pleased with the work they produced. He was particularly pleased with the work one of his students produced. He said:

> 'I will use this idea again - its fairly easy to setup, although grading is quite a challenge. It's effective because it allows students to show what they have learnt and it always easily differentiates between students' abilities. Answering a question on a test can be algorithmic, writing a challenging question (with a mark scheme) can show greater understanding'.

Both teachers reported that they used more open small tasks at the beginning of the lesson (sometimes called starter tasks in the UK). For example, Barbara said she might present a diagram and ask students to write a statement about it; she remarked that previously she would probably have asked a more direct question. She said she allowed them to make any points they wanted before she directed the discussion towards her main teaching points. She chooses some starter tasks in order to promote discussion, such as asking the students to find a number with exactly five factors, which led to a discussion of the fact that numbers with an odd number of factors are a special sort of number (square). She said that in the past she would probably have given the class a more closed starter such as 'What are the factors of 16?' Peter provided an example, saying he might say 'The answer is a quarter, what is the question?' and he said this provided the students with opportunities for creative thinking.

Barbara told us that in order to share students' work she obtained a visualiser (a device which projects anything put under its lens onto a whiteboard) for her classroom. She now regularly shares student work in lessons. She also told us that because of her participation in the course, she has talked freely with her team about her own learning and she thinks this is good for the team. When our researcher spoke briefly to the second in charge in the department, he reported that the whole department had benefited from Barbara's CPD because she shared new ideas with them and encouraged them to experiment in their own classrooms.

Peter says that since he has been doing this CPD his teaching has changed. He says that he tries hard not to talk to the students from 'high up' and that he likes to get down to them (physically). He has started to move away from writing the lesson objectives on the board, and now has primary and secondary objectives (skills-based and content-based respectively). Sometimes he leaves an objective blank and asks the students at the end of the lesson what it they thought it was. This is an idea that came from someone at the CPD.

## THE INTERRELATED FACTORS CONTRIBUTING TO EFFECTIVE CPD

The discussion above provides some evidence that for both teachers some learning and changes in practice took place. In-line with the learning and changes the course leaders intended (see page 5), both teachers took some risks, using more open and challenging tasks in the classroom, and developing more relaxed interactions with their students. Barbara appears to have developed confidence to allow pupils to follow their own directions more and she had begun to think more critically about her own practice. We argue that this demonstrates that, to some extent at least, the CPD was ‘effective’.

This raises the questions of the factors that may have contributed to this effectiveness, and what barriers may have been present to reduce effectiveness. First, both teachers confirmed the importance of experimenting in the classroom as suggested in the literature (see for example, Guskey), and what is perhaps interesting is how the CPD is set up to encourage this experimentation. We suggest that teachers involved in this CPD felt they have to try something new in their classroom, because it is expected and because of the need to report back to the group. There was also some encouragement from the leaders’ comment that attending the course gave permission to take risks. It is interesting that Barbara chose to do the gap tasks, whereas Peter decided to try something suggested by one of the other teachers participating in the CPD. This may demonstrate that, although it was expected to do something between meetings, it seems that the way the task was set up allowed a great degree of personal choice in the selection of gap tasks.

The differences between the gap tasks chosen by the two teachers may be explained by the differences in their experience and positions in their respective schools and by the culture of the schools. For Barbara, as an experienced teacher and head of department it may have been much easier to implement the gap task suggested by the leaders of the CPD, but as Peter told us, he was not able to experiment and try out new things in the classroom as much as he wanted (this was partly because of an intervention programme that has been put in place in his school to address the whole school emphasis on raising attainment).

Second, being part of the CPD group was important to both teachers. This does not surprise us, as again the literature suggests that working collaboratively may contribute to effective CPD. However, we are interested in what it was for the two teachers that they valued. What seemed to be important for Peter was having access to new ideas, whereas Barbara’s emphasis was on the sharing of what she had done and the out-loud reflecting on it.

Thirdly, and again unsurprisingly (Borasi, Fonzi, Smith, & Rose, 1999; Day, 1999; Olson & Barrett, 2004), it seems that having time away from school to think and discuss was important to the teachers, although we cannot tell what contribution this discussion made to the professional development of the teachers. However, our suggestion is that they found it stimulating and enjoyable, and that this sort of discussion has an important role in retaining the interest and motivation of teachers.

As a final point, our observation of two of the meetings suggests that the participants enjoyed ‘doing’ the mathematics and our suggestion is that this is an important factor contributing to ‘effective’ CPD. However, interestingly, neither teacher commented on the enjoyment they experienced when they were given the mathematical gap tasks to work on in the meetings.

## REFERENCES

Bishop, K., & Denleg, P. (2006). Science learning centres and governmental policy for continuing professional development (CPD) in England. *Journal of In-Service Education*, *32*, 85-102.

Borasi, R., Fonzi, J., Smith, C. F., & Rose, B. J. (1999). Beginning the Process of Rethinking Mathematics Instruction: A Professional Development Program. *Journal of Mathematics Teacher Education, 2*(1), 49-78.

Cobb, P. (2008). District Development as a Means of Improving Mathematics teaching and Learning at Scale. In K. Krainer, & Wood, T. (Ed.), *International handbook of mathematics teacher education: Vol. 3. Participants in mathematics teacher education: Individuals, teams, communities and networks.* (Vol. 3). Rotterdam: Sense Publishers.

Cooney, T. J. (2001). Considering the paradoxes, perils, and purposes of conceptualizing teacher development. In F. L. Lin & T. J. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 9-31): Springer

Day, C. (1999). Professional development and reflective practice: purposes, processes and partnerships. *Pedagogy, Culture and Society, 7*(2), 221-233.

Goodall, J., Day, C., Lindsay, G., Muijs, D., & Harris, A. (2005). *Evaluating the impact of continuing professional development*. London: DfES.

Guskey, T. R. (2002). Professional Development and Teacher Change. *Teachers and Teaching: Theory and Practice, 8*(3), 381-391.

Harwell, S. H. (2003). Teacher professional development: It’s not an event, it’s a process. *Waco, TX: CORD. Retrieved January, 21*, 2004.

Kennedy, A. (2005). Models of Continuing Professional Development: a framework for analysis. *Journal of In-Service Education, 31*(2), 235-250.

Muijs, D. (2008). Where are we at? An empirical study of levels and methods of evaluating continuing professional development. *British Educational Research Journal*, *99999*(1), 1-17.

Olson, J., & Barrett, J. (2004). Coaching teachers to implement mathematics reform recommendations. *Mathematics Teacher Education and Development, 6*, 73-91.

Rogers, M., Abell, S., Lannin, J., Wang, C.-Y., Musikul, K., Barker, D., et al. (2007). Effective Professional Development in Science and Mathematics Education: Teachers' and Facilitators' Views. *International Journal of Science and Mathematics Education, 5*(3), 507-532.

# TEACHERS' PERCEPTIONS ABOUT INFINITY: A PROCESS OR AN OBJECT?

Kattou Maria, Michael Thanasia, Kontoyianni Katerina,
Christou Constantinos, & Philippou George

Department of Education, University of Cyprus

*The present study aims to examine elementary school teachers' perceptions about the notion of infinity. In particular, the two aspects of the concept- as a process or as an object- were examined through participants' responses. In addition, teachers' reactions during the comparison of infinite sets or numbers with infinite decimals were analyzed. Data were collected through a self-report questionnaire that was administered to 43 elementary school teachers in Cyprus. Data analysis revealed that the majority of teachers comprehend infinity as a continuous and endless process; thus, teachers confront difficulties and hold misconceptions about the concept.*

*Key words: infinity, teachers' perceptions, misconceptions, actual and potential infinity*

## INTRODUCTION

A major component of the research in mathematics education in the last decades has been the study of students' and teachers' conceptions and reasoning about mathematical ideas. Most of the research purported to examine the existence and persistence of alternative conceptions (preconceptions, intuitions) which diverge from the accepted mathematical definitions (e.g. Monaghan, 1986; Tall, 1992). The concept of infinity may be seen as a mathematical idea that causes various obstacles to learners due to the duality of its meaning, as an object and as a process (Monaghan, 2001). Thus the present study examines how primary school teachers conceive the notion of infinity in an attempt to define the notion, to provide suitable examples and to comprehend numbers or sets with infinite elements.

## THEORETICAL FRAMEWORK

### Definition of the concept of infinity

The notion of infinity constitutes an intuitively contradictory concept that has long occupied many philosophers and mathematicians. Concretely, infinity emerged as a philosophical issue in the work of Aristotle, who separated the concept in two different aspects- potential and actual- that correspond to the ways of looking at infinity- as a process or as an object (Sacristàn & Noss, 2008; Tirosh, 1999). According to Aristotle the potential infinity can be conceived as an ever lasting activity that continues beyond time, while the actual infinity as the not finite that is presented in a moment of time (Dubinsky et al., 2005). The former category of infinity appears as something that qualifies the process, whereas the latter category refers to an attribute or property of a set (Moreno & Waldegg, 1991).

The acceptance of potential infinity elicited a mathematical way of thinking that gave rise to great accomplishments in Greek mathematics - such as, the Eudoxus method of exhaustion– but ruled out the possibility of developing an actual conceptualization of infinity (Moreno & Waldegg, 1991). In the 19th century, actual infinity through Cantorian set theory has profoundly contributed to the foundation of mathematics and to the theoretical basis of various mathematical systems (Tsamir & Dreyfus, 2002).

According to Galileo and Gauss, the use of actual infinity leads to inherent contradictions since it cannot be included in a logical, consistent reasoning. Due to the fact that the human brain is not finite, individuals cannot consciously focus on all the information at a given time- and therefore conceive infinity as an object- but they move between different aspects- and conceive infinity as a process (Tall, 1992). Usually, learners define infinity as "something that continues and continues" and not as a complete entity (Monaghan, 2001; Tirosh, 1999) or they conceive infinity using the limit notion, referring to a process of "getting close", with the limit perceived as unreachable (Cornu, 1991). On the other hand, the concept of actual infinity ascribed to learners through the reference to large finite numbers or to collections containing more than any finite number of elements (Monaghan, 1986).

**The construction of the N set**

From the time that Aristotle introduced the two meanings of infinity- potential and actual- difficulties in the understanding of the set of natural numbers were provoked. For example, regarding the formation of the set of natural numbers, a simple, not finite process begins from number 1 and adds one in each step indefinitely without stopping. This results to a line of infinite sets ({1}, {1,2}, {1,2,3}, …), which is an instance of potential infinity, a series of sets without end (Lakoff & Núñez, 2000). On the contrary, someone may consider the set of all natural numbers, without having the ability to enumerate all the elements of the set. By the encapsulation of the process, the object of N= {1,2,3,…} is created, that corresponds to the set of natural numbers (Monaghan, 2001). That is an instance of actual infinity - a completed infinite entity (Lakoff & Núñez, 2000).

**Comparing infinite sets**

One of the misconceptions that appears in the comparison of infinite sets is the application of properties that apply only to finite sets. Tsamir and Tirosh (1999) mentioned that methods used by learners for comparing infinite sets are largely influenced by the methods they tend to use when comparing finite sets. As Galileo (1945) pointed, a finitist interpretation that prevails upon the comparison of infinite sets is the use of the inclusion idea: that a set and a proper subset cannot be equivalent (Sacristàn & Noss, 2008; Tirosh, 1999). For instance, every natural number has its square and vice-versa, which means that the set of natural numbers and the set of their squares are equivalent, although the set of squares is a subset of natural numbers. Such a conclusion is not consistent with simple logic since the whole and the part cannot be equivalent. Therefore, an individual, in an attempt to reinforce his/her beliefs that a set has a different cardinality from any of its subsets, uses the justification of "part-whole" (Singer & Voica, 2003) than the one-to-one correspondence among the elements of sets that determines the equivalence between infinite sets (Tirosh & Tsamir, 1996).

Furthermore, many researchers (e.g. Tirosh, 1999; Tirosh & Dreyfus, 2002) explored the impact of different representations on the comparison of the same infinite sets. Researchers have focused on students' inconsistencies in relation to the concept of infinity using four different representational registers: horizontal, vertical, numeric explicit and geometric. Tirosh and Tsamir (1996) found that a numerical horizontal representation- in which the two sets are horizontally situated one next to the other- encouraged part-whole argumentation. On the contrary, the geometrical representation that is constituted of a schematic drawing of sets, triggered equivalent responses and "matching consideration" through a notion of pairing elements (Tirosh & Tsamir, 1996). It seems that geometrical representation prevents access to higher levels of conceptualisation and allows better understanding of one-to-one correspondence among the elements of infinite sets (Moreno & Waldegg, 1991).

**Conceptualising the equalities 0.999…=1 and 0.333…=1/3**

Various obstacles are presented with limiting processes that deal with the properties of the set of real numbers and of the continuum (Sacristàn & Noss, 2008). In particular, difficulties are observed during the comparison of irrational numbers which consist of infinite repeating and non-repeating decimals (Vinner & Kidron, 1985).

Many studies focused on the conceptualisation of the equalities 0.999…=1 and 0.333....=1/3 (Edwards, 1997; Monaghan, 2001). The majority of students tend to reject the former equality, on the ground that the two numbers have a negligible difference from one another (Monaghan, 2001) and with the limit being viewed as a boundary, rather than as the value of infinity (Cornu, 1991). With respect to the second equality, students seem to accept that 0.333… tends to 1/3, as it may result by dividing 1 by 3, something unfeasible in the case of the equality 0.999… =1 (Edwards, 1997). This happens because most students conceive number 1 more as an object, as an entity, while 0.999… is conceived as a process (Monaghan, 2001).

So far, several studies have examined learners' perceptions and misconceptions about infinity (Tsamir & Tirosh, 1999; Monaghan, 2001; Edwards, 1997). However, there is a lack of research studies that examine teachers' perceptions about infinity and this fact has served as a motivation to conduct this study. Namely, the purpose of the present study is threefold. Firstly, this study aims to examine the perceptions of elementary school teachers regarding the concept of infinity. In particular, the two aspects of the concept- as a process or as an object- are examined through the definition and participants' responses. Secondly, misconceptions that participants have during the comparison of infinite sets or numbers with infinite decimals will be discussed. Finally, the impact of different representations in the comparison of infinite sets will be investigated.

# METHODOLOGY

## Sample

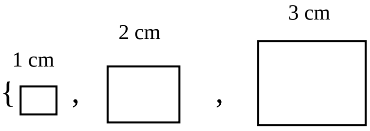
The present study involved 43 participants, 25 pre-service and 18 in-service primary school teachers, 12 men and 31 women. The experience of in-service teachers in instruction varied from one to 32 years. In addition, 25 participants possessed a master degree and one of them was a PhD degree holder. It is worthy to notice that the participants were randomly selected from a seminar offered in Mathematics Education at the University of Cyprus during the fall semester 2007-2008, without taking into consideration if they were pre-service or in-service teachers.

## Instrument

Data were collected through a self-report questionnaire (Figure 1), which took 20 minutes to complete. The questionnaire was comprised of four tasks that aimed to identify perceptions related to the concept of infinity.

1. a) Please give a definition of the concept of infinity.
   b) Give two examples for the concept of infinity.

2. How many elements are there in the set S= {-3, -2, -1, 0, {1, 2, 3,…}}?

3. Which of the following sets has the bigger cardinality? Please justify your answer.
   a) The set of natural or the set of even numbers?
   b) The set A= {1, 2, 3, 4, …} or the set B={1, 3, 5, 7,…}?
   c) The set A= {1, 2, 3, 4,…} or
   the set B = {1,½,1/3,¼, …}?
   d) The set of squares A= { 

   , … },
   or the set of numbers B= $\{1^2, 2^2, 3^2, \ldots\}$?

4. a) Is the equality 0.999…=1 true? Please justify your answer.
   b) Is the equality 0.333…=1/3 true? Please justify your answer.

**Figure 1: The tasks of the questionnaire.**

The first task aimed at eliciting teachers' perceptions about the concept of infinity. Participants were asked to report a definition for infinity and to present two examples that would involve the particular concept. The definitions were not coded as right or wrong answers according to formal mathematical concepts and notations, since the goal of the task was to address the underlying conceptions of infinity as a process or as an object.

The examples suggested by participants were grouped as mathematical or empirical examples according to their context. In particular, the examples that referred to mathematical concepts were categorized as mathematical examples. At the same time, the examples related to personal experiences or knowledge from real life were considered as empirical.

The second task examined teachers' understanding about the construction of an infinite set. Specifically, participants were asked to determine the cardinality of the set S={-3,-2,-1, 0,{1,2,3, ...}}, in which the infinite set of natural numbers appeared as an element of a different set. Moreover, the task attempted to investigate teachers' understanding about the construction of the N set as an entity or as a process.

The third task aimed to investigate the methods that teachers use during the comparison of infinite sets: the part-whole and the one-to-one correspondence. In addition, this task examined the impact of different representations in the selection of a criterion to determine the equivalence of infinite sets. The impact of four representations- horizontal, vertical, numeric explicit and geometric- were investigated in the comparison of infinite sets (Tirosh & Tsamir, 1996).

Finally, the fourth task included two sub-tasks that examined teachers' comprehension of the equalities 1=0.999… and 1/3=0.333… (Fischbein, 2001; Dubinsky et. al, 2005). The task aimed to observe the way teachers understand numbers with infinite digits and to compare the answers of the sample between the two equalities. The comparison was based on the different nature of the numbers, since the division of 1/3 can result to 0.333…, in contrast to 1 that can not be produced directly by 0.999…

The questionnaire required teachers to complete the four tasks and to justify their responses. Quantitative data were analyzed with the statistical package SPSS using descriptive statistics. The justifications and the examples provided by the sample were analyzed using interpretative techniques (Miles & Huberman, 1984), as evidence of teachers' perceptions about the concept of infinity.

## RESULTS

### Task 1. Definition of the concept of infinity

Two out of three participants (72.1%) defined infinity as an endless process. Teachers used phrases such as: "it goes on forever", "it's a process that never ends", "it has no beginning and no end…always follows another number", "keeps going and increasing". The remaining teachers (27.9%) defined infinity as an object. In their own terms: "it is an infinite whole", "it is something countless", "it is a set with unlimited elements", "it is an undefined set".

The majority of teachers (79.1%) were able to provide two examples for the concept of infinity, either mathematical or empirical, while 11.6% provided only one. The remaining 9.3% of the participants were unable to provide at least one example. Specifically, 62.8% of teachers presented two mathematical examples and 86.1% provided at least one mathematical example. The mathematical examples that were provided can be grouped as: (a) sets of numbers (e.g. natural, odds), (b) infinite sequences and series, (c) numbers that can be expressed as an infinite sequence of decimal digits (e.g. √2, 1:3), (d) geometrical examples (e.g. the set of straight lines through a point, the set of rectangles with perimeter 20 cm) and (e) trigonometric examples (e.g. the tangent of $90^0$).

On the other hand, only 30.3% of the participants gave empirical examples. The empirical examples that were provided in their own words were: “sunrays”, “earth’s rotation about its axis” and “the number of a satellite’s tracks in the void”. Participants provided wrong examples for the concept of infinity using objects the quantity of which is a large finite number, as stars, universe, sounds, grain of sands, and the number $10^{10^{\wedge}10}$. In addition, it is worthy to notice that 2.3% of the participants did not provide any example at all. One interesting statement was the following:

> “There are no specific examples for the concept of infinity. By the moment you define it, it stops being infinity any more”!

**Task 2. The construction of the N set**

In the second task, that referred to the cardinality of the set S={-3,-2,-1,0,{1,2,3,...}}, two different answers emerged. Even though it may seem to be striking, 38 out of 43 teachers (88.4%) considered the cardinality of the set S as infinity, while the rest of them (11.6%) considered that the cardinality is 5. The majority of the participants used explanations such as:

> “Set S has infinite elements, since it is an overset of {1, 2 , 3…} that is infinite.”
> “The set consists of infinite elements, because this (showing the N set) is unlimited.”
> “The cardinality of S is infinity because if you add 4 elements to infinity, you get infinity again: $\infty + \alpha = \infty$. ”
> “Elements included in S are: -3, -2, -1, 0 and all natural numbers.”
> “S is an infinite set in its positive direction.”

**Task 3. Comparing infinite sets**

The third task aimed to investigate the way different representations influence the comparison of infinite sets. As expected, the geometric representation helped the comparison more than the others, since 76.7% of teachers realized that the two sets presented, had the same cardinality. The respective percentages of correct answers for the other representations were: 46.5% for verbal, 51.2% for horizontal, and 55.8% for vertical representation.

As Table 1 shows, the geometric representation facilitated the participants to understand the one-to-one correspondence among the elements of the two sets rather than the remaining representations. Nevertheless, none of the teachers showed a coherent reasoning that connects infinite sets to confirm their explanation.

| Justifications | Representation | | | |
|---|---|---|---|---|
| | Verbal | Horizontal | Vertical | Geometric |
| 1-1 correspondence | 3 (7.0%) | 3 (7.0%) | 5 (11.6%) | 13 (30.2%) |
| Part-whole | 18 (41.9%) | 17 (39.5%) | 15 (34.9%) | 6 (14.6%) |
| None | 22 (51.2%) | 23 (53.5%) | 23 (53.5%) | 24 (55.8%) |

**Table 1: Justifications for the comparison of infinite sets**

Moreover, the geometric representation reduced the misconception “the whole is greater than the part” that in other cases causes false answers. Some indicative false answers using the “part-whole” justification are presented below:

> "There are more natural numbers than odd numbers. Odd numbers are only a part of natural numbers."
> "Set A={1,2,3,4,…} has more elements than set B={1,3,5,7,…}, because set A contains also even numbers."
> "Set B={1,½,1/3,¼,…} has additional elements than A={1,2,3,4,…}, since you can find many fractions between two natural numbers. "

**Task 4. Conceptualising the equalities 0.999…=1 and 0.333…=1/3**

Participants conceived the above equalities differently, providing three categories of answers. Specifically, 41.9% of teachers thought that the equality 0.333…=1/3 is right in contrast with 4.7% that accepted the equality 0.999…=1 as correct. The majority of the teachers (58.1%) used the concept of limit to confirm the correctness of the equality 0.999…=1, while only 27.9% of them used a similar explanation for the equality 0.333…=1/3. The difference between the two conceptions was supported by the following statement:

> "0.333…=1/3 because if you divide 1 by 3 you get 0.333… but you don't get 0.999… if you divide 1 by 1."

A considerable number of participants answered that the two equalities are false (34.9% for 0.999…=1 and 27.9% for 0.333…=1/3). Some indicative false explanations offered by teachers regarding the equality 0.999…=1 were the following:

> "Number 1 will always be larger than the largest decimal number 0.999…"
> "In daily life, the equality can be right due to rounding-up, but in mathematical contexts, the numbers 0.999… and 1 are different."
> "There is an infinitesimally small difference between the two numbers."
> "An equality is not right unless a=a is valid."

Teachers' explanations for the equality 0.333…=1/3 were similar to the former ones.

## DISCUSSION

The present study examined elementary school teachers' conceptions about infinity. Specifically, the aim of the study was threefold: to examine teachers' perceptions about the nature of infinity as an object or as a process, to investigate teachers' misconceptions during the comparison of numbers or sets with infinite elements and to discuss the impact of different representations in the comparison of infinite sets.

The majority of teachers comprehend infinity as an unlimited process as indicated by their responses on tasks 1, 2 and 4. This finding is in accordance with the work of many researchers (Tall, 1992; Monaghan, 2001; Tirosh, 1999) who stated that a person's comprehension regarding the notion of infinity is supported by the strength of his intellectual finite schemes that are mainly referred to the process that creates infinity than to the completed entity. The intuitive interpretation of infinity as potential constitutes a cognitive obstacle in the understanding of the concept and therefore individuals confront difficulties and hold misconceptions about the concept (Fischbein, 2001).

Teachers mainly conceive infinity as a mathematical idea with limited applications to daily life. The fact that teachers quoted examples from various fields of mathematics (e.g. geometry, trigonometry, and series) indicates that the concept of infinity is presented throughout the mathematics curriculum. Although some empirical examples were provided, these included large finite numbers. According to Singer and Voica (2003), due to the human's disability in counting the grain of sands or in computing the number $10^{10\wedge 10}$, the person correlates them with the concept of infinity. Indeed, when an individual cannot observe something with his/her senses totally, then this thing is connected with the notion of infinity, which is by definition something unreachable.

The results of the study reveal the correlation between the definitions of infinity with its mathematical implications during the construction of an infinite set, as the N set. Although teachers were expected to determine that set S={-3,-2,-1,0,{1,2,3,.. .}} is identical to set S={-3,-2,-1,0,N}, it seems that they couldn't perceive {1,2,3,...} as a single object, as an entity. According to Dubinsky and his colleagues (2005), an individual is able to construct a completed idea for the concept of infinity after interiorizing repeating endless actions, reflecting on seeing an infinite process as a completed totality, and encapsulating the process to construct the state at infinity, understanding that the resulting object transcends the process.

Teachers' decisions as to whether two given infinite sets have the same cardinality depend on the specific representation in the problem (Tirosh & Tsamir, 1996). Geometric representation yielded one-to-one correspondence during the comparison of infinite sets and helped teachers avoid the justification "part-whole". The schematic drawing, in combination with the vertical representation, facilitated teachers to understand that infinite sets had the same cardinality. In contrast, the use of horizontal and verbal representations caused misconceptions of the form "part-whole" similar to those reported by Singer and Voica (2003). This particular finding shows that teachers give contradicting answers during the comparison of the same sets that are presented in different representations, not acknowledging that incompatible responses are not acceptable in mathematics.

Participants' responses about the equalities 0.999…=1 and 0.333…=1/3 confirm the results of previous researches (Monaghan, 2001; Cornu, 1991; Fischbein, 2001). Although the aim and the context of the two equalities were similar, they caused different answers. The equality 0.333…=1/3 was accepted as valid easier than the equality 0.999…=1 which reinforced the use of limit. As Edwards (1997) stated, 0.333… equals to 1/3 because it might result from the division 1 by 3. Indeed, the number 0.333… can be constructed from a process, in contrast with 0.999… that is not intuitionally or visually understandable (Dubinsky et al., 2005). For this reason, the concept of potential infinity is used in the first case, while in the second case there is a mixed understanding of potential (0.999… as an infinite sequence of 9's) with actual infinity (object conception for the number 1).

The present study offers teachers an opportunity to consider the misconceptions related to the concept of infinity. If these misconceptions are reproduced during teaching, then students' misconceptions about the concept of infinity will be empowered and in turn become very difficult to overcome. The notion of infinity is related with important mathematical concepts, such as number configuration, number comparison and the numerical line, that are important for arithmetic and algebra. For this reason, teachers must be aware of the difficulties encountered regarding the specific concept, in an attempt to avoid "problematic" teaching. In addition, it is important for teachers to develop conceptual understanding of the notion of infinity that is to connect potential and actual infinity with concrete examples from real life **(**Singer & Voica, 2003**).**

Furthermore, the present study offers educators an opportunity to consider the abovementioned misconceptions and to propose ways to overcome them. In particular, academic programs offered to teachers should include mathematical knowledge regarding to infinity in combination with instructional approaches related to the concept. A proposed teaching approach could include the following steps: presentation with several typical tasks aimed at uncovering teachers' intuitions about the concept, discussion about infinity's applications in real life, introduction of the formal definition of infinity and the two aspects- potential and actual- and attempt to distinguish them in examples. Furthermore, students' difficulties for the concept, comparison of the intuitive beliefs in light of the formal definition, and explanation of the symbols and other representations of the concept may be presented. Thus, in the framework of the training program teachers could be exposed to opposing views of the concept that may be used to develop a more coherent appreciation of the formal definition and to the refinement of intuitions (Mamona-Downs, 2001). As Fischbein (2001) noted, appropriate teaching may help the learners to cope with counter intuitive situations while it makes them aware of intuitive constraints and of the sources of the mental contradictions.

## REFERENCES


Cornu, B. (1991). Limits. In D.O. Tall (Ed.), *Advanced Mathematical Thinking*, Kluwer, Dordrecht, 153-166.

Dubinsky, Ed., Weller, K., McDonald, A., & Brown, A. (2005). Some Historical Issues and Paradoxes Regarding the Concept of Infinity: An APOS-based analysis: Part 2. *Educational Studies in Mathematics, 60,* 253-266.

Edwards, B. (1997). An undergraduate student's understanding and use of mathematical definitions in real analysis. In J. Dossey, J.O. Swafford, M. Parmentier, & A.E. Dossey (Eds.), *Proceedings of the 19th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 17-22). Columbus: OH.

Fischbein, E. (2001). Tacit models and infinity. *Educational Studies in Mathematics, 48*, 309-329.

Galileo, G. (1954). *The New Science.* Dover, New York.

Lakoff, G. & Núñez, R. (2000). *Where mathematics come from*. Basic Books, New York.

Mamona-Downs, J. (2001). Letting the intuitive bear on the formal: didactical approach for the understanding of the limit of a sequence. *Educational Studies in Mathematics*, 48, 259 - 288.

Miles M.B., Huberman A.M. (1984). *Qualitative Data Analysis: A Sourcebook of New Methods*. Newbury Park, CA: Sage.

Monaghan, J. (1986). *Adolescents' understanding of limits and infinity*. Unpublished Ph.D. thesis. Mathematics Education Research Centre, University of Warwick, UK.

Monaghan, J. (2001). Young peoples' ideas of infinity. *Educational Studies in Mathematics, 48,* 239-257.

Moreno, L., & Waldegg, G. (1991). The Conceptual Evolution of Actual Mathematical Infinity, *Educational Studies in Mathematics, 22* (5) 211-231.

Sacristàn, A.I., & Noss, R. (2008). Computational Construction as a Means to Coordinate Representations of Infinity. *International Journal of Computers for Mathematical Learning*, 13, 47-70.

Singer, M., & Voica, C. (2003). Perception of infinity does it really help in problem solving? In A. Rogerson (Eds.), *Proceedings of the 6th International Conference of The Decidable and the Undecidable in Mathematics Education* (pp. 252-256). Brno, Czech Republic: The Mathematics Education into the 21st Century Project.

Tall, D. (1992). The transition to advanced mathematical thinking: Functions, limits, infinity and proof. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning (pp. 495-5 I 1).* New York: Macmillan.

Tirosh, D. (1999). Finite and infinite sets: Definitions and intuitions. International Journal of Mathematical Education in Science & Technology. 30(3), 341-349.

Tirosh, D., & Tsamir, P. (1996). The role of representations in students' intuitive thinking about infinity. *International Journal of Mathematics in Science& Technology, 27*(1), 33-40.

Tsamir, P., & T. Dreyfus (2002). Comparing infinite sets — a process of abstraction The case of Ben. *Journal of Mathematical Behavior*, 21, 1–23.

Vinner, S., & Kidrom, I. (1985). The concept of repeating the non-repeating decimals at the senior high level. *In the Proceedings of the 9th International Conference for the Psychology of Mathematics Education.* Noordwijkerhout, The Netherlands, 357-361.

# PERCEPTIONS ON TEACHING THE MATHEMATICALLY GIFTED

Kontoyianni Katerina, Kattou Maria, Ioannou Polina, Erodotou Maria, Christou Constantinos, & Pittalis Marios

Department of Education, University of Cyprus

*The aim of this study is to describe and analyze the structure of the perceptions of elementary school teachers concerning mathematically gifted students. The study was conducted among 377 elementary school teachers, using a questionnaire of 21 statements on a 5-point Likert type scale. The results of the study revealed that teachers' perceptions regarding gifted students in mathematics can be described across four dimensions based on the following factors; teachers' needs, teachers' self-efficacy beliefs, characteristics of the gifted and the different services delivered to meet the needs of the gifted. Implications for teachers, researchers and policy-makers are discussed.*

*Keywords: giftedness, teachers' perceptions, teacher training, self-efficacy beliefs, special education*

## INTRODUCTION

Gifted students differ from their classmates. Therefore, differentiated instruction is required, in order to maximize their talents. However, according to Archambault et al. (1993), as well as Westberg et al. (1993), very few instructional or curricular modifications are made in regular elementary classrooms in order to enhance gifted students' abilities.

The present study purports to examine the perceptions of elementary school teachers regarding gifted students, with reference to mathematics. In particular, in this paper we firstly aim to confirm that teachers' perceptions can be defined accross four dimensions which correspond to teachers' needs, teachers' self-efficacy beliefs, the characteristics of the gifted and the different services delivered to meet the needs of the gifted, as described in the model developed specifically for this study. Secondly, we intend to investigate the structure of teachers' perceptions about the ways to address the needs of gifted students, the characteristics of mathematically gifted students and the importance of  the teacher in order to be able to provide the appropriate support and guidance to these students.

Investigating the views of teachers regarding gifted students is expected to provide valuable information on the aspects which are susceptible of improvements. In addition, this study could serve as a starting point for the development of inservice programs for teacher education concerning mathematical giftedness.

## THEORETICAL FRAMEWORK

### Characteristics of gifted students in mathematics

Mathematically gifted students are characterized by an expanded cognitive base and are more capable of exploiting knowledge in order to realize their objectives. A necessary trait of a teacher of the gifted should be the knowledge of their characteristics and needs, as stated by Kathnelson and Colley (1982). Several characteristics of mathematically gifted students have been discussed in previous studies. Maker (1982) pointed out three key areas in mathematics that gifted students differ from their peers; pace at which they learn, depth of their understanding and their interests.

Regarding the first area, gifted students are capable of providing answers with an unusual speed and precision (Heid, 1983), namely they are able to solve mahematical problems faster (Hettinger & Carr, 2003). Their ability in identifying relationships in subjects, concepts and ideas without previous related teaching (Heid, 1983), increases the pace at which they learn. The fact that gifted students are flexible in using different strategies and they are able to select the most suitable strategy for each situation in compination with the possession of complex metacognitive and self-regulative skills (Hettinger & Carr, 2003) proves the depth of their understanding. In addition, Johnson (2000) reported that mathematically gifted students give original explanations and have the ability to organize data, transfer knowledge and generalize ideas. It has also been observed that gifted students are often more interested and perform better in tasks that require mathematical reasoning than computational processes (Rotigel & Lupkowski-Shoplik, 1999). As far as their interests are concerned, gifted students prefer to discuss with adults and to be involved with professionals. They are more favorable to advanced issues than their classmates, e.g. mathematical proof, politics, space.

## Nurturing gifted students

A number of methods have been proposed and developed to fulfill the needs of gifted students. Among them, enrichment activities, differentiation of teaching, flexible grouping, acceleration and increased use of technology are the most common ones. Research by Rotigel and Pello (2004) has shown that a combination of the aforementioned approaches is the best solution for the gifted.

Enrichment refers to the presentation of content in more depth, width, complexity or abstraction related to the curriculum delivered to all students (Florida Department of Education, Bureau of Exceptional Education and Student Services, 2003; Rotigel & Pello, 2004). According to Lewis (2002) and Renzulli (1976), new content is added to the curriculum, existing content is explored in more depth and the curriculum is expanded with additional tasks that require cognitive and research abilities.

Acceleration is defined as the practice of presenting content sooner or in a faster pace. Brody and Benbow (1987) argued that acceleration can be obtained in a variety of ways. For example, acceleration can be achieved in one or many subjects or by skipping grades. In addition, university courses offered to secondary education gifted students or early graduation from secondary education and early enrolment in a higher institution may be considered as acceleration options (Brody & Benbow, 1987). Acceleration provides the appropriate level of challenge in order to avoid boredom from repeated learning and to decrease the time required to graduate from an educational level (National Association for Gifted Children, 2004).

Useful suggestions about ways teachers can use in their classrooms in order to differentiate teaching to fulfill the needs of gifted students are provided by Johnson (2000). In particular, Johnson (2000) pointed that gifted students need inquiry-based learning approaches that emphasize open-ended problems with multiple solutions, as an opportunity to show their abilities. To this end, the teacher should pose a variety of higher-level questions during justification and discussion of problems. Moreover, technology can serve as a means for the gifted student to reach the appropriate depth and width of the curriculum (Johnson, 2000).

**Teachers' needs**

There is a prevailing myth that gifted students do not need special attention since it is easy for them to learn what they need to know (Johnson, 2000). On the contrary, their needs require a deeper, broader, and faster paced curriculum than the regular one. Due to the complexity of giftedness, it is of great importance that teachers have specialized preparation in gifted education, namely in identifying and nurturing the mathematically gifted (Johnson, 2000; VanTassel-Baska, 2007). Not only strong pedagogical knowledge is needed, but also a strong background in mathematical content. Providing a more general framework, Jenkins- Friedman and her colleagues (1984) argued that an effective teacher should have five kinds of skills; managerial-facilitative, pedagogical, social-consultative, directive and planning and interactive skills.

In this direction, Gear (1978) observed that teacher effectiveness can be improved with specific training. VanTassel-Baska (2007), commented that teachers of the gifted need to be able to address multiple objectives at the same time, recognize how students might manipulate different higher level skills in the same task demand, and easily align lower level tasks within those that require higher level skills and concepts.

Despite all recommendations and efforts in providing appropriate support to gifted students, previous studies have shown that the majority of teachers have neither the time, qualifications nor sources to develop and implement a differentiated curriculum (Tyler-Wood et al., 2000). In addition, low teacher efficacy beliefs in meeting the needs of gifted students, their lack of relevant teacher training which is partially originated by the lack of preparation for this task during their graduate studies (Lee & Bailey, 2003), reveals the intensity of this phenomenon.

**Teachers' perceptions regarding gifted students**

Teachers' perceptions about teaching and learning have a powerful influence on the ways teachers act in the classroom and interact with their students (Bain et al., 2007).

Despite their importance, little is known about the current perceptions of individuals in teacher-education programs regarding the educational practices for gifted children (Bain et al., 2007). Particularly in the case of gifted students, there is a disparison between teachers' perceptions; on the one hand teachers are overwelmed to work with gifted children and on the other hand they are negatively prejudiced towards them.

Regarding positive perceptions held by teachers about gifted students, Rothney and Sanborn (cited by Martinson, 1972) noted that teachers believe that the gifted will reveal themselves through academic grades and they need all existing content plus more. Therefore, teachers should add to the existing curriculum material requirements rather than delete anything. Studies conducted by Justment and colleagues (cited by Martinson, 1972) revealed that teachers experienced with special programs were generally enthusiastic to work with gifted students, since the experience with training programs produces more favorable attitudes toward gifted children (Martinson, 1972).

Nevertheless, teachers of the gifted often feel threatened by these students since they are sometimes confronted with students with more knowledge and abilities than themselves (Shore & Kaizer, 1989). In addition, the often stated misconception, as suggested by Bain and her colleagues (2007), namely that gifted children will find their way on their own, provides an alibi for educational system to continue neglecting their needs.

## METHODOLOGY

### Subjects

The sample consisted of 337 elementary school teachers. Table 1 presents demographical data of the study sample. The percentage of each category is presented in parenthesis.

| Years of service | Men | Women | Total |
|---|---|---|---|
| **1-10** | 39 (11.57) | 174 (51.63) | 213 (63.20) |
| **>10** | 26 (7.72) | 98 (29.08) | 124 (36.80) |
| **Total** | 65 (19.29) | 272 (80.71) | 337 (100.0) |

**Table 1: Sample demographic data**

### Data Collection

In order to collect data for this study, a questionnaire was administered to 337 elementary school teachers in Cyprus. The questionnaire consisted of 21 statements in a 5-point Likert scale with number 1 referring to the complete disagreement of the teacher and number 5 represented complete agreement with the statement. Participants indicated the degree that better expressed their opinion. In addition, empty space was provided to optionally add any remarks.

### Data analysis

Data collected were analyzed in an effort to explore the perceptions of elementary school teachers regarding mathematically gifted students. In particular, the statements focused on four aspects; teachers' role, teachers' self-efficacy beliefs, ways to meet the needs of gifted and their characteristics. Given that on the theoretical part of the study several issues regarding mathematical giftedness have been highlighted, an effort was made to assess whether a theoretically driven model would fit to the data. To achieve this, confirmatory factor analysis was performed.

The statistical modeling program MPLUS (Muthen & Muthen, 2007) was used to test for model fitting in the present study. Three fit indices were calculated, before evaluating model fit: The ratio of chi-square to its degree of freedom ($x^2$/df ), the comparative fit index (CFI), and the root mean-square error of approximation (RMSEA). According to Marcoulides & Schumacker (1996), in order to support model fit, the abovementioned indices required to be verified. In particular, the observed values for $x^2$/df should be less than 2, the values for CFI should be higher than 0.90, and the RMSEA values should be close to or lower than 0.08.

## RESULTS

In this study, we hypothesized an a-priori structure of teachers' perceptions regarding the mathematically gifted and then tested the ability of a solution based on this structure to fit the data. The proposed model consists of four first-order factors: teachers'needs (F1; statements 15, 17, 18 and 21), teachers'self-efficacy beliefs toward teaching the mathematically gifted (F2; Statements 5 and 13), ways to meet the needs of these students (F3; Statements 9 and 20) and characteristics of gifted students in mathematics (F4; statements1, 2 and 3) that form the second-order factor of teachers' perceptions concerning the mathematically gifted.

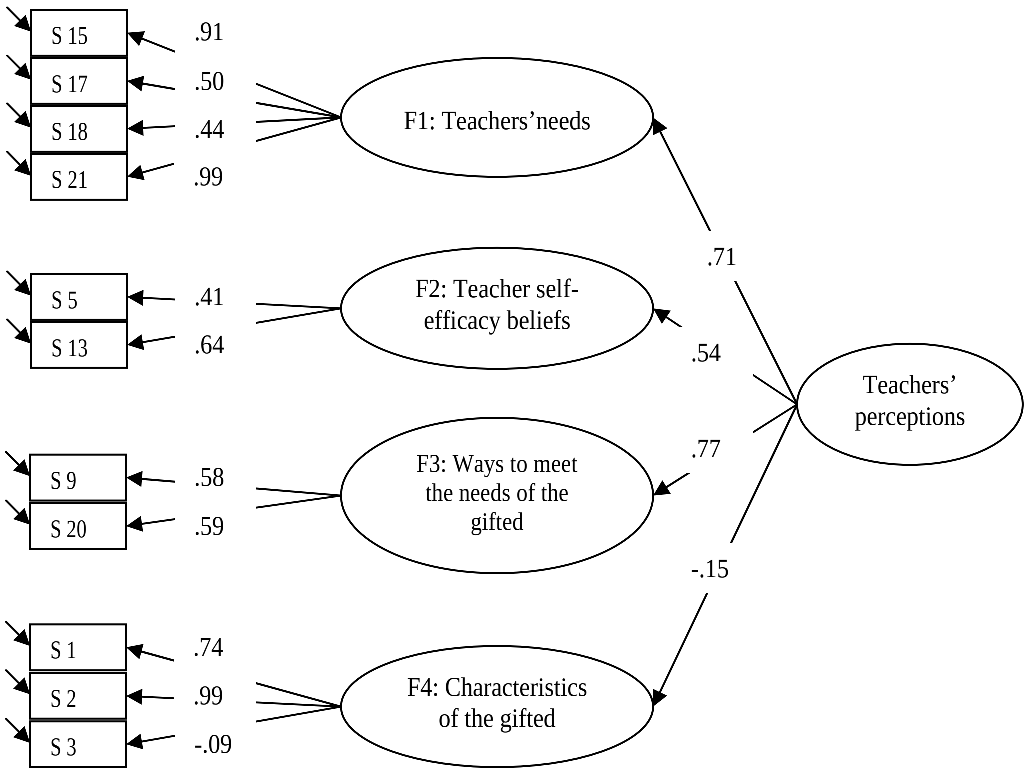



**Figure 1: The structure of teacher perceptions about gifted students in mathematics.**

Figure 1 presents the structural equation model with the latent variables of teacher perceptions regarding mathematically gifted students and their indicators. The descriptive-fit measures indicated support for the hypothesized model (CFI=0.97, $\chi^2$=66.07, *df*=40, $\chi^2/df$=1.65, $p<0.05$, RMSEA=0.04). The parameter estimates were reasonable in that almost all factor loadings were statistically significant and most of them were rather large (see Figure 1). Several statements were excluded from the model due to their low factor loadings compared to the remaining statements. The 11 statements included in the model are shown in Appendix 1.

In particular, the analysis showed that each of the statements employed in the present study loaded adequately only on one of the four factors (see the first order factors in Figure 1), indicating that the four factors can represent four distinct aspects of teachers' perceptions concerning gifted students in mathematics.

Teachers' comments that were written in the empty space provided are presented below to enhance the proposed model, after being categorized in the four factors formed by the abovementioned model.

| **Factor** | **Teachers' comments** |
|---|---|
| **1. Teachers' needs** | - It is necessary for the teachers to receive training in teaching gifted students. Having a counselor in each school will be very helpful for the teachers.<br>- The ideal is to have special teachers for gifted students in each school. |
| **2. Teacher self-efficacy beliefs** | - Gifted students might ask difficult questions that I will not be able to answer. I prefer not to have one in my classroom.<br>- I am not aware of the criteria to identify a truly gifted child. |
| **3. Ways to meet the needs of the gifted** | - The Ministry of Education should send material for the gifted in order to differentiate their work.<br>- The school should support gifted students, not only students who experience difficulties. They should be given opportunities to take advantage of their talents and experiences according to their interests. Challenging activities should be provided in order to avoid boredom.<br>- It is difficult for them to follow a mechanical learning path. Thus, the learning process should conform to their personality and allow for creative activities.<br>- Gifted students do not always prefer to have differentiated work. Sometimes they prefer to work like the others. Particularly in the first grades, they do not want to differ.<br>- They should help low-ability students and facilitate teacher's work.<br>- They can develop their talents out of school motivated and supported by their parents.<br>- The fact that they have different potentials than those of their classmates, is enough. They do not need any other differentiation. |

| | |
|---|---|
| **4.Characteristics of the gifted** | - Being gifted does not mean being perfect in everything. Usually, you meet students gifted in one or more domains.<br>- I think that nowadays it is difficult to talk about gifted students. Many children have special abilities-talents in specific domains. Intelligence is defined from various factors. A student may be gifted in mathematics, while another student may me gifted in art. I would say that there is no general giftedness.<br>- In my opinion, the term "gifted" does not exist or it is used erroneously.<br>- There are no objective criteria to define a student as gifted. |

**Table 2: Teachers' comments**

## DISCUSSION

Given the importance of the role of the teacher both in identifying and nurturing gifted students, the aim of this study was to examine the structure of the perceptions of elementary school teachers concerning gifted students in mathematics. The study reported in this paper provided evidence that teachers' conceptions about mathematically gifted students can be described across four dimensions based on the following factors. Specifically, the first factor is teachers' needs to appropriately cater this special group of students. The second factor refers to self-efficacy beliefs held by teachers, such as considering themselves able to provide adequate support to mathematically gifted students and help them realize full potential. The third factor is the different ways used during teaching to meet the needs of the gifted, i.e., providing them with more challenging activities than those of their peers. The fourth factor consists of the characteristics of the gifted; for instance, that gifted students prefer to reason than proceed to computational processes. The abovementioned structure suggests that teachers need to work not only on their knowledge regarding the characteristics of gifted students and the different approaches that proved to be useful in providing appropriate services, but also knowledge and skills required for the teachers to possess, as well as their self-efficacy beliefs. Based on this assumption, we could speculate that programs aimed at educating teachers in the domain of gifted education and more specifically in the field of mathematics, should focus on these four aspects.

The high factor loadings of the statements regarding the existence of counselors of the gifted in schools (S15 and S21) to the corresponding factor might be explained by the fact that teachers receive no guidance or training regarding educating the gifted. This is also reported in the remarks provided by teachers after completing the questionnaire. In Cyprus, there is no provision for gifted students stated in the mathematics curriculum. Therefore, the need for gifted education programs inside or outside the school boundaries is apparent. The teachers' concerns about the absence of relevant support by the state is also evident by the factor loadings of F1 and F3 in the second-order factor which is the teachers' perceptions. The results verify similar findings by Tyler-Wood et al. (2000) as well as by Lee and Bailey (2003).

It is evident from teachers' remarks related to the ways of meeting the needs of gifted students, that although they are aware of various approaches, such as differentiation as suggested by Johnson (2000), enrichment discussed by Lewis (2002) and Renzulli

(1976), they also hold various misconceptions. In particular, a remark that was noted by a teacher is that gifted students should help low-ability students and facilitate teacher's work. Another view held by a teacher is that the fact that gifted students have different potentials than their classmates is already enough and they do not need any other differentiated teaching. The aforementioned perceptions contribute to the prevailing myth that gifted students do not need special attention since it is easy for them to learn what they need to know (Johnson, 2000). Another teacher pointed out that students can advance their talents out of school motivated and supported by their parents. It is also important to note that no teacher mentioned anything about the use of technology as a way of supporting mathematically gifted students as proposed by Johnson (2000).

The results reveal that teachers are also concerned about their efficacy. In fact, a teacher acknowledged the fact that he is not able to identify a gifted student, while another teacher stated that gifted students might ask difficult questions, thus embarrassing the teacher and causing negative attitudes towards the gifted. This remark enhances the findings of Lee and Bailey (2003), according to which teachers have low efficacy beliefs in meeting the needs of the gifted.

At the same time, the characteristics that distinguish mathematically gifted students do not seem to be of great significance to the teachers. This could be owed to the fact that teachers are more interested in providing suitable experiences and activities for their students, without being aware of their distinctive characteristics. This implies that whether teachers have high ability or gifted students in their classrooms, they treatall students in the same way. In order to succesfully deliver the appropriate services to gifted students, teachers need first to identify them. Therefore, a solid understanding of characteristics observed in gifted children should be a requirement for teachers.

The present study extended the literature in a way that a model was validated examining the structure of teachers' perceptions concerning the mathematically gifted. The model proposed in this study offers teachers, researchers and policy makers a means to examine mathematical giftedness as it is experienced through the eyes of the teachers. From the perspective of teachers, the model may be used in order to aknowledge their lack of knowledge regarding behaviors that characterize gifted students and receive the appropriate support to feel confident to help mathematically gifted students realize their potentials. From the perspective of researchers and policy makers, it is likely that the model could serve as a starting point for the development of appropriately designed teacher training programs for the identification and nurture of the gifted. As a consequence, the change observed to teacher beliefs towards the gifted could be examined by researchers, as well as their shift in using various instructional approaches regarding mathematically gifted students. Finally, policy makers could exploit the results of this study by adding a special section in the curriculum for gifted students, acknowledging the fact that they have special needs that should be met.

## REFERENCES


Archambault, F., Westberg, K., Brown, S., Hallmark, B., Zhang, W., & Emmons, C. (1993). Classroom practices used with gifted third and fourth grade students. *Journal for the education of the gifted, 16*(2), 13-28.

Bain, S., Bliss, S., Choate, S., & Brown, K. (2007). Serving children who are gifted: Perceptions of undergraduate planning to become teachers. *Journal for the education of the gifted. 30*(4), 450-478.

Brody, L. E., & Benbow, C. P. (1987). Accelerative strategies: How effective are they for the gifted? *Gifted Child Quarterly, 31,* 105-110.

Florida Department of Education, Bureau of Exceptional Education and Student Services (2003). *Acceleration of Gifted Students.* Retrieved February 10, 2007, from http://www.firn.edu/doe/commhome/pdf/gift_accel.pdf

Heid, M. K. (1983). Characteristics and special needs of the gifted student in mathematics. *Mathematics Teacher, 76*, 221-226.

Hettinger, H., & Carr, M. (2003). Cognitive development in gifted children: toward a more precise understanding of emerging differences in intelligence. *Educational Psychology Review, 15*(3), 215-246.

Jenkins- Friedman, R., Reis, S. & Anderson, M.A. (1984). Professional training for teachers of the gifted and talented. Retrieved August 3, 2008, from http://eric.ed.gov/ERICDocs/data/ericdocs2sql/content_storage_01/0000019b/80/2e/f3/ed.pdf.

Johnson, D.T. (2000). *Teaching mathematics to gifted students in a mixed-ability classroom*. Retrieved July 9, 2008, from http://www.ericdigests.org/2001-1/math.html.

Gear, G.H. (1978). Effects of training on teachers' accuracy in the identification of gifted children. *Gifted Child Quarterly, 22*, 90-97.

Kathnelson, A., & Colley, L. (1982). *Personal and professional characteristics valued in teachers of the gifted*. Paper presented at California State University, Los Angeles.

Lee, L., & Bailey, J. (2003). Rethinking Practices for Gifted Young Children: A Collaborative Action Learning Project. *Curriculum Perspectives, 23(*1*),* 1-8.

Lewis, G. (2002). Alternatives to acceleration for the highly gifted child. *Roeper Review,* 24, 130-134.

Maker, C.J. (1982). *Curriculum development for the gifted*. Rockville, MD: Aspen.

Marcoulides, G.A., & Schumacker, R.E. (1996). *Advanced structural equation modelling: Issues and techniques*. NJ: Lawrence Erlbaum Associates.

Martinson, R.A. (1972). *Education of the gifted and talented* (Vol.2). Background papers submitted to the U.S. Office of Education. Washington DC: U.S. Government Printing Office.

Muthen, L.K., & Muthen, B.O. (1998-2007). *Mplus User's Guide*. Fourth Edition. Los Angeles, CA: Muthen & Muthen.

National Association for Gifted Children (2004). *Acceleration: Position Paper.* Retrieved February 10, 2007, from http://www.nagc.org/index.aspx?id=383

Renzulli, J. S. (1976). The enrichment triad model: A guide for developing defensible programs for the gifted and talented. *Gifted Child Quarterly, 20,* 303-326.

Rotigel, J. V., & Lupkowski-Shoplik, A.(1999). Using talent searches to identify and meet the educational needs of mathematically talented youngsters. *School Science and Mathematics,* 99, 330-337.

Rotigel, J. V., & Pello, S. (2004). Mathematically gifted students: How can we meet their needs? *Gifted Child Today, 27*(4), 46-65.

Shore, B. & Kaizer, C. (1989). The training of teachers for gifted pupils. *Canadian Journal of Education, 14*(1), 74-87.

Tyler-Wood, T. L., Mortenson, M., Putney, D., & Cass, M. A. (2000). An effective mathematics and science curriculum option for secondary gifted education. *Roeper Review* 22, 4.

VanTassel-Baska, J. (2007). Ten things all administrators should know about gifted children. Handout from *Superintendents' Forum on Gifted Education 2007 Annual NAGC Convention.*

Westberg, K., Archambault, F., Dobyns, S., & Salvin, T. (1993). The classroom practices observation study. *Journal for the education of the gifted, 16*(2), 29-56.

**Appendix 1: The 11 statements included in the model.**

| | |
|---|---|
| **S1** | Mathematically gifted students solve problems faster. |
| **S2** | A mathematically gifted student prefers to reason than compute. |
| **S3** | Gifted students might have attitude problems. |
| **S5** | I believe that I have the appropriate means to provide adequate support to gifted students. |
| **S9** | Gifted students should be provided with more challenging activities compared to their classmates. |
| **S13** | Having a gifted student in my classroom makes me feel very nervous. |
| **S15** | It is important to have at least one specially trained teacher for gifted students in each school. |
| **S17** | It is important to use identification procedures for gifted students. |
| **S18** | University programs should include teacher training regarding teaching gifted students. |
| **S20** | Acceleration of gifted students should be permitted through grade-skipping. |
| **S21** | I believe that there should be councelors/mentors for gifted students. |

# THE NATURE OF NUMBERS IN GRADE 10: A PROFESSIONAL PROBLEM

Mirène Larguier and Alain Bronner

LIRDEF, IUFM de Montpellier, Université Montpellier 2

*Teachers who teach grade 10*[1] *in France have to ensure the continuity of the mathematics taught between Junior High and Senior High*[2] *without doing any systematic revision. It seems to be a difficult task as teachers have to elaborate on reprise gestures*[3] *(Larguier, 2005) to go over knowledge already taught in Junior High while also introducing new knowledge. It is thus this problem of the profession (Cirade, 2006), which we analyze through direct observations of classes and data collected, about the way teachers tackle this. This study has allowed us to show some characteristic elements of this teaching problem. For example, the determination of the nature of numbers is a type of tasks between the two institutions; it can also be gone over as a reprise in various niches of the syllabus throughout the year. However, we show that teachers do not seem to take advantage of these opportunities.*

**Keywords:** reprise, professional gestures, the filter of the numeric

## A PROBLEM IN THE PROFESSION OF TEACHING THE NUMERIC

Going into grade 10 in France is a threshold to be crossed between Junior High and Senior High; it is an important passage between the two institutions. The mathematics syllabus states that in grade 10 students have to master the knowledge and know-how that most of them have already been taught in Junior High. A question then becomes central: the relationship between the *professionnal reprise gestures* and the knowledge and know-how. It takes us to the broader question of *interweaving* (Bucheton, 2009). We analyze the kind of gestures about the synthesis of numbers encountered during Junior High which must be done thoroughly during Senior High. We update the problems for teachers even though this part of the syllabus does not seem to be problematic for them to teach.

## THEORETICAL FRAME

To study the question of the construction of numeric space in grade 10, we essentially use the framework of the "anthropological theory of didactics" which has been developed by Chevallard (2007) and studies concerning the numeric and the algebraic

[1] In France, there are two distinct institutions after primary school: "collège" for students aged 11 to 15 and "lycée" for students aged 15 to 18. The first Senior High class is called "seconde" and corresponds to grade 10.

[2] Junior High school will be used for "collège" and Senior High school will refer to "lycée".

[3] "Reprise" can mean to go over, to patch together, to interweave. We shall use the word "reprise" for reasons of economy.

in Alain Bronner's works (1997, 2007). Bronner developed a tool for the study of numeric space: the “filter of the numeric”. The function of this filter is to " pursue" the numeric, whether it is at a practical level or an institutional level. Various elements of a numeric space can be identified:

- **The objects**: the number systems, the set operators (taking the square root) and comparators (< …);
- **The types of practices** (exact calculation, approached calculation and mixed calculation) as well as the various institutional contracts of calculation;
- **The articulations and the dynamics** of the numeric domain with the other domains as well as the underlying contracts;
- **The rationales** (“raisons d’être” in French) of the numeric.

Analysis of the numeric domain is completed by the identification of the “mathematical organizations” of the numeric. Together they make up a numeric space. The observation of the numeric space also includes the “didactic organization” to say what is specifically numeric. We also take from Chevallard (1999) the notion of praxeology which is broken down into four elements: type of tasks, technique, technology, theory. It permits us to model a teaching task which we indicate by professional gestures. We also use the levels of didactic determination defined by this author (1999) to question the conditions and restrictions of various origins which weigh on the didactic choices of the teachers. These levels as defined by Chevallard are: civilization, society, school, pedagogy, discipline, domain, sector, theme, and subject.

The study of the *reprises* can be analyzed according to different criteria (Larguier, 2005). The principal criteria of all the *reprises* can be represented on an axis, the extremes of which are:

- on the one hand, systematic revisions which do not meet with the new knowledge required by the syllabus;

- on the other hand the *reprises* which link up with new knowledge. In other words, the new learning and knowledge are the continuation of the study which began in the previous classes. This first criterion can also vary between systematic revisions (a kind of repetition of the same), a form denounced by the official curriculum; and *reprises* in accordance with the syllabus which introduce something new.

The second criterion of analysis of the *reprises* concerns the mathematical contents institutionalized at the end of the learning experience. It involves the targeted mathematical praxeologies, in other words the mathematical organization. This establishes a connection with the objectives of the teacher with regard to the types of mathematical tasks which are given. These objectives are:

- techniques to be reproduced by imitation and without a justification, so that technologico-theoretical elements of the praxeology are missing;
- know-how only for action, legitimized only by explanations which do not allow for updating mathematical rationales. Technologico-theoretical elements of the

praxeology are then incorrect towards the epistemology of the discipline;
- knowledge constituted with complete praxeologies that supposes that four elements of the praxeology are present and based on mathematical rationales.

This second criterion is called completeness of the praxeologies. It identifies the degree of completeness between two extremes: they are complete, and it seems that they are mathematically valid; otherwise they are incomplete.

## METHODOLOGY

Our research on the teaching praxeology concerning the *reprises* of the numeric leans on the study of grade 10 with a particular methodology. It differs from usual methods in the didactics of mathematics; in fact the analysis of the teaching practices in the classes is not conditioned by the objectives and the expected behavior of the researcher. This would have been clarified by an analysis a priori according to the research project. Here, observation in class comes first, permitting discovery and access to the knowledge taught, without any interaction between the teacher and the researcher. From elements revealed to the researcher in the dynamic of the teaching, an analysis a priori is elaborated. This is done by taking into account the previous experiences of the students, the didactic memory (so called by Brousseau) of the class and the requirements of the syllabus. It is then possible to make parallels between this analysis a priori and the project of the teacher reconstituted by the researcher after the session. In the same way, parallels can be drawn between this analysis a priori and the analysis a posteriori of the observed session. The collected data by observing sessions in a class throughout the school year are completed by interviews with teachers and with some students representing various levels, as well as by all the written traces of the year (exercises, lessons, homework …). Teachers and students only knew that the researcher was interested in the teaching of mathematics. They did not know about our interest for numerical domain. So the interviews with teachers and students were open and the focus of research was hidden. This condition was important to capture ordinary practices with the least possible influence of the researcher. Two experimented teachers (but not experts) agree to the researcher's presence in their classes, Mathieu in 2006 2007 and Clotilde in 2007 2008. This research follows a study in the framework of a Master 2 qualification (Larguier, 2005) which had made it possible to track down the difficulty of *reprises* at the beginning of the school year for novice teachers in grade 10, notably Rosalie.

## THE PROBLEMATIC OF NUMERIC

In the document which accompanies the syllabus (June 2000) we found the following commentary concerning the sector “numbers” and the theme “nature and writing of numbers”: “We will make a summary of the knowledge encountered so far by the students and we will introduce the ordinary notations of the different sets. The students will have to know how to identify which numbers belong to which set”. So, the recognition of the nature of the numbers is a well-defined task in the syllabus and is faithfully followed by the teachers according to the researcher's observations. We are going to develop our analyses concerning the following task: “recognizing which sets the given numbers belong to”. This type of tasks is emblematic of the numeric domain worked on at the beginning of the year during the resumption of the school year. It is also equally symbolic of the Junior High/Senior High link by allowing a *reprise* of former knowledge and at the same time working on completely new knowledge (like the nomination of sets). This type of tasks will be written as T, this represents an essential problematic to the numeric domain. This restriction is found at the level of the discipline in Chevallard's terminology.

In Clotilde's and Mathieu’s classes many specimens of T are worked on in the first chapter. In general the justifications are not asked for. In Clotilde’s workbook the following affirmations without any justification are found: $\sqrt{18}$ irrational or 1/3 rational. The decision theory made in the relative class to this type of task T is incomplete. The technologico-theoretical block elements are absent, the expected response of the teacher rests on the numerous implicit elements which are certainly not shared by all the students.

The same observation concerning the incompleteness of the praxeologies relative to T was carried out on the 17th of September 2004 in Rosalie’s class. We will take the same example which has been indicated and which concerns written numbers under the quotient form of two whole numbers. Rosalie does a particular study of two specimens $\frac{22}{7}$ and $\frac{103993}{33102}$ prompting this study with the fact that they are approximations of $\pi$. In other words, a cultural condition which is not based on a real mathematical problem.

For the first example, a possible technique known from Junior High, is to carry out the division of 22 by 7 in order to prove that the decimal writing of the number is unlimited and periodical. Rosalie expected this proof from the students as a relative technique to 22/ 7, which corresponds to an interesting *reprise* to continue to work on the concept of decimal numbers as is seen in this extract:

> A student wrote his answers on the board. Rosalie hears another student in the class:
> **Alexis:** It’s a rational number
> **Teacher:** Why?
> **Alexis:** Because it’s a fraction and the decimal part is infinite
> **Teacher:** How do we know that? …. It’s best to write down the division because the calculator will always give a finite amount of numbers…of terms since it shows the numbers it has on its screen. Now this one here (she points out “$\frac{103993}{33102} \in R$” written on the board by a student) who doesn’t agree?

The proof for the first quotient 22/7 is brought up orally, but it is not carried out effectively by the students, or the teacher. With the calculator experiment, Rosalie does not leave the students enough time to do it themselves. In doing this, she also avoids a debate which could have taken place on the nature of numbers displayed on the calculator screen. This certainly would have allowed her to consolidate the necessary learning of this tool and the numbers in play (moreover, registered learning in the syllabus as one of the numeric themes). The mathematical decision theory linked with T is just a draft, it is not completely developed yet. We can therefore ask ourselves what is going to remain of this for the students. We equally make a hypothesis that the personal relationship between the students and the mathematical activity in general runs a risk of not conforming to the institutional relationship. Rosalie may let her students believe that it is enough to bring up a possible proof during a demonstration.

For the second example, the possibility of articulation with the new parts of the Senior High syllabus is interesting. Indeed, the two rational numbers 22/7 and 103993/33102 are both *idecimal numbers*[4] (Bronner, 1997) but the choice of numerator and denominator for 103993/33102 makes it necessary to change the technique compared to the previous example. The technique expected by Rosalie for the first number, to know the division "by longhand" of 22 over 7 cannot lead to the underlining of idecimality for the second number. The quotient obtained for the first number is 3,142857 while the length of the period from the second quotient is too big for the quotient to be calculated by longhand. We see a change of the didactic variable between the two tasks. We wonder if this is really what the teacher anticipated. Indeed, in the observed session, the fact that the second number is idecimal is not shown and is not even questioned:

> **Teacher:** (…) Now this one (she points out $\frac{103993}{33102} \in$R written on the board by a student) who doesn't agree? Yohan, Kamel?
>
> **Kamel:** I agree but it's also a rational number
>
> **Teacher:** It is, that's true but the answer to the question lies in Q. It's the R of real and it's the Q from quotient (she corrects what is on the board at the same time). But we suppose that Xavier is using the notations that he knows. Now the last one... (she points out $\frac{167}{80}+\frac{\sqrt{10}}{3} \in$R).

The study of the nature of numbers, beyond knowing whether a number is rational or not, is not made. There is not even a technique brought up contrary to what is brought up for 22/7. Consequently there is no implementation of a new decision theory, it is avoided. A possible technique in grade 10 uses a theorem which is in the syllabus (optional). It is not available to the class at this moment of the year. The question of knowing if the number belongs to **D** is thus left aside. In the second case, the demonstration of the *idecimality* of the rational number is not even brought up, it is simply completely avoided.

Nevertheless a decision theory corresponding to the syllabus could have been built into this class for task T. Here is the description: a possible technique in grade 10 is

[4] Idecimal: in Bronner's terminology, following the model of rational/irrational, decimal/idecimal

to determine the irreducible fraction which is equal to the given quotient. In this case, Euclide's algorithm allows us to demonstrate that the numerator and the denominator are coprime, and that the given fraction is irreducible. The denominator has a decomposition in product of prime numbers $2\times3^3\times613$, it is not a product of powers of 2 and 5, the number is *idecimal*. This technique is possible only from grade 10 onwards, but it also uses tools which are taught in Junior High, like the idea of irreducible fractions. This also permits another way of conceiving the decimal number in the register of fractional writings (Duval, 1995). Therefore, it gives us the opportunity to really strengthen our knowledge of numbers. So, T is indeed in a moment of *reprise* in the numeric space, which allows us to connect past knowledge, and new knowledge.

The comparison between what could have been done with T and what was effectively done clearly shows what is avoided in the targeted mathematical organization. We wondered why Rosalie made these choices:
- Is it about a lack of reflection in the analysis of the session?
- Is it the decision about the mathematical theory regarding the syllabus which is seen as not being a suitable teaching form in this class?
- Does Rosalie anticipate that the technique is too difficult to set up and might discourage students at the beginning of school year? This technological element of the professional gesture was confirmed in an interview with her. She said that she does not want to put students off learning mathematics.

This observation brings to light one of the difficulties that teachers have in building numeric space. The work in this numeric domain assumes a very precise study of the mathematical decision theory in accordance with the knowledge of the students. Another symptom of the problem of the profession is probably the misunderstanding of teachers on these difficulties. It asks the following question: what is the knowledge necessary for teachers in order to achieve the process of didactic transposition between the reference mathematical knowledge and the knowledge to be taught (Bosch et al., 2005)?

But what are the *raisons d’être* of this emblematic task? What essential mathematical problem for the discipline motivates the mastery of decision theory linked to T? By asking these types of questions, we refer to Yves Chevallard who denounces the teaching of mathematics as being like a museum visit, or the traditional way of teaching answers, even when the original questions have been lost (Chevallard, 2000). He questions what motivates the calculation of numbers in order to express them under these particular forms. He makes us become aware of the problem which legitimizes this work in the numeric domain:

> “We come to […] a big problem in mathematics: how to recognize if two mathematical objects of a certain type are or are not the same object? How to know for example if $7\times5-8 = 23$? Or if $\frac{60}{84} = \frac{380}{532}$? Or again if $\frac{n(n+1)(2n+1)}{6} - \frac{(n-1)n(2n-1)}{6} = n^2$? There is one solution to this one generic, universal problem: to respond to the question asked. We

need to use a considered type of written system for the objects, where each of these objects has a writing expression and a written expression of its own. The calculation of the «canonic» writing of the objects to be compared therefore allows us to answer: so we have $7\times5 - 8 = 35 - 8 = 27$, which shows that $7\times5–8 \neq 23$. Similarly it comes from a part $\frac{60}{84} = \frac{4\times15}{4\times21} = \frac{3\times5}{3\times7} = \frac{5}{7}$, from another part $\frac{380}{532} = \frac{190}{266} = \frac{5\times19}{19\times7} = \frac{5}{7}$, meaning that we can positively conclude this time that we have equality $\frac{60}{84} = \frac{380}{532}$."

In this citation Chevallard wishes to show that the only question about numbers which is important is to know how to write a number in relation with its nature. Different kinds of writing are possible, and we have to know the canonic one, useful to compare and calculate with several numbers. So it is not the knowledge of the nature of the number that is important, but the knowledge of the canonical writing given for a type of number. This necessity is backed up by another necessity of mathematical work, which is the rule of the *institutional contract of calculation* (Bronner, 2007). For demonstration work in mathematics, we are obliged to use exact values. The following reasons explain then why it is important to know the exact values of trigonometric lines of particular angles such as: $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ and why we keep this way of writing with a radical. We are going to further develop this example, various types of numbers appearing within the framework of trigonometry, a *reprise* of work on the numeric is then possible.

## THE EMBLEMATIC TASK AND TRIGONOMETRY

In the part concerning irrational numbers we are going to come across "products" (Bronner, 2007) within the framework of trigonometry, but neither their appearance nor their nature is questioned. In Mathieu's and Clotilde's classes, the chapter on trigonometry was approached late in the year, for Mathieu from May 23rd, 2007 and for Clotilde from April 30th, 2008. By using our methodology, a work of comparative analysis was able to be carried out.

The comments of the syllabus of grade 10 state: *"The definition of sin x and cos x for a real x will be made «rolling up* ***R*** *» on the trigonometric circle. We will make the link with sine and cosine of 30°, 45° and 60°".*

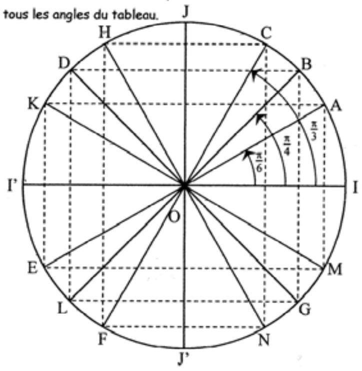
During Clotilde's lesson on May 16th, 2008, at the end of the sequence on trigonometry, she gives out a table which the students have to complete.

**Exercice :** On a donné les valeurs exactes du sinus et cosinus de quelques angles remarquables entre 0 et $\frac{\pi}{2}$.

| Point | | | | | | | | I | A | B | C | J | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | $-\frac{5\pi}{6}$ | $-\frac{3\pi}{4}$ | $-\frac{2\pi}{3}$ | $-\frac{\pi}{2}$ | $-\frac{\pi}{3}$ | $-\frac{\pi}{4}$ | $-\frac{\pi}{6}$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\pi$ |
| cos $x$ | | | | | | | | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | | | | |
| sin $x$ | | | | | | | | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | | | | |

a. Retrouver le point qui correspond à chaque angle.

b. En déduire les valeurs exactes des cosinus et sinus de tous les angles du tableau.




This document presents an extraordinary showcase of numbers which appear in the numeric space of grade 10 with whole relatives, decimals, irrationals formed with the typical examples often used like $\pi, \sqrt{2}$ and $\sqrt{3}$. We observed that it does not become the student's responsibility to know that it is necessary to keep complex writings of these numbers, for example $\frac{\sqrt{2}}{2}$. If the teacher had given the responsibility of this question to the students, then he would have been able to carry out a *reprise* of the emblematic task T to justify the canonical writing of these numbers. But the awareness of the nature of the numbers is completely absent in this entire sequence even though it is very rich in respect to possible work on the numeric. The only justifications are under the form of conventional rules not referred to as necessities of the discipline. So, Clotilde does not accept the answer $\frac{1}{\sqrt{2}}$ and transforms it into $\frac{\sqrt{2}}{2}$ by arguing that: *"as we already said we did not like the roots of 2 under the line of fraction, we write it like that"*.

Thus, teachers accustom the students to practices of exact calculation, which are governed by conventional rules only decided on by the teacher, while epistemological reasons support them. The institutional contract of calculation remains in this context of trigonometry entirely the responsibility of the teacher. Nevertheless, the underlying questions could be seen by the student as being an aspect of the mathematical work.

The numeric space elaborated in grade 10 is so enriched by new elements which are *operators* (Bronner 2007), namely the operators cosine and sine, generators of tables of real numbers containing many irrational numbers. These operators allow a production of numbers in a procedural way. The interest is centered on the way of obtaining the numerical values, and not on their nature. In the same way, there is no interest in the change of status of the number which must be seen as a variable of the function cosine.

The *dynamic* implemented by both teachers is a *numerico-geometrical dynamic* (Bronner 2007). Numbers of various natures are generated by the operator cosine from the trigonometric circle and from the right-angled triangle. However, another dynamic remains implicit, it is an *inter-numeric dynamic.* This one could exist thanks to the numeric resumption of work at the beginning of the year linking with the symbolic task and the canonical writing of the numbers according to their nature. However, it would seem that this symbolic task is not exportable except the sector “Numbers” of the domain “Calculations and functions”. This place of trigonometry in grade 10 would allow the numeric to work, because irrationals come “naturally”. But, the awareness of the nature and the writing of these numbers is not the responsibility of the student. Nevertheless, it would be interesting to ask the question about the exact value of a number like for example cos17 and to make the students aware that the writing of the exact value is cos17, in the same way that the exact value of $\sqrt{34}$ cannot be written without using a radical. These examples could enrich the usual prototypes used as irrationals. Nevertheless, from the synthesis of numbers encountered in the vast *mixed-bag* of school, this type of number has been popular and can be reused as an example.

## IDENTIFICATION OF A PROBLEM IN THE PROFESSION

We asked the question of the *reprise gestures* concerning the study of the nature of numbers by focusing our gaze on an essential problem in mathematics: writing numbers according to their nature. Obviously, this question takes its meaning only in the context of a problem. The most relevant register of writing is conditioned by the work to be done with these numbers. But what we also observed with the teachers was the absence *reprise* whenever the problem arose. The notions are only worked on as objects, the “raisons d’être” posed about the writing of the numbers becomes nothing more than a question of habit.

In the reality of our observations, the teachers introduce T to the students at the beginning of the year in a certain number of cases in accordance with the syllabus. They do this without taking into account the specific problems of the discipline, nor is it used later to pursue the study of synthesis relative to numbers. Nevertheless we have seen that a *reprise* of T is possible during the grade 10 syllabus (we have only quoted the case of trigonometry). Teachers do not see these new niches for reactivating this type of tasks no matter how essential it is to work on the numeric. Our study opens new ways for identifying specific teachers' knowledge in the matter of numeric domain. It is especially useful for the formation of teachers and the necessary practice of particular *gestures of interweaving*.

## REFERENCES

Bronner, A. (1997). *Etude didactique des nombres réels, idécimalité et racine carrée.* Grenoble: Thèse de doctorat, Université J. Fourier.

Bronner, A. (2007). *La question du numérique: Le numérique en questions.* Mémoire d'HDR, Université Montpellier II.

Bronner, A. &. Larguier, M. (2004). Symposium coordonné par Bucheton D. : Analyse didactique de la séance "carte de géographie". *Actes du colloque international de l'AIRDF*. Québec.

Bucheton, D. (2009). *L'agir enseignant: des gestes professionnels ajustés.* Octarès.

Chevallard, Y. (1999). Organiser l'étude. Cours 3. Ecologie et régulation. *Actes de la Xème Ecole de didactique des mathématiques.* Corps: La pensée sauvage.

Chevallard, Y. (2000). *Enseignement insensé, enseignement raisonné et créativité sociale.* www.aix-mrs.iufm.fr/formations/filieres/mat/

Chevallard, Y. (2004). *La place des mathématiques vivantes dans l'éducation secondaire: transposition didactique des mathématiques et nouvelle épistémologie scolaire.* Saint-Flour: actes de la 3ème université d'été Animath.

Chevallard, Y. (2007). Passé et présent de la Théorie Anthropologique du Didactique. Dans *Sociedad, escuela y matematicas, Aportaciones de la Teoria Antropologica de lo Didactico (TAD).* Publicaciones de la Universidad de Jaen.

Chevallard, Y., & Joshua, M.-A. (1991). *La transposition didactique. Du savoir savant au savoir enseigné.* Grenoble: La Pensée Sauvage (2e édition)).

Cirade, G. (2006). *Devenir professeur de mathématiques : entre problèmes de la profession et formation en IUFM.* Thèse de doctorat, Université Aix-marseille 1.

Duval, R. (1995). *Sémiosis et pensée humaine.* Peter Lang S. A.

Larguier, M. (2005). *Les reprises des domaines numérique et algébrique en classe de seconde.* Mémoire de DEA, Université Montpellier II.

# A EUROPEAN PROJECT FOR PROFESSIONAL DEVELOPMENT OF TEACHERS THROUGH A RESEARCH BASED METHODOLOGY: THE QUESTIONS ARISEN AT THE INTERNATIONAL LEVEL, THE ITALIAN CONTRIBUTION, THE KNOT OF THE TEACHER-RESEARCHER IDENTITY

Nicolina A. Malara
Modena & Reggio E. University, Italy

Roberto Tortora
Naples "Federico II" University, Italy

*The European project* (PDTR)[1]*, which this paper deals with, is aimed at the development of research based methodologies for teacher training to promote new classroom approaches in the sense of PISA competences. After a short description of the Project, we present in some details the cultural choices, the work methodology and the outcomes of the Italian teams. Some reflections are made about the main problems involved, in particular on the intense attempts to clarify the meaning of the figure of the teacher-researcher, the true core of the Project. In a few final remarks we discuss the validity and the potentialities of the Project.*

Key-words: European cooperation. Teachers' professional development. Educational methodologies. Teacher-Researcher figure.

## INTRODUCTION

The PDTR is a project finalized to induce in teachers structured view and knowledge of mathematics, in coherence with new pedagogical approaches and social needs, and to promote, by means of suitable classroom practices, motivation and sense-making in students involved in mathematical activities. A key idea of the project is that of *Teaching-Research*, based on the principle of inseparability of classroom practice and educational theory in the context of the action aimed at the improvement of learning. The intention is to build a formation and teaching path where instruction, research and professional development mutually support each other. The underlying hypothesis is that the involvement of teachers in "mentored" collaborative study within a research team and a familiarity with theoretical studies increase their awareness as school teachers, and bring them to change their beliefs, to conceive their professional development as a life-long process and to assume a scientific inquiring approach in their classroom pedagogy.

The central aim of the Project has been to initiate a process of transformation of the ways to teach mathematics, while respecting the standards and contents of national curricula. The main specific goals have been: a) introducing Teaching-Research into daily classroom practice, with special emphasis on the integration of mathematical and didactic knowledge; b) developing instructional research based materials, which improve students' understanding and mastery of mathematical competences as assessed by the OCSE-PISA tests, while, at the same time, increase their enjoyment of mathematics; c) promoting in teachers the attitude to give more weight to students' process of thinking than to formal skills and knowledge.

The Project has lasted three years: the first one mainly devoted to the study of general methodological-curricular choices, that be coherent with the approach to mathematical competences in OCSE-PISA tests; the second one centered on designing, implementing and analysing didactic experiments and producing shared materials; the third one devoted to a critical review and refinement of experimentations, and to the production of reports to be published. An additional task has been the study of the English language, to favour exchanges among participants.

## THE ITALIAN CONTRIBUTION WITHIN PDTR

In Italy, many research projects were promoted by the National Research Council since the seventies, for the renewal of mathematics teaching. This implied the birth in several universities of the '*Nuclei di ricerca didattica*' (that is, groups formed by university and school teachers of all levels, working jointly) and contributed to the emergence of a new "bivalent" figure of teacher: the '*insegnante-ricercatore'*. Such a figure can be considered the result of a slow evolution of a motivated and able teacher through *stages of active involvement at different levels*, stages which can be said the steps of a *process of training to research*. This process, starting from simple experimentations, brings gradually the teacher to collaborate in the formulation of research hypotheses and in the theoretical analysis of research data, until to be able to autonomously realize a research project and to write scientific papers. This national frame constitutes the background of our cultural and methodological choices within PDTR, and of our way of conceiving the participants as *perspective teachers-researchers,* novice in research.

The two Italian (Modena and Naples) teams share not only this general framework, but also common research themes and a long habit of mutual collaboration. Therefore their work has been done along the same lines. Here, we want to report in some details three aspects of our activity: the theoretical and laboratorial work, the conduction of teaching experiments, the production of the final reports.

### The work at theoretical level and the laboratory-based activities

We worked at three levels, facing: theoretical questions concerning mathematics education, with particular reference to the teacher figure; questions related to mathematical contents and questions devoted to a renewal of classroom practice.

We have taken inspiration from two related models of teacher, as *resonance mediator* (Guidoni, Iannece & Tortora, 2005) and as *decision maker* (Malara & Zan, 2002). In our view, teachers are influenced by important factors that the research should not neglect, such as knowledge, beliefs and emotions. Thanks to close contacts with Math Education ideas and theories, they can become more and more aware of all these components and to be able to possibly change them. For this reason we have devoted special sessions to introduce teachers to selected literature samples, in order to clarify our theoretical reference framework. These include epistemological studies, mathematics oriented papers, and papers focused on didactic-methodological aspects and on classroom practices.

For what concerns didactic and methodological aspects, we have assumed a socio-constructive approach, with particular emphasis on studies about the mathematical discussion, the didactic contract and the classroom norms. A particular importance is also assigned to reflections on class processes, and to the role of teachers (and of their beliefs, actions, wordings, …); for this we refer to Mason studies (see for instance Mason, 1998). Moreover, we have taken into account the linguistic and communication dimensions, as described by Pimm (1987) and Sfard (2000).

As to mathematical content we worked in Shulman’s sense (1986). We privileged the arithmetic-algebraic field, directing our attention towards the competences promoted by the PISA tests.

For the renewal of classroom practice, we studied the units of the ArAl Project, which can be seen as models for socio-constructive teaching, and some protocols of classroom processes on them, highlighting the incidence of different variables in the process (teacher’s behaviours, students’ participation, affective relationships, gender issues).

**The work related to teaching experiments and the methodology adopted**

The work with teachers has been carried out in small groups and has been structured through: design and planning of teaching sequences, experimental setting in the classes, critical analysis of the enacted didactic processes, editing of reports for dissemination. The chosen themes concerned: a) problem solving, according to the theoretical framework of the PISA tests and with reference to the development of proportional thinking; b) the approach to the algebraic language as an instrument to represent relations, to interpret graphs, to solve optimization problems and to solve proof problems. Teacher were engaged in teaching experiments for at least two years, and in the second year the experiments were broadened and refined on the basis of the initially implemented ones. They involved students of school grades between 6 and 11, with a high concentration of grades 6-8.

In order to implement a given teaching sequence, we faced: a joint study of selected research papers on the chosen theme for the clarification of didactic key points and hypotheses to be tested; the construction (or adjustment) of tasks constituting the main steps of the path and the *a priori* analysis of pupils’ potential difficulties. This work was not easy, due to: a) the need to combine the progressive development of the mathematical set of questions with curricular time constraints; b) the analysis of the difficulties of the tasks from both linguistic and mathematical points of view; c) the planning of discussions related to questions to be tackled and solved collectively.

In the classroom the teachers worked constructively, stimulating and orchestrating pupils' interventions, promoting reflections on what was gradually being carried.

They promoted verbalisation, by always inviting the pupils to write down their ideas, conjectures, reasons for their procedural choices, etc. Moreover, they (video) recorded classroom discussions, transcribed them, adding local and general comments on classroom processes.

**The driven analysis of classroom processes and the birth of the 'multi-commented diaries'**

We carried out a complex activity of critical analysis of the transcripts, looking at the relationships between the knowledge constructed by students and the teacher's behaviour in guiding them to such achievements. Our main aim has been to lead teachers to get a higher and finer control over their own behaviours and communicative styles and to observe the incidence of a critical analysis on both classroom processes and pupils' behaviours and learning. This critical and reflective activity, based on the classroom transcripts commented by the teachers (shortly called diaries) developed along different moments of comparison between: the pair 'teacher-mentor'; the teachers involved in the same teaching sequence; the whole team (teachers, mentors and the leader). Within some projects – due to participants' different locations and therefore to the difficulty to meet – the diaries have been commented by at least three people: the mentor assigned to the teacher; the co-ordinating mentor; the head of the project. The diaries, so enriched by a multiplicity of written comments, reflect a variegated range of points of view and interpretations, which highlight crucial points of the process as well as critical elements in the teacher's behaviour.

They allowed us to identify five key areas of teachers' weakness concerning: *beliefs on cultural and/or educational issues*; *pedagogical content knowledge*; *bifurcation between theory and practice* (e.g. difficulties in realising what has been studied or planned, and in working on the basis of relational thinking); *linguistic issues* (massive use of operative linguistic expressions coming from the received model of teaching; difficult balance between colloquial language and language of scientific teaching; scarce attention to word paraphrases in view of an algebraic translation); *management of classroom discussions* (dialogues mainly between teacher and pupil; widespread prompting; yes/no questions; lack of attention to the development of 'social intelligence' in the classroom). But two issues seem to be crucial and dramatic at the same time: *the teacher's language* in communication, often imprecise, not correct, full of slang expressions and rich in not always appropriate metaphors; *the conception of mathematics,* too often *operative,* where 'calculate' and 'find' often prevail over 'represent', and 'do' over 'reason' and 'reflect' (for more details see Malara's contribution, in Czarnocha, 2008).

**The reports editing**

In the third year of the project teachers were asked to produce written report about their teaching experiments following the rules of the Mathematical Education community. This phase of teachers' work turned out to be a true pivot toward the acquisition of a *researcher behaviour*. In fact, teachers are used to report their classroom experiences within their own community, but this kind of "internal" communication, having its focus on students' performances, leaves behind any information about one's own role in the process and about the choices made for its development. In the first version of the report, almost all teachers applied this model of communication to the new situation, in spite of the attitude, developed in two years of participation in the project, to reflect on the influence of their own role in the development of a discussion, and more in general, on the relationship between teacher and pupils, with a special focus on the impact of their own knowledge, beliefs and emotions on the process itself (see next Section). The experts faced the problem, trying to change this communication praxis. Several individual and collective comparisons were needed to lead teachers to become aware they had to change their usual point of view and to include, in their writings, themselves as determinant components of the process itself. This way, by means of successive approximations, always mediated by interaction with the experts, teachers succeeded in writing their reports. Then these reports were reviewed by international reviewers before being accepted for publication (in the books edited within the project[2]).

From the point of view of the research training, this final phase has been crucial to attain project aims: the necessity of communicating lead teachers to make explicit for other people, but for themselves too, the key points of change in their classroom behaviours.

## Reflections on the project spin-off for teachers

The project turned out to be a great opportunity for teachers to engage with a new way of conceiving and teaching mathematics and to reflect on their own conceptions and ways of being in the classroom. Teachers met major difficulties in transposing in their practice what they had learned at theoretical level, especially concerning the didactic-methodological aspects.

Here is a list of the main problems concerning the role of the teacher in managing class-based activities, in particular discussions: the problem of the language used, often misleading for the pupils; the problem of the pertinence and consistency of the indications provided at crucial moments of the discussion; the problem of listening to pupils and being unable to grasp the potentiality of interventions that diverge from predicted ones (especially when they come from pupils who are not viewed as leaders); the problem of a real social knowledge construction: the issue of sending back ideas to the class so that they might be validated and shared, the issue of institutionalizing knowledge, the issue of individual learning (the teacher often took for granted that pupils had understood or intuited something, only on the basis of reassuring 'yes' in chorus); the problem of checking that participation is actually collective (discussions often developed with the contribution of a few pupils and there were no interventions aimed at involving everybody).

Nevertheless, at the end of three years, several appreciable improvements can be noticed in teachers' classroom practice, as well as changes in their beliefs and a better awareness of their professional role. All this is also witnessed by the teachers themselves within their final essays. In the Appendix we will report a few excerpts from these essays.

## THE INTERNATIONAL ACHIEVEMENTS. THE FIGURE OF THE TEACHER-RESEARCHER

At the international level, the Project did not fully meet our expectations. Many substantial disagreements emerged along the common work, concerning first of all different views about Math Education research contents and methodology, between Eastern and Western countries and, as a consequence, disagreements emerged on the way to conceive a teaching experiment. Therefore, only in the last year a first true international collaboration, a bilateral teaching project between Italy and Hungary, occurred (see Navarra, Malara & Ambrus, 2008).

The main points of difference concerned: variables to be observed (students vs the pair "teacher-students"); time (short vs long term experiments); types of intervention (simple proposals of PISA question vs insertion of suitable PISA problems into didactic paths designed for the whole year workplan); way to refine a teaching experiment (proposals of 'corrective tasks' for students vs critical analysis of classroom processes with/for teachers); and, *dulcis in fundo*, the figure of the teacher-researcher.

The question of defining what the word "(mathematics) teacher-researcher" means is by no means a rhetorical one and, well beyond the limited range of the Project, is of deep interest for the whole Math Education research community. Indeed, for some authors, the two domains of academy and school are incommunicable worlds, and therefore the unique possible concern of the teachers is their school-practice (Crawford & Adler, 1996). For others the two roles are still separate, even if there are teachers who are able to investigate about their practice; but it is very rare that a teacher can identify by himself a research question (Jaworski, 2003; Brenn quoted by Peter-Koop, 2001). Some other authors believe that the teachers can become true researchers, provided they frequent for enough time an academic environment (Malara & Zan, 2002).

One of the, so to call, side achievements of the project PDTR, but, in our opinion, a valuable one, has been that of trying to share a common view on this question, naturally arisen in order to achieve the main goals of the Project. So here we want to report some conclusions about it, reached at the end of several discussions and collaborative work, together with some reflections of ours. The question has received several interpretations and answers by the members of the PDTR staff, due to their different views deeply dependent on different theoretical frameworks and social and cultural traditions.

Moreover, the following related questions have arisen: “How do the double roles of teacher and researcher acting simultaneously in concrete situations accord to each other? How can the possible conflicts between the two roles, each embodying its own objectives and its own ethics rules, be managed? How can one harmonize the two roles in the different real situations or perhaps in the different phases of the work?” Of course, all the above questions are open ones. But the wide debate developed has given some contribution to them, witnessed by specific papers devoted to these items in the two books edited within the Project. It seems to us that they well represent the variety of positions.

The main task remained of reconciling the different views about the crucial point: when a teacher can be identified as a teacher-researcher. A shared conclusion has been that of recognizing some steps by which a teacher can become a teacher-researcher. Teachers teach following textbooks and external indications. Good (or excellent) teachers utilize natural skills and their own intuition to obtain good results from their students, following textbooks and other resources filtrated by their personality. A teacher-researcher adds to this a *personal aspect*: the habit to reflect upon one’s own teaching action and to utilize such reflections to interpret and to improve practice (one can also recognize this habit in a reflective teacher); and a *social aspect*: the readiness to face a matching, comparing one’s own actions with others’ actions, to identify and to clearly formulate research questions, to be able to communicate with other people according to the rules of an evolving scientific community. In particular, what surely characterizes teacher-researchers and distinguishes them from, may be, excellent teachers, is the capability to share ideas within a scientific community. This implies to follow some general and specific rules, for example to put well identified research questions into a general theoretical framework, to utilize experience and materials in order to argue about some well declared thesis, to accept criticism and to be continuously well disposed to changes.

We believe that to fix some minimal condition that characterize a teacher-researcher is necessary in order to satisfy the standard of a scientific community: in this sense it is important to have shared criteria to carefully distinguish an acceptable contribution for a research journal, from more freely written, though interesting, accounts of a teaching activity. At the same time we are aware that pretending to strictly satisfy those requirements as a necessary goal of the enterprise of forming reflective teachers or perhaps teacher-researchers could entail the risk of discouraging willing young teachers from realizing their urges for improving their professional behaviour. This recommendation has been one of the main points of discussion in the Project.

## SOME FINAL REMARKS

The outcomes of the international meetings allowed us to understand the depth and the multiplicity of problems to be overcome, in order to achieve an effective collaboration between researchers belonging to different cultures. A necessary condition for such a collaboration goes through: *a real willingness of sharing* *problems*; listening to others and taking into account the working and operating conditions of a certain group (in order to understand and to search for solutions after common studies and efforts).

In our opinion, the main result of the project might be considered *a deeper awareness of the problems that make an effective collaboration between Eastern and Western countries difficult*. By making these problems explicit, we might help others to overcome the rigid barriers we met. It is not an easy task, due to the weak common background, which makes actual interests often diverge.

## NOTES

1. The Project PDTR (*Transforming Mathematics Education through Teaching-Research Methodology)* has been realized in 2005-2008 under the leadership of S. Turnau (Rzeszów University, Poland), with the help of B. Czarnocha and the expertise of H. Broekman, J. Mason, N.A. Malara. It has involved seven teams of mathematics teachers, apprentices in the craft of "teaching-research", from Hungary, Italy, Poland, Portugal and Spain.

2. The two books (Czarnocha, 2008) and (Turnau, 2008) are downloadable from http://www.pdtr.eu/index2.php

## REFERENCES

Crawford, K. & Adler, J. (1996). Teachers as Researchers in Mathematics Education, in Bishop, A.J. *et al.* (eds), *International Handbook of Mathematics Education*, 1187-1205.

Czarnocha, B. (ed) (2008). *Handbook of Mathematics Teaching Research*, Rzeszów University Press, Rzeszów (Poland).

Guidoni, P.; Iannece, D. & Tortora, R. (2005). Forming Teachers as Resonance Mediators. *Proceedings of PME 29*, Melbourne, 3, 73-80.

Jaworski, B. (2003). Research practice into/influencing mathematics teaching and learning development: towards a theoretical framework based on co-learning partnerships, *Educational Studies in Mathematics*, 54, 249-282.

Malara, N.A. & Zan, R. (2002). The problematic relationship between theory and practice, in English, L. (ed.), *Handbook of International Research in Mathematics Education*. LEA, NJ, 553-580.

Mason, J. (1998). Enabling teachers to be real teachers: necessary levels of awareness and structure of attention. *Journal of Mathematics Teacher Education*, 1, 243-267.

Navarra, G.; Malara, N.A. & Ambrus, A. (2008). A problem posed by John Mason as a starting point for Hungarian-Italian bilateral teaching experiment within the European project PDTR, paper presented at *CERME 6* -WG 13.

Peter-Koop, A. (2001). From "Teacher Researchers" to "Student Teacher Researchers" – Diagnostically Enriched Didactics. *Proc. of PME 25*, 1, 72-79.

Pimm, D. (1987). *Speaking Mathematically*. Routledge, London.

Sfard, A. (2000). Symbolizing Mathematical Reality Into Being – or How Mathematical Discourse and Mathematical Objects Create Each Other, in Cobb, P. *et al.* (eds.), *Symbolizing and Communicating in Mathematics Classrooms*. LEA, Mahwah, NY, 37-98.

Shulman, L.S. (1986). Those who Understand: Knowledge Growth in Teaching. *Educational Researcher*, 15, 4-14.

Turnau, S. (ed) (2008). *Handbook of Mathematics Teaching Improvement*, Rzeszów University Press, Rzeszów (Poland).

## APPENDIX

## Excerpts from the teachers' final global reflections revealing the impact of the project on them

NG (Primary school teacher). Thanks to PDTR project I have understood that my professional growth is still at the beginning, and it is a process that has never to be considered concluded. To sum up, in these three years I have learned to reflect on: cognitive processes (How have I done? How does my mind work when I learn? How does children's mind work when they learn? etc.); metacognitive activities of control (I have learned how to carry out this activity… I have used these strategies… such strategies allowed me to… Which structures or models do my pupils construct? How do they use these structures?…); the disciplinary structures on which I've been working with my pupils (above all arithmetical structures and "proportional thought").

RF (Middle school teacher). Transcriptions, that have demanded time and energy, allowed for a self-evaluation of my own professionalism, a critical meta-reflection on my own way of managing collective discussions, on my way to send pupils' suggestions back to the class, to intervene and direct the discussion itself. After this process I got to a higher professional awareness: I became aware of the need to refine my capacity of grasping immediate feedback by pupils in a meaningful way, always keeping in mind the aims of the route I undertook. I also reached a higher awareness of the need for a careful control of didactic methods and of knowledge about the discipline. This has led an empowerment of my professional awareness on the pedagogical sensitiveness that needs to be used in order to favour pupils' cognitive, relational and affective increase.

MP (Middle school teacher). Through the training activities I actually saw the relevance of linguistic obstacles, which make the interpretation of texts with a mathematical content problematic well before their translation into the most typical languages of this discipline (numerical, algebraic, tabular, graphic). For many students this process implies an extremely hard move from a narrative context to a logical relational one. This aspect is often neglected in the ordinary mathematics teaching activity, whereas it would require an in-depth reflection by teachers. … Since the whole teaching sequence was video recorded, the careful analysis of the recordings strongly highlighted the main features of my *modus operandi* in the classroom. It is embarrassing and instructive at the same time to see yourself during the class, to find out that you did not grasp immediately the opportunities offered by students to guide the lessons towards fertile grounds for a discussion.

MB (Secondary school teacher). The *a-posteriori* analysis of my lessons sometimes meant realizing the inefficacy of my own didactical methodologies and behaviours. During this project of research on our own practice we had the possibility to learn to consider failures, not as negative events to be cancelled without trying to find a remedy, but as “launching pads” to bring ourselves into question. During this phase, the work with the mentor particularly helped me. The numerous pre and post class activities meetings and the crossed analysis of excerpts of class discussions represented a further source of reflections. Cooperating with the mentor gave coherence to my work, aimed at reaching prearranged objectives: the didactic ones, those related to the relationship to be established with my students and those correlated with the research on my practice. In these three years I gradually acquired more confidence in the tutoring-relationship with the mentor, who initially was an “uncomfortable” presence and quickly became an important reference.

SD. (Secondary school teacher). The relationship with the mentor and the coordinator must be particularly taken into account because, with their experience, they helped us in keeping the coherence between the path we planned and the objective of the project. Their advices concerned not only the theoretical framework of reference, but also the planning of the different phases of the path, the organization of the methodology of work in our classes and the a priori and a posteriori analysis of class activities. Thanks to this collaboration, I understood the importance of considering the didactic action as a set of measured choices of contents, proposals, methodologies and teacher’s behaviours. In this perspective, students’ contributions are interpreted as a resource, rather than a dreadful unforeseen event… Numerous aspects have made my participation in the Project significant, even if I am aware that I have only taken a little step in the professional development of a teacher, which is full of shades and potentialities.

# WHY IS THERE NOT ENOUGH FUSS ABOUT AFFECT AND META-AFFECT AMONG MATHEMATICS TEACHERS[1]?

Manuela Moscucci - University of Siena (I)

*The role of affect in the teaching and learning of mathematics is widely recognised by researchers in the field of mathematics education, and a plethora of literature has been published on the subject. However, the related issue of meta-affect has been addressed only minimally. This paper aims to increase awareness of its importance within the community of mathematics teachers and mathematics teacher trainers. More specifically, it suggests how a meta-affective approach may be usefully adopted by mathematics teachers in the classroom as well to catalyse the personal and professional growth of current or future mathematics teachers.*

Keywords: affect, awareness, belief, emotion, meta-affect.

## Introduction

The realm of affect is an especially rich area of research in mathematics education. However, the impressive scientific achievements in both qualitative and quantitative terms have failed to adequately influence practice among mathematics teachers or moreover, to drive investigation into the application of scientific research to practical mathematics instruction in the classroom. To no avail, Burkhardt and Schoenfeld (2003) invited researchers to "make progress on fundamental problems of practice". With twenty-five years of experience imparting in-service training for mathematics teachers and ten years of experience as a mathematics teacher trainer (in Italy a two-year postgraduate degree leading to teacher certification was launched ten years ago), the author has investigated the relationship between affect, meta-affect and changes in teaching practice among mathematics teachers. The adoption of a teaching methodology based on the resulting experience would appear to offer considerable promise.

## Theoretical framework

McLeod (1992) identified beliefs, attitudes and emotions as the constructs upon which affect regarding mathematics is based. De Bellis & Goldin (1997) also recognised the role of values in this sense. Research into affect has evolved considerably since then, with growing investigation into the issues involved and a broadening of the theoretical background, to the point where multiple theoretical frameworks have emerged. We may thus address affect as a system of representation and communication (Goldin, 2002) in which beliefs, attitudes, emotion and values – the four elements in Goldin's *"tetrahedral model"*- are viewed as a sub-domain; as a

[1] The author hopes the title doesn't sound disrespectful to Schoenfeld (Schoenfeld, A. H.(1987). What's all the fuss about metacognition?. In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189-215). Hillsdale, NJ: Lawrence Erlbaum Associates), who wrote the paper in question when asked to explain 'metacognition'.

system “strongly, naturally and in a dynamical way” linked to cognition (Malmivuori, 2004); within a socio-constructivist framework (Op ‘t Eynde, 2004) or with an embodied cognition approach (Brown & Reid, 2004). The various theoretical frameworks highlight two elements which should attract the attention of researchers. The first of these regards the frequent appearance of the terms ‘metacognition’, ‘consciousness’, ‘awareness’, ‘self-awareness’ and ‘meta-level’ in relevant literature. An important step in developing the debate and research field would be taken by investigating the meta-levels of the four constructs, their theoretical collocation and their correlations with metacognition. Hannula (2001) offered an approach to the issue, but there remains much more to be learned. The importance of metacognition in the learning processes was first highlighted by Flavell (1976). LeDoux (1998) and Damasio (1999), by conducting investigations based on fMRI (functional Magnetic Resonance Imagining), CAT (Computerized Axial Tomography) and PET (Positron Emission Tomography), have demonstrated that the functioning of the cognitive and emotive systems are closely related. In light of these studies one might plausibly wonder whether the term metacognition still means anything, or what its role might be within the new scientific framework. Must it be accompanied by the term meta-emotion, must a new term be coined to comprise the two, or must yet other terms be coined? The second element to emerge from the theoretical frameworks of affect is how consistently they display links between affect and neuroscientific research (Schlöglmann, 2003). This has made it possible to create a neuroscientific basis for the interdependence of affect’s four constructs, so frequently emphasized in research. It has also afforded clarification of other hotly contested issues, such as the nature of beliefs, which must necessarily be hybrid (i.e.Furinghetti & Pehkonen, 2002): that is, both cognitive and emotive. This supports author’s hypothesis (Moscucci, 2007) beliefs are the ‘best’ element, among the four constructs of affect, which to act on, and this is the reason why, in this contest, the author is particularly interesting in ‘beliefs’, which seem, together with emotions, to shape attitudes (Hart, 1989). The matter of defining ‘belief’ remains unresolved within the research field. Hence, here the term ‘belief’ will be taken to represent some sort of ‘primitive entity’, and every belief some sort of ‘axiom’ assumed as a result of personal experience; basically an affirmation which is accepted without proof. Furthermore, different mathematics-related belief systems (Schoenfeld, 1992; Leder, Pehkonen & Törner, 2002) are in some way all correlated. So we might say, by adopting terminology from algebraic structure language, that the individual’s beliefs regarding mathematics (although the choice of subject is inconsequential) do not make up a ‘set’ of beliefs but rather a ‘structure’ of beliefs. Researchers have not simply investigated the role of student beliefs in their learning processes, but also the role of the beliefs of mathematics teachers. As regards definitions, Richardson (1996) identifies teacher beliefs with their theoretical perspective of teaching methodology. This underlines the effect of teachers’ beliefs on their teaching practices. It would seem logical to deduce that teachers’ beliefs determine the quality of their practices (Cooney, 2001). However, almost twenty years ago, Cobb, Wood and Yackel (1990) noted that these influence each other reciprocally, rather than in terms of 'side of the implication'. The interrelations among teacher beliefs and student beliefs are equally complex and controversial (Beswick, 2005) and it appears currently impossible to hypothesize the entity of these relations, given that student beliefs have not been proven to be the product of teacher beliefs, nor *vice versa*. Nevertheless, although the theoretical issue has not been resolved, the impact of belief systems on the classroom behaviour of teachers has been recognised in numerous studies involving mathematics teachers (for instance, Pehkonen, 1994; Chapman, 1997, 1999).

## From the realm of theory to didactic practice

As mentioned in the introduction, this proliferance of scientific research has failed to produce significant developments that may be of direct use to mathematics teachers in the classroom. And yet, such developments are sorely needed by mathematics teachers, students, school systems and indeed society in general. Thus any efforts to impact on the belief systems of teachers, and especially on any beliefs that are damaging to students, are more than welcome. Damaging ideas might be identified as 'inefficacy beliefs' (*e.g*."A special inclination is needed to be good at maths in school"), in contrast with 'efficacy beliefs' in teaching mathematics, which have been investigated and illustrated (Philippou and Christou, 1998; 2002). The question to be answered is how to progress from inefficacy beliefs to efficacy beliefs and efficacy teaching practices. An approach addressing meta-affect may well prove useful. Goldin (2002) considers meta-affect as a key construct, "including affect about affect, affect about and within cognition that may be again about affect, monitoring of affect, and affect as monitoring". The potential of meta-affect as a vehicle for the development of the professional profile of mathematics teachers has been confirmed throughout ten years of successful[2] mathematics teacher training carried out by the author with teachers undergoing training and already in service. Due to space restrictions, only in-service teachers will be considered here.

## Towards a holistic approach to maths teachers affect

Fifteen or so years of training courses proved that, in spite of apparent success, the impact on classroom practice was undeniably disappointing, with the didactic practices of the teacher participants evolving only rarely. Few teachers could bear the prospect of giving up the "school mathematics tradition" (Cobb et al., 1992) (frontal lessons aimed at the introduction of the new technique, presentation of examples and setting of exercises), even if the main goal of the courses was precisely didactic quality. Indeed, within the Italian school system the proportion of failures in mathematics with respect to all academic subjects has been and continues to be

[2] Training is reputed successful when: 1) the participating teachers express their satisfaction with the training by means of their responses to a survey presenting questions in a 4-point Likert scale format; 2) the participating teachers begin to modify their teaching practices as suggested during the training course; 3) the modification of classroom practices by teachers produces positive effects (in the sense that the students benefit both in terms of their affect toward mathematics and their actual performance in this discipline).

telling: the Ministry sets this year's figure at 42%. Moreover, the 'discomfort' (lack of success but also, for instance, 'negative' emotions and 'inefficacy' beliefs towards maths) of Italians with mathematics was believed (and subsequently proven (Moscucci et al. 2005) to be an 'endogenous cause' (*e.g.* arising within the school system itself) of student dropouts. This alarming situation called for the creation of an intervention scheme based on the following principles: 1) teaching methodology and teacher affect are closely linked (this was contextualized above from a theoretical perspective); 2) dealing with beliefs as a purely psychological construct is limiting, as mathematics teachers work together with their colleagues within a social context that tends to perpetrate traditional, time-tested teaching techniques (Op 't Eynde, 2004); to consequently avoid marginalising teachers who attempt to update their approaches, the teacher educator needs to undertake group work as has been carried out during well-documented experimentation (Jaworski, 2003); 3) the teacher trainer must obviously make use of the same didactic methods that are presented to the teachers for use with their students. The outcome of these considerations was the creation of an intervention scheme (Moscucci, 2007), in which beliefs systems role was highlighted. Meantime, the author has understood the synergy springing out the contemporaneous work about emotions and beliefs. As has been repeatedly debated within the theoretical framework, the affect of an individual (be it a student or teacher) is a complex structure comprising closely-linked constructs. Therefore any effort to influence it must simultaneously address all the elements on which it is based. So, perhaps, the success of that intervention scheme is due to the global – we would say 'holistic' – approach to teacher affect.

## Meta-affect: a 'tool' not enough used by mathematics teachers?

About thirty years have passed since Flavell (1976; 1979) published his metacognition research and the importance of this concept to the learning process has been proven and reported (for instance, Hartman (1998)). However, it is rare to meet mathematics teachers who make use of didactic techniques informed by the abundance of metacognition research. The big step in the field of metacognition might involve equipping maths teachers with tools of observation and intervention that could be applied first and foremost to themselves: "...increasing metacognitive activity through private reflection and shared conversations increases teachers' awareness of their subjective knowledge… beliefs are often challenged through this process, which lays the groundwork for the construction of new knowledge and for real change in teaching practice" (Hart, 2002). The training courses for mathematics teachers conducted by the author over the last ten years were structured by means of a method (Moscucci, 2007) that seeks to achieve meta-affective goals with the teachers prior to addressing discipline-specific issues. The distinguishing characteristic of this method is its emphasis on awareness (Marton & Booth, 1997): the teachers are put in a position to autonomously become aware of their own belief systems and emotions, without being obliged to openly declare their beliefs and emotions. There are two reasons for this. The first, as regards beliefs, is the well-known distinction between "beliefs espoused and beliefs in practice" (Schoenfeld, 1989). What's more, teachers often are not conscious or even aware of the beliefs underlying their teaching practice. The second regards emotions. Awakening the emotions that have accompanied teachers during the development of their professional capacity is extremely beneficial. The emotions experienced almost certainly influence their beliefs regarding mathematics learning and teaching. Even memories of what it was like to be a maths student as far back as primary school need to be evoked. Remembering is the first step. Then the emotion recalled must be elaborated to try to analyse its immediate impact and understand any eventual lasting repercussions. This means engaging teachers in 'meta-emotive' activity without attempting to place educators in the role of psychologist, but rather assisting teachers to self-analyse their memories. Let us briefly examine the close link between meta-emotion, meta-cognition and the awareness of beliefs. Emotion[3] is a personal response to an event signalled by physical symptoms such as an accelerated heart rate, blushing and facial expression. With time (a matter of seconds or minutes) these symptoms lessen and eventually disappear. There is consciousness of the emotion, but awareness takes hold only as the intensity of the physical reaction diminishes and it again becomes possible to 'think rationally', as we say. If the emotion has been particularly intense or is part of a series of emotions related to a single situation (such as learning mathematics), it begins to generate thoughts regarding the emotion's cause, origins, consequences and responsibilities. These spontaneous or subsequent thoughts may set off a chain of further thoughts as well as further emotions. The initial emotion and its related physical manifestations have only short-term effects, thus failing to directly influence an individual's future. However, the resulting chain of thoughts and emotions may lead to the creation of certain beliefs that are known to be highly influential. Most beliefs are generated in this way. Thus awareness of this process is a fundamental step in controlling negative emotions, neutralising their impact on the present and re-elaborating the beliefs generated by them. When considering this process, a distinction must be made between maths teachers with a mathematics degree and those with a different degree (in Italy this is not only possible but predominantly the case with teachers of the grade 6-9 levels). With this latter group a greater effort must be dedicated to developing awareness of emotions**,** as such teachers often experienced difficulty with mathematics, as student, at school or at university. As also regards teacher attitudes, activities that develop awareness of them must be provided, and teachers can be left free to define 'attitude' as they wish. Awareness of one's attitudes is intended as awareness of what teachers consider to be their attitudes toward mathematics both as a learner in the past and as a teacher presently. To give an example, the following activity frequently proves useful. Teachers are asked to put down in writing – informally, without attention to composition – how they perceive their attitudes. Then their students are asked to repeat the exercise anonymously by the researcher - trainer. The students may find it

[3] When especially intense, the amygdale may come into play (LeDoux, 1995).

easier to express their opinions if they are provided with a guideline such as the beginnings of sentences to complete. The teachers observe the opinions expressed by their students and, following a personal analysis, are asked to put in writing their comments regarding both their and their students' tasks. As this brief description illustrates, this approach concentrates on beliefs and emotions, inasmuch as they are considered to shape attitudes, as underlined in the theoretical framework. The aim of this approach is to create a virtuous cycle between the re-elaboration of beliefs and emotions on one hand, and the adoption of non-traditional methods on the other (the non-traditional methods are, in certain cases, 'discovered' by the teachers in a socio-constructivist learning environment, in other cases by questioning their classroom practices). The first feeble attempts to make use of new methodologies and non-traditional disciplinary approaches produce initial resources that encourage teachers to progress in their development. The teachers begin to experience new emotions, thus they re-elaborate their beliefs, and recontextualise their previous emotions. This is how the virtuous cycle is catalysed. The awareness of one's own awareness represents another step toward quality in a teacher's meta-affective competence.

## A short description of one experience

Of many cases observed, the following - chosen to give a 'hint'- offers elements to ponder as far as different teacher typologies are concerned. In 2005 the author was invited by the principal of a vocational school to set up and implement a three-year project aimed at reducing student failures in mathematics, which regarded over 60% of students (official data provided by the School Administration). The situation was in line with that of all schools of this kind, so it was actually no worse than average. Due to the lack of space, it is impossible to describe the details of the project. Briefly, it consisted in conducting activities based on meta-affect, as described in the previous section. The author worked with the teachers and the teachers worked with their students. As for subject teaching, the teachers were required to 'embrace' a socio-constructivist teaching methodology. The author personally met the students with special difficulties (three-four times -two hours- for each class involved) in order to diagnose their nature. The school's three mathematics teachers -all of them- were more or less of the same age, between forty and forty-five, while their psychological and professional profiles varied. One teacher, who will be called Victoria, was very cordial and outgoing, had a degree in mathematics, attended mathematics teaching conferences regularly, had previously participated in various innovative mathematics teaching projects and had always attempted to put into practice the developments presented in mathematics teaching journals. In spite of her efforts to improve her students' results, she had never been successful. She participated in the project with great expectations. Another teacher, who will be called Angela, had a degree in mathematics and was disappointed by the poor results and scarce interest of her students, to the point where she simply wanted to retire. Angela was more insecure than Victoria but sincerely wanted to help her students. Perhaps it was a sense of impotence that made her want to retire. Although without great hopes, she participated in the project willingly. The third teacher, who will be called Bill, had a degree in IT and had taken the teaching job following a frustrating experience as an IT technician. He had acquired a reputation for strictness with the students. He commented that “his students didn’t work enough” or “lacked the basics”, and that “some of them simply couldn’t be helped”. He participated in the project only following the insistence of the principal. As questions came up during the initial meetings (What is the role of school in educating individuals? And what is the role of mathematics? What is ‘school mathematics’?), his interest seemed to grow. “The answers to certain questions should be obvious to a teacher while they may not be; most answers are simply rhetorical!”. The three teachers attended an introductory course (about 30 hours, as a whole), using the intervention scheme mentioned in the previous paragraph (Moscucci, 2007), during the month of September 2005, prior to the beginning of the school year. They worked as usual together with their mathematics-teaching colleagues, but in an atmosphere of “contrived collegiality” (Hargreaves, 2004), while in this new context they began to appreciate the value of ‘collaborative work’, undoubtedly benefiting from collaboration in “small groups”, as underlined by Santos (2007). They used the same methodology with their first -and second- year classes (involving more than 150 students). Throughout the year their work in class was supported by means of meetings with the author, every two weeks during the first three months of the year, later monthly, as well as long phone calls to provide emergency help. The author decided not to attend teachers’ lessons not to intrude a ‘strange’ element in the ‘classroom ambience’ and it was impossible to organize recording tools (but author’s meeting with the students in special difficulties). Unbeknown to the teachers and the author, the project was monitored by the principal through inspection of the attendance registers. At the end of the first school semester, appreciable improvements were noticed of the average final marks for the same level classes with respect to preceding years (data, and the following ones, from the Minutes of Class Meetings). The only change undertaken regarded the teaching methodology introduced in the project, so it is ‘highly’ likely that this was precisely the reason of these improvements. Victoria and Angela’s classes proved to be the most successful in the project, as, at the end of the first year, the number of failures in mathematics was reduced by about 90%. Angela also regained enthusiasm in her teaching. Bill encountered greater difficulty than his colleagues in applying the initial methodology focussing on meta-affect and the subsequent content methodology: while Victoria and Angela showed their enthusiasm for the activities suggested by the author, Bill always needed additional time to accept the proposals, and, above all, he was hesitant to update the activities in his classes. In any case his students achieved much better results with respect to previous years. Even if each teacher made up their own test, they were very similar except for insignificant details. Overall, at the end of the project’s first year, the only students to fail mathematics had also failed most other subjects and consequently had to make up the year. At the end of the year the school’s vice principal conducted a school-wide survey (completely unrelated to the project), and the results showed mathematics to be the students’ favourite subject. Undoubtedly the aspect of the project regarding course content played a part in the project's success, but it would have been impossible to even address course content without first eliminating the negative preconceptions towards mathematics of most students. In the third year of the project Victoria was transferred to a scientific high school renowned for its strictness and traditional methodology. The classes she adopted the method with achieved better results than all the other classes of the same year on a standardized test administered to all. In the last year of the project Angela suffered the lack of (mostly psychological) support from Victoria and lost some enthusiasm, but is still convinced of the method's validity. Bill seems to have become less strict and perseveres in trying to apply the method. The author has obtained such surprising outcomes as those described in this paper on many other occasions. Now she is planning to monitor wider experimentation in a vocational school. At present it seems important, at first, to spread a research hypothesis: the awareness of one's own belief systems accompanied by a personal reworking of the emotions felt during mathematics tasks, may be key in removing 'inefficacy beliefs' and 'recontextualising' past emotions so that they are innocuous in the present. Secondly, the author hopes other researchers, teacher trainers and teachers will try to adopt these teaching methods and schema so as to confirm or contrast the hypothesis.

## Remarks

The positions of numerous researchers on meta-affect recognising its central role in affect, the relationship between meta-affect and metacognition revealed by neuroscientific research and the success of certain teaching methods based on meta-affective methodology should encourage researchers to investigate this subject from a theoretical perspective. After all, like many fields of education science, mathematics education displays distinct characteristics. In disciplines such as medicine or pharmacology, before a treatment such as pharmacological therapy can be applied, various levels of experimentation must be carried out. Instead, in the field of education it is possible and often especially effective to alternate research and the application of research outcomes to practice. Or better, this is a very fruitful way to proceed. This makes it particularly important to spread the use of practices with a high potential for success. The resulting discussion, rebuttal and development can only contribute to furthering research and increasing didactic quality.

## REFERENCES

Beswick, K. (2005) . The beliefs practice connection in broadly defined contexts. Macmillan, *Mathematics Education Research Journal*, 17 (2), 39-68.

Brown, L. & Reid, D.A. (2004) . Emotional orientation and somatic markers: Implications for mathematics education. *Proceedings of PME 28*, 1, 123-126.

Burkhardt, H. & Schoenfeld, A.H. (2003) . Improving educational research: Toward a more useful, more influential, and better-funded enterprise. *Educational Researcher*, 32 (9), 3-14.

Cobb, P., Wood, T. & Yackel, E. (1990) . Classroom and learning environments for teachers and researchers. In R.B. Davis, C.A. Maher & N. Noddings (Eds.), *Constructivist Views of the Teaching and Learning of Mathematics*, National Council of Teachers of Mathematics, Reston, VA (pp. 125-146).

Cobb, P., Wood, T., Yackel, E. & McNeal,B. (1992) . Characteristics of classroom mathematics traditions: an interactional analysis. *America Educational Research Journal*, 29, 573-604.

Cooney, T.J. (2001) . Considering the paradoxes, perils and purposes of conceptualizing teachers development. In F.L. Lin (Ed.), *Making sense in Mathematics Teacher Education*, (pp. 9-31) Dordrecht: Kluwer.

Chapman, O. (1997) . Metaphors in the teaching of mathematical problem solving. *Educational Studies in Mathematics*, 32, 201-228.

Chapman, O. (1999) . Inservice teachers' development in mathematical problem solving. *Journal of mathematics teachers education*, 2, 121-142.

Damasio, A.R. (1999) . *The feeling of what happens: body and emotion in making of consciousness.* New York/San Diego/London: Harcout.

DeBellis, V. & Goldin, G.A. (1997) . The affective domain in mathematical problem solving. In E.Pehkonen (Ed.), *Proceedings of PME 21* (Lahti), 2, 209-216.

Flavell, J.H. (1976) . Metacognitive aspects of problem solving. In R.B. Resnick (Ed.), *The Nature of Intelligence*, Hillsdale, NY: Erlbaum.

Flavell, J.H. (1979) . Metacognition and cognitive monitoring: a new area of cognitive-developmental inquiry. *American Psychologist*, 34(10), 906-911.

Furinghetti, F. & Pehkonen, E.: 2002, 'Rethinking Characterizations of Beliefs', in Leder, G., Pehkonen, E. & Törner, G. (eds.), *Beliefs: A Hidden Variable in Mathematics Education?*, Kluwer, Dordrecht.

Goldin, G.A. (2002) . Affect, meta-affect, and mathematical belief structures. In G. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp.59-729) Dordrecht: Kluwer.

Hannula, M.S. (2001) . The metalevel of cognition-emotion interaction. In M. Ahtee, O. Björkqvist, E. Pehkonen & Vatanen (Eds.), *Research on mathematics and science education. From beliefs to cognition, from problem solving to understanding.* Univ. of Jyväskylä, Institute for Educational Research, 55-65.

Hargreaves, A. (2004) . *Changing teachers, changing times, teachers work and culture in the postmodern age*, London: Cassell.

Hart, L.E. (1989) . Describing the affective domain: saying what we mean. In D.B. McLeod & V.M. Adams, (Eds), *Affect and mathematical problem solving* (pp. 37-45), NY: Springer Verlag.

Hart, L. (2002) . A four year follow-up study of teachers' beliefs after participating in a teacher enhancement project. In G, Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 161-176), Dordrecht: Kluwer.

Hartman, H.J. (1998) . Metacognition in teaching and learning: An introduction. Instructional sciences. *International Journal of Learning and Cognition*, 26, 1-3.

Jaworski, B. (2003) . Inquiry as pervasive pedagogic process in mathematics education development, *Proceedings of CERME 3*, Bellaria (I).

Leder, G., Pehkonen, E. & Törner, G. (Eds.) (2002) . *Beliefs: a hidden variable in mathematics education?*, Dordrecht: Kluwer.

LeDoux, J. (1998) . *The emotional brain*. Phoenix: Orion Books Ltd.

Malmivuori, M. L. (2004) . A Dynamic viewpoint: affect in the functioning of self-system processes. In *Proceedings of PME 28(1)*, pp. 114-117, NY: Bergen.

Marton, F. & Booth, S. (1997) . *Learning and Awareness*, Hillsdale, NY: Lawrence Erlbaum Associates.

McLeod, D. (1992) . Research on affect in mathematics education: A reconceptualization. In D.A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 575-596) NY: Macmillan.

Moscucci, M., Piccione, M., Rinaldi, M.G., Simoni, S. & Marchini, C. (2005) . Mathematical discomfort and school drop-out in Italy. In *Proceedings of CERME4*, (pp.245-255).

Moscucci, M. (2007) . About mathematical belief systems awareness. In *Proceedings of CERME5* (pp. 298-308).

Op 't Eynde, P. (2004) . A socio-constructivist perspective on the study of affect in mathematics education. In *Proceedings of PME 28(1)* (pp. 118-122), Bergen.

Pehkonen, E. (1994) . On teachers' beliefs and changing mathematics teaching. *Journal für Mathematik-Didaktik*, 15 (3/4), 177-209.

Philippou, G. & Christou, C. (1998) . The effects on a preparatory mathematics program in changing prospective teachers' attitudes toward mathematics. *Educational Studies in Mathematics*, 35, 189-206.

Philippou, G. & Christou, C. (2002) . Mathematics teaching efficacy beliefs. In *Beliefs: A hidden variable in mathematics education?* (pp-202-231). Dordrecht: Kluwer.

Richardson, V. (1996) . The role of attitudes and beliefs in learning to teach. In J. Sicula (Ed.), *Handbook of Research on Teacher Education* (pp.102-119). London: Prentice Hall International.

Santos, L. (2007) . The project work and the collaboration on the initial teacher training. In *Proceedings of CERME5* (pp.1974-1983).

Schoenfeld, A.H. (1989) . Exploration of students' mathematical beliefs and behaviour, *Journal of Research in Mathematical Education*, 20, 4.

Schoenfeld, A.H. (1992) . Learning to think mathematically: problem solving, metacognition and sense making in mathematics. In *Handbook of Research on Mathematics Teaching and Learning* (pp.334-370). NY: Macmillan.

Schlöglmann, W. (2003) . Can neurosciences help us better understand and affective reactions in mathematical learning?. In *Proceedings of CERME3, TG2*, Bellaria (I).

# THE ROLE OF SUBJECT KNOWLEDGE IN PRIMARY STUDENT TEACHERS' APPROACHES TO TEACHING THE TOPIC OF AREA

Carol Murphy

University of Exeter

*This study reviews the relationship between student teachers' subject knowledge in the topic of area and their approaches to teaching that topic. The research was carried out with four primary student teachers and examines the similarities and differences between the nature of their subject knowledge and their plans to teach the topic. In this paper results of two of the four student teachers are focused on to illustrate the contrasts in planning and subject knowledge. The intention is not to generalise relationships but to examine the phenomena presented. It raises questions related to the variables in connecting student teachers' subject knowledge and their knowledge of how to teach.*

Key-words: subject knowledge; area; student teacher; approaches to teaching; understanding

## INTRODUCTION

The importance of subject knowledge in the preparation of teaching activities is clearly recognised (Ball, Lubienski & Mewborn, 2001). If we see teaching fundamentally as an exchange of ideas it would seem evident that a teacher's understanding of a topic will impact on how the idea is 'shaped' or 'tailored' when presented in a classroom. As such "teaching necessarily begins with a teacher's understanding of what is to be learned and how it is to be taught" (Shulman, 1987, p.7). Shulman emphasised the transformation of a teacher's knowledge of a subject into 'pedagogical content knowledge' and consequent pedagogical actions by "taking what he or she understands and making it ready for effective instruction" (p.14). In this way mathematical content knowledge is 'intertwined' with knowledge of teaching and learning (Ball & Bass, 2003).

It is generally accepted that mathematics should be taught with understanding (Hiebert & Lefevre, 1986; Skemp, 1976). In the topic of area it would seem that children often rely on the use of formulae with little understanding of the mathematical concepts involved (Dickson, Brown & Gibson, 1984). They are unable to see the reasonableness of their answers and so are unable to monitor their use of these formulae. There is also evidence that student teachers have a similar reliance on formulae (Baturo & Nason, 1996; Tierney, Boyd & Davis, 1990).

It would seem that a student teacher with limited understanding of the mathematical topic such as area would not be effective in developing children's understanding. This study aims to investigate the impact of primary student teachers' subject knowledge on approaches to teaching the topic of area. As an interpretive study the intention is not to generalise any relationship but to examine phenomena related to differences and similarities in the student teachers' understanding of the topic and in how they plan activities to teach the topic.

## DEVELOPING UNDERSTANDING IN THE TOPIC OF AREA

Measuring area is based on the notion of 'space filling' (Nitabach & Lehrer, 1996). However, unlike children's other common experiences of measure such as length, the use of a ruler in measuring area is indirect. In this way instruction that focuses on procedural competence with measuring tools such as rulers "falls short in helping children develop an understanding of space" (p.473) and it is not surprising that many children confuse area and perimeter (Dickson *et al*., 1984). Instruction that models the counting of squares on grids provides more success and may represent the notion of 'space filling'. However this does not represent the full nature of area. As Dickson *et al*. (1984) commented the possible restriction to a discrete rather than a continuous view of area measure might not lead to the notion of Π and the formula of the area of a circle.

Further to this, figures used as representations in the classroom often provide a static view rather than a dynamic view. That is, as a boundary approaches a line, the area approaches zero (Baturo & Nason, 1996). This may lead to misconceptions about the conservation of perimeter and area. The recognition of such a misconception goes back at least to the 1960s with Lunzer's (1968) notion of 'false conservation'. This false notion has more recently been cited by Stavy and Tirosh (1996) as an example of the intuitive rule 'more A, more B', in that as the perimeter increases so the area will increase. Alternatively the intuitive rule can be manifested as 'same A, Same B' in that the same perimeter will mean the same area.

It would seem that once introduced to the formulae, children have a tendency to use these regardless of the success of their answers (Dickson, 1989). Studies such as Pesek and Kirshner (2000) and Zacharos (2006) suggested that, where instruction involved procedural competence and use of formulae, children would insist on repeating strategies that caused errors and they often had difficulty in "interpreting the physical meaning of the numerical representation of area" (Zacharos, p. 229). Where instruction was based on measuring tools such as dividing rectangles into squares children demonstrated flexible methods of constructing solutions and often achieved more success. The studies suggested that the early teaching of formulae presented 'interference of prior learning' (Pesek and Kirshner) or 'instructive obstacles' (Zacharos).

Such 'interference' or 'obstacles' could explain why many children at the beginning of secondary school take algorithmic approaches to the solution of area measurement problems (Lehrer & Chazan, 1998). It follows that student teachers are likely to have a similar reliance on algorithms. If we refer back to Shulman's model of transformation and Ball and Bass's idea of 'intertwining' content and teaching knowledge, then a student teacher's understanding of the nature of area would seem key to the way they would teach it. Studies that have examined student teachers' subject knowledge in the topic of area (Baturo and Nason, 1996; Tierney et al, 1990) found that student teachers often demonstrated a lack of understanding of how practical concrete experiences could relate to the use of formulae and how area measure evolves from linear measure. They were often uncertain about the reasonableness of their answers and were unable to explain how formulae were related. A study that has examined student teachers' lesson plans for teaching the topic of area (Berenson, Van der Valk, Oldham, Runesson, Moreira, and Brockman's, 1997) found that many student teachers represented the topic of area through the demonstration of procedures and use of formulae rather than focusing on the activities that would support understanding. What we do not know from these studies is whether the student teachers that planned to teach the topic through the demonstration of procedures were the students who demonstrated a lack of understanding of the topic.

## THE STUDY

The four student teachers involved in this study had varied backgrounds in mathematics. At the time of the study they had completed the taught university based element of a one year Post Graduate Certificate in Education (PGCE) and they were about to start their final teaching practice. The student teachers had attended workshop seminars on the teaching of primary mathematics. All four student teachers had the same course tutor so would have followed the same content in their mathematics seminars. The student teachers were also reassured that the work for this project would not be used as part of their course assessment.

Clinical interviews were carried out with each of the student teachers to reveal underlying processes in their understanding (Swanson, Schwatz, Ginsburg and Kossan, 1981; Ginsburg, 1997). The first part of the interview examined the development of the student teacher's lesson plan and the second part of the interview involved the use of mathematical tasks to investigate the nature of their understanding in the topic of area. The mathematical tasks were equivalent with some standardisation of probing questions but further interrogation was managed flexibly in order to be contingent with the student teachers' responses. The interviews were audio taped and transcribed.

### The use of lesson plans

Planning is central to teaching and the development of lesson plans is a key aspect of teacher training. Lesson plans provide a source of data in assessing student teachers' professional development. They can also provide useful cues in follow up interviews when the activities, explanations and questions used by the student teachers help to generate further descriptions (John, 1991, Berenson et al, 1997). Although lesson plans are limited to demonstrating the student teacher's 'espoused' theory of action

(Argyris and Schon, 1974) they can be seen as effective in indicating the student teacher's perceptions of teaching.

The student teachers were asked to plan a lesson to introduce the topic of area to a Y4 class (8 to 9 year olds). The student teachers were advised that they could use any sources they normally would to help plan the lesson. The only restriction being the ideas would be their own or their own interpretation of teaching ideas from other sources. The student teachers were questioned about the following:

1. How they had developed the activities
2. How they felt the activities would facilitate the children's learning
3. The instructions or explanations they intended to give
4. The questions they intended to ask the children
5. The difficulties that they felt the children would encounter

**Area Tasks**

The second part of the interview involved four tasks adapted from Baturo and Nason's (1996) and Tierney *et al*. (1990) studies to ascertain the subject knowledge of the student teachers.

Task 1 (Baturo and Nason, p.245) includes both open and closed shapes to test student teachers' understanding of the notion of area (see fig 1). Shapes G and F were included to test the ability to differentiate between area and volume, shapes J and K test the notion of area as the amount of surface that is enclosed within a boundary and shapes E, H and L test the understanding of area from a dynamic perspective.

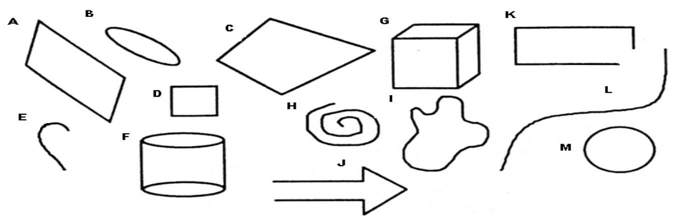



**Fig 1: Task 1**

Task 2 (adapted from Baturo and Nason) was designed to test the ability to compare areas, initially without the use of formulae (see fig 2). The student teachers were presented with two pairs of cardboard shapes. Dimensions were not given. Comparison by visual inspection alone would be inconclusive so the student teachers were asked to consider ways to compare area. This was used to determine if the student teacher was able to use measuring processes other than external measures and use of the formulae.

Pair A: Pair B:

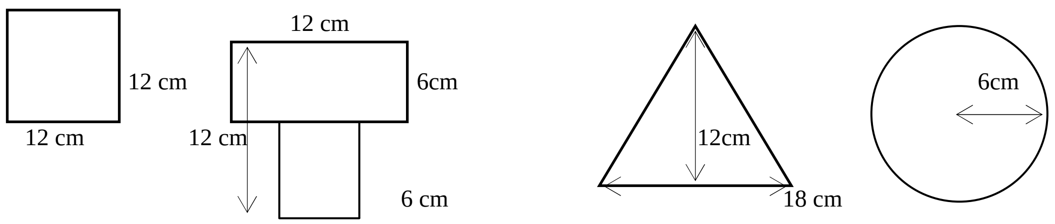



**Fig 2: Task 2 (shapes not drawn to scale)**

Task 3 (adapted from Tierney *et al*.) was intended to determine a dynamic view of area and the ability to consider changes in area and perimeter (see fig 3). The student teachers were given three cardboard shapes. Dimensions were not given.

1. a rectangle 9cm by 4cm
2. a parallelogram where the area is the same as the rectangle but the perimeter has changed (base 9 cm and height 4 cm)
3. a parallelogram where the perimeter is the same as the rectangle but the area has changed

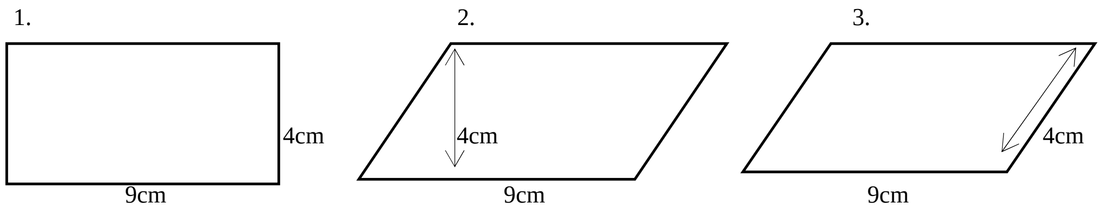



**Fig 3: Task 3 (shapes not drawn to scale)**

Task 4 (adapted from Baturo and Nason) aimed to test the correct use of formulae. It also tested for an understanding of the relationship with non-rectangular figures, including the use of the ratio Π (see fig 4).

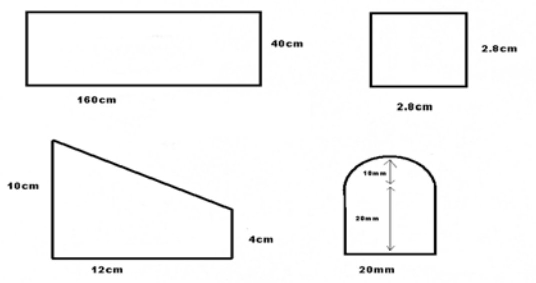



**Fig 4: Task 4 (shapes not drawn to scale)**

## RESULTS AND ANALYSIS

In this paper it is presented the results of two of the four student teachers, Alan and Charlotte, are focused on to illustrate the contrasts in planning and subject knowledge.

### Alan

Alan's highest qualification in mathematics was an 'A' level taken over 5 years ago. He felt that his confidence level was moderate to high. In his lesson plan he intended to model the use of the formula using a transparent grid over a rectangle and by, "thinking out loud", would state, "Find this side, this side and multiply together". He would then show the children how to check by counting the squares. He was concerned that the children might confuse area and perimeter and that they might add the lengths rather than multiply. In order to overcome this he would show how to use a ruler to measure the lengths and repeat the instructions from the introduction. He felt that he would have to tell the children what units to use and that the '2' means squared. Alan would continue the lesson with further practice of the formula with other rectangles and with shapes composed of rectangles. He suggested using a 'real-life' context by extending the use of units to square metres and finding the area of the classroom.

Alan's use of formulae and calculations in Tasks 2, 3 and 4 were quick and accurate. He used the formulae as a first resort in comparing areas of shapes in Task 2 and Task 3 rather than reasoning or comparing by placing the shapes on top of each other. Alan gave a clear definition of area related to the covering of surfaces. He was also aware of the relationships between formulae and the notion of Π as a ratio in finding the area of circles. He was able to consider the dynamic view of area with the parallelograms in Task 3 but did not identify the area of the open shapes as zero in Task 1.

### Charlotte

Charlotte had obtained a grade C GCSE qualification in mathematics, the minimum entry requirement for a primary PGCE course, and she spoke of lacking confidence in mathematics. Charlotte stated that she found the lesson difficult to plan and had researched pedagogy based texts. Charlotte intended to introduce the topic with a large paper rectangle and ask, "How many children can fit onto this shape?" She would use these arbitrary units to determine the area of other shapes and then draw rectangles on the board and pretend that each child is a centimetre square. Charlotte felt that the activities would "lead naturally" to a definition of area as the "amount of space within a shape" and she intended to note the strategies that the children used. She also intended to set an activity to investigate the area of rectangles and changes in perimeter. She would encourage the children to talk together about the patterns they had found. Charlotte would ask, "What do you notice about the perimeter and area of the two classrooms?" (sketches on the board) and "Can you draw different shaped rectangles with an area of 12 squares?".

Charlotte's notion of area from Task 1 seemed inconsistent. Although she stated that the area was the amount of space inside a shape she attempted to include some of the open shapes as those that had an area. She was uncertain as to whether the three-dimensional shapes would have an area, and if so, how to measure it. She was, however, secure in the relationships between the formula for the area of a rectangle and the area of a triangle and was aware of an activity to determine Π as a ratio. Charlotte was aware of the dynamic view of area from Task 3 and was able to compare the areas of the parallelograms with little difficulty. Charlotte made errors in using the dimensions and formulae for calculating areas in Task 4. She was also not aware of the correct units and confessed that she never knew when to use $cm^2$ or $cm^3$.

## ANALYSIS AND DISCUSSION

Performances on the mathematical tasks suggested that Alan had a good understanding of the nature of the topic of area. In particular Alan demonstrated quick and accurate use of formulae. In contrast Charlotte's performance on the tasks demonstrated limited knowledge in the use of formulae and units. Her understanding of the nature of the topic of area appeared to be inconsistent.

Charlotte based her intended introduction to the topic of area on the counting of regions. Charlotte initially started with arbitrary units that would be used later to introduce the square unit. Charlotte was aiming to provide children with activities and problems that would help them realise the notion of area 'naturally'. On the other hand, Alan's lesson was focused on teaching the use of the formula. He was concerned that the children would not use the correct formula for area and he would articulate explicitly how to do this. There was an attempt to relate the use of the formula to 'real-life' by finding the area of the classroom.

According to the review of research above, Alan's intended focus on the use of the formula from the start of his lesson might suggest a premature introduction that would create 'interference' or 'obstacles'. However Alan was a confident mathematician who demonstrated accurate use of formulae and secure understanding of the nature of the topic. In contrast, the activities that Charlotte planned to use would be more likely to support children in developing a notion of area as 'space filling'. This might reduce the children's reliance on the use of formulae and consequently support their understanding. However Charlotte was less confident in mathematics and she demonstrated weaker subject knowledge.

Ambrose (2004) has suggested that student teachers may often believe that teaching mathematics is straightforward. They assume that, if they know the mathematics they need to teach, and then all that is needed is to give clear explanations of this knowledge. Further to this the student teacher may believe that the aim of teaching mathematics is to explain useful facts and skills to children to help them become skilful and efficient in their use and to know when to apply them. The analysis of Alan's lesson plan indicates that he may have this belief of teaching. Stipek, Givvin, Salmon and MacGyvers's (2001) referred to this belief as a traditional 'knowing' orientation. They suggested that a shift away from such a traditional orientation towards an 'enquiry' orientation where mathematics is seen as a tool for problem solving, would be more effective. Analysis of Charlotte's lesson plan suggests that she may have been more inclined towards an 'enquiry' orientation.

In order to avoid the 'interference' or 'obstructions' that might become apparent by focusing on the procedures of area measurement we would want student teachers to move towards this 'enquiry' orientation. Stipek et al.'s empirical study indicated that teachers' beliefs about mathematics predicted their instruction. However they also suggested that less confident teachers were more likely to be oriented towards mathematics as 'knowing' due to lack of confidence in dealing with the questions that might be asked through an enquiry based approach. If we interpret Alan's orientation as 'knowing' and Charlotte's approach as moving towards 'enquiry' then this suggests an anomaly as Charlotte was less secure and lacked confidence in her knowledge of the content.

It could be said that as Alan used the formulae with particular ease and accuracy his aim was to support the children in developing such a use. Although he was able to realise relationships he did not see this as an important aspect of mathematics and hence he did not focus on this pedagogically. Charlotte's emphasis was not on ensuring clear explanations were given but that the children arrived at an understanding through the activities. She suggested that the children would use their own strategies and she intended to employ activities that would 'lead naturally' to their understanding. Could it be that Charlotte's lack of confidence and knowledge meant that she was uncertain of how to explain the mathematical ideas to the children? In this way she may have researched pedagogical approaches further. Or could it be that Charlotte's beliefs in the teaching of mathematics differed from that of Alan? Despite a lack of knowledge in mathematics, Charlotte's pedagogical approach may have been based on a belief that children develop understanding through active engagement in activities and that this belief has been carried over from her view of what is important in mathematics.

This is not to suggest that Charlotte would be more effective in teaching the topic. This study has not investigated how the student teachers responded to the children's learning in the classroom and Charlotte's misunderstandings are likely to inhibit her ability to develop the children's learning at some point.

## CONCLUSION

Hill, Rowan and Ball (2005) have suggested that it is not knowledge of content but knowledge of 'how to teach' the content that is influential in considering teacher effectiveness. What remains a question is how this knowledge of 'how to teach' is arrived at? Although this research does not provide any generalisable evidence it does raise questions regarding the nature of subject knowledge in relation to the knowledge of ‘how to teach’, and whether there may be other variables at play, such as orientations and beliefs about what is important in mathematics.

## REFERENCES

Ambrose, R. (2004). Initiating change in prospective elementary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of mathematics Teacher Education, 7,* 91 - 119.

Argyris, C. and Schon, D. (1974). *Theory in Practice.* San Francisco: Jossey-Bass

Ball, D. L. & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In E. Simmt & B. Davis (Eds.), *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group* (pp. 3-14). Edmonton: CMESG/GCEDM.

Ball, D. L., Lubienski, S. & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers’ mathematics knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (pp. 433–456). New York: Macmillan.

Baturo, A. & Nason, R. (1996). Student Teachers' Subject Matter Knowledge within the Domain of Area Measurement. *Educational Studies in Mathematics, 31,* 235 - 268.

Berenson, S., van der Valk, T., Oldham, E., Runesson, U., Queiroz Moreira, C. & Broekman, H. (1997). An International Study to Investigate Prospective Teachers' Content Knowledge of the Area Concept. *European Journal of Teacher Education, 20(2),* 137 - 150.

Dickson, L. (1989). Area of a Rectangle. In D. Johnson (Ed.), *Children's Mathematical Frameworks 8 - 13: A Study of Classroom Teaching* (pp. 89–125). Windsor, Berkshire: NFER-Nelson.

Dickson, L., Brown, M. & Gibson, O. (1984). *Children Learning Mathematics: A Teacher's Guide to Recent Research.* London: Cassell.

Ginsburg, H. (1997). *Entering the Child’s Mind: The Clinical Interview in Psychological Research and Practice*. Cambridge: Cambridge University Press

Hiebert, J. & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: an introductory analysis. In J. Hiebert (Ed.), *Conceptual and Procedural Knowledge: the case of mathematics* (pp. 1-27). Hillsdale: Lawrence Erlbaum.

Hill, H., Rowan, B. & Ball, D. L. (2005). Effects of Teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42(2),* 371 - 406.

John, P. (1991). A Qualitative Study of Lesson Plans. *Journal of Education for Teaching, 17(3),* 301-320

Lehrer, R. & Chazan, D. (Eds.) (1998). *Designing Learning Environments for Developing Understanding of Geometry and Space*. Mahwah, New Jersey: Lawrence Erlbaum Associates Publishers.

Lunzer, E. (1968). Formal Reasoning. In E. Lunzer & J. Morris. (Eds.) *Development in Human Learning*. New York: Elsevier.

Nitabach, E. & Lehrer, R. (1996). Developing Spatial Sense through Area Measurement. *Teaching Children Mathematics, 2(6),* 473 - 476.

Pesek, D. & Kirshner, D. (2000). Interference of Instrumental Instruction in Subsequent Relational Learning. *Journal for Research in Mathematics Education, 31(5),* 524 - 540.

Shulman, L. (1987). Knowledge and Teaching: Foundations of the New Reform. *Harvard Educational Review, 57(1),* 1 - 22.

Skemp, R. (1976). Relational Understanding and Instrumental Understanding. *Mathematics Teaching, 77,* 1-7.

Stavy, R. & Tirosh, D. (1996). Intuitive rules in science and mathematics: the case of ‘more of A – more of B’. *International Journal of Science Education, 18(6),* 653 - 667

Stipek, D., Givvin, K., Salmon, J. & MacGyvers, V. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17(1),* 213 - 226.

Swanson, D., Schwartz, R., Ginsburg, H. and Kossan, N. (1981). The Clinical Interview: Validity, Reliability and Diagnosis. *For the Learning of Mathematics, 2(2),* 31-38.

Tierney, C., Boyd, C. & Davis, G. (1990). Prospective Primary Teachers’ Conceptions of Area. In G. Booker, P. Cobb & T. D. Mendecuti (Eds.), *Proceedings of the 14th Conference of the International Group of the Psychology of Mathematics Education* (pp. 307–315). Mexico: IGPME

Zacharos, K. (2006). Prevailing Educational Practices for Area Measurement and Students' Failure in Measuring Areas. *Journal of Mathematical Behaviour, 25,* 224-239.

# DEVELOPING OF MATHEMATICS TEACHERS' COMMUNITY: FIVE GROUPS, FIVE DIFFERENT WAYS

Regina Reinup

Tallinn University, Estonia

*Developing a mathematics teachers' learning community is one of the in-service teacher training methods in the university. At the beginning of 2006, from the initiative of some teacher training educators, a mathematics teachers' community formed at Tallinn University. The aim of the project was to focus on two of the main problems in school mathematics: teaching percentages and functions. Although all the groups were given the same problem by the tutors, a different approach was used by each group. The article presents an overview of the division of task inside the groups at the end of the first stage of the whole process, and also in what way each group reached its final decision with the matter of how to teach percentages. It turned out that at this stage the workgroups had developed differently.*

## INTRODUCTION

By Wenger's (1998) theory, working in the *communities of practice* is one of the most common and natural ways of cooperation and it can be seen in every sphere of social life where there is communication between colleagues. The aim of the communication is to solve a certain problem, and in this solving process there occurs constant intercommunication between the group members and the participants learn from each other (Wenger, 1998; Olson & Kirtley, 2001). Communities of practice are mostly informal groups. In a well-formed community of practice people have to know each other well, which implies that the following qualities apply: (Q1) the members of the community know each other's abilities, (Q2) they can be set to work quickly, (Q3) there is a quick flow of information inside the community, (Q4) there is a fluent exchange of information, (Q5) there is a good grounding for finding new strategies, (Q6) the group finds original solutions to problems that have been solved already (Wenger, 1998; see also McGraw, Arbaugh & Lynch, 2001).

The mathematics teachers' learning community, as a part of the in-service teacher training method has, according to a number of researchers (e.g. McGraw, Arbaugh & Lynch, 2001; Goodchild & Jaworski, 2005; Olson & Kirtley, 2005), proved to be successful. The exchange of different opinions and views in the course of discussions gives the participants a chance to view the problems from different angles, and therefore it is instructive for every member (Olson & Kirtley, 2005). Jarowski (2005) points out the importance of disputes and constructive discussions inside the circle, as it is by this process that all conclusive decisions are made. Grossman, Wineburg & Woolworth (2001) also warn that at the initial stage of work the group is liable to become a *pseudocommunity*, as discussions lack subject matter and the members reach agreement too easily when trying to find solutions.

A mathematics teachers' learning community (referred later as *MMM*-project [1]) was assembled at Tallinn University for the first time in 2006. The *MMM*-project was part of a wider project of enhancing mathematics teaching in Estonia (Hannula, Lepik & Kaljas, 2007). A preparation period of about seven months preceded the assembling of the *MMM*-project, during which mathematics educators at the university acquainted themselves with research on mathematics teachers' communities worldwide (e.g. Olson & Kirtley, 2005; Jaworski, 2005; Goodchild & Jaworski, 2005) and thereby planned the *MMM*-project. The project awoke great interest among mathematics teachers – there were 34 applicants (initially it was planned for 10 teachers), and all of them were invited. I was one of these mathematics teachers. The planning of the *MMM*-project and its initial stages has been described by Hannula *et al.* (2007).

The teachers participating in the *MMM*-project were divided into groups of 6 or 7 members (referred to as G1-G5 in the text) at random. I was a member of G1. At the first two seminars we discussed the problems which resulted from the teaching process of percentages. We worked in the groups only at the seminars, as more rather individual homework was given by the tutors (designing and mediating artifacts). At the third seminar in October 2006 the groups were given a collective task: to make a detailed schedule for 20-25 lessons, about teaching percentages for grades 6-9 (pupils aged 13-16), and producing worksheets for them. The present article focuses on the fourth seminar of the *MMM*-project, which took place seven weeks later where the groups presented their respective views on teaching percentages. Most of the groups (G1, G2, G4 and G5) also gave reasoning in their presentation of how they reached their conclusions, and how they divided the tasks between the group members. In principle, the fourth seminar also marked the end of the first stage of the project, as at the next seminar the groups had to present their completed work of teaching percentages, and then to start discussing a new topic.

This paper seeks answers to the following questions: (1) did any similarities occur in the division of task inside the groups and (2) on what did each group base their approach (the ideas given by the university mathematics educators, scientific articles, or the participants' own experience). According to Jaworski (2005), the approach is only taking shape at the first stage of the learning community's work. Therefore, it would be interesting (3) to analyse whether the groups, as learning communities, had acquired qualities of a solid community of practice at the end of the first stage of the *MMM*-project (Wenger, 1998), or if they were still pseudocommunities (Grossman *et al.*, 2001).

## METHODS

In the study I analyze the division of labour inside the groups, the level of development of the groups at the end of the first stage of the *MMM*-project, and on what were the groups approaches based, apart from each member's own thinking and experience. Unfortunately there isn't much authentic evidence of the division of task inside the groups. The participants were not interviewed about it by the tutors, although the results might have proved interesting, and there are no video recordings of the process of working in the group(s). One of the authentic materials is a video recording of the fourth seminar (hereafter Video), where the representatives (or a representative) of each group tell(s) about the work inside the group and what conclusions they have reached. Parts of this recording have been used as the material to warrant conclusions and as illustrative examples in the present article. At the Estonian mathematics teachers' annual conference (November 2-3, 2007) every group gave its view on how percentages should be taught at school, and each group also had an article about it in the proceedings of the aforesaid conference. These articles were another source that I could use. As the third source I used the teaching materials in each member's folder on the *MMM*-project's home page [2] and also in the folders of the different groups. In the autumn of 2008 I sent an e-mail to the participants of the *MMM*-project in which I asked them to explain the first stage of our project as they recalled it. In this paper I use excerpts of some of their answers to me.

By comparing the above sources it is possible to make some conclusions about the work inside the groups. I searched for certain similarities in the division of task. I also tried to specify the level of development within each group by seeking the qualities of Wenger's (1998) community of practice (see in the introduction, hereafter Q1 to 6). I got data based on each group's approach from their articles (used references), and from the video (the tutors' suggestions to groups).

## RESULTS

### Division of task

The majority of the groups (G2, G4 and G5) used division of task so that each member of the group had to prepare one subtopic in depth. The unified form and structure was either agreed upon earlier or at the fourth seminar during the group work.

> "… We also divided the material by the topics so that each teacher could have one topic to think over more thoroughly … what it might consist of. And this is exactly what all our members have been doing. And today we tried to unify a little … what items to put down and where …" (member of G2, Video).

> "… On the basis of it we divided the lessons between us…who is taking what part of these lessons to analyse, and we realised that we had to put down worksheets for the pupils and worksheets (with answers, R. R.) for the teacher, and we agreed on what it should look like. And now we will start writing them, as we do not have anything else today," (member of G4, Video).

> " … First we relied on our division of tasks as we had agreed earlier … we had divided the topics between us as we had previously, and how many lessons might be

> reasonable… Then we gave to every member of our group – we chose it ourselves – which topics for whom to analyse in detail. Each teacher ... or a colleague here can choose a topic to his own liking and then we write a program for pupils and for the teacher. We communicate by e-mail; we are trying to put our materials in the internet (*MMM*-project's home page, R. R.)," (member of G5, Video).

G1 compiled their own home page on how to learn and teach percentages, and how to go over the material, which refers to Q6 of Wenger's (1998) community of practice. This group had chosen a slightly different way of dividing tasks, although here also each member was responsible for a certain part of the whole work (Q1). One of its members had knowledge of the program *eXe-Learning*, which he used in making their home pages. There were two experienced teachers in the group with good teaching methods and they prepared the theoretical part. Others prepared exercises and searched for some tests in the web, and my task (as I was the member of G1) was to find visual material and suitable games in the internet.

> "Visualization is very important and … we had one member who specialized on this…" (member of G1, Video).

G1's teamwork can be characterized as very active. In other groups the report was made by one member and all the others were only listeners, whereas in G1 all the members took part in the discussion by reporting (Video).

The division of tasks is not clear in G3. There were two members who gave a report and one of them gave an overview of how he had taught percentages at school (Video). G3's folder on the project's home page is empty; there are some teaching materials in the group members' folders, but they do not follow the principles set for the group work.

**Different approaches**

The university mathematics educators gave all the groups the same task: to make (1) a detailed schedule of 20-25 classes and (2) worksheets to help pupils to understand percentages better. Yet every group had a different approach.

G1 did not give any detailed schedule of classes. Their group website was meant first and foremost for repetition, so that both the teacher and the pupil can go over the sub-themes (Pihlap, Aluoja, Kopli, Koppel, Lepik & Reinup, 2007; Video; *MMM*-project's home page).

> "We had one more idea; we wanted to introduce something new, to do it this way as to put the picture and the text side by side, running simultaneously. So that those pupils who do understand the text perhaps do not need it, while others have difficulties with it and so the text keeps running alongside the picture." (member of G1, Video).

The university mathematics educators gave G1 an idea to add to the homepage a test on the basic knowledge and skills of multiplicative thinking (Video). The group work of G1 on the *MMM*-project's home page and the sketch which they presented at the fourth seminar are very similar.

G2 gave a schedule of classes for teaching in different grades as suggested. The group presumed that the teacher would be using current textbooks and workbooks, and concentrated on making additional worksheets to them. G2 planned to present the most important items of their theory in a *PowerPoint* slideshow (Video). Their work on the *MMM*-project's home page is left unfinished and the group's folder is empty, although there is a lot of different teaching materials (*PowerPoint* slideshows as well) in the members' folders.

In G3 there were two teachers who had been teaching percentages in differing ways for a number of years. This explains why the approach in G3 was influenced by these two teachers.

> "For the beginning I must say that it seemed to me that in other groups there have been attempts to teach percentages as it has been suggested; as to our group it is interesting to notice that we happen to have two teachers here (A and H) who have already practiced teaching in the way we advocate now. … We have tried to have percentages together with fractions, or more precisely: finding a part. … And now A, who practiced this in his class, is playing his videotape," (H, member of G3, Video).

The presentation of this group's research work was the longest of all. The report was very interesting in my opinion, and full of subject matter. Yet, as mentioned before, one of the members of the group presented his own personal view of how to teach percentages (Video).

> "And therefore I consider it very important that, namely, to began with, I do not ask the pupils to do any operations, I take simple numbers and you will have to say quickly – three quarters, a half, one quarter or ten percent as well," (A, member of G3, Video).

The group's article (Ojasoo, Kaasik, Lahi & Pärnamaa, 2007) is based mainly on the same report (Video). The group does not have a collective folder on the *MMM*-project's home page.

G4 based its work on Merrill's taxonomy (Gagne & Merrill, 1990; see Matiisen, Kalda, Kasendi, Tamm & Vahtramäe, 2007). The proportional number of classes was not fixed, and the work was divided into three major subdivisions: (1) immediate understanding (grade 6), (2) arithmetic/basic rules of calculation (grade 7), and (3) "life itself" (grades 8 & 9). On the given theoretical basis this group created entirely novel teaching material – different worksheets for pupils and for teachers (Q5 and Q6). The possibility to use current textbooks and workbooks was excluded (Video).

> "As far as I understood we were given such a task … we cast aside all schoolbooks and we have that batch, and the teacher goes in front of the class with that batch and the pupils will learn how to do percentages." (member of G4, Video).

In an e-mail a member from G4 brought to mind the period when they had dealt with percentages in the *MMM*-project.

> "I had read about and also practiced in my classes the heuristic approach that has been used in schools, and as it sounded interesting to my colleagues they were willing to try it. … About specialised literature. It is difficult to tell now from which sources exactly. …

> Anyway, some articles written by our mathematics educators are among them." (member of G4, from e-mail to R. Reinup, Sept.10th 2008).

The results of G4's work in full are on the *MMM*-project's home page in the group's folder.

G5 based its program of teaching percentages on the official program for schools. The group members' experience in teaching at school was their main starting point. In addition to this they read articles written by different researchers and thereby got an overview of the main problems teachers have when teaching percentages at school in Estonia (see Laanpere, Kattai & Sasi, 2007). The group decided to make some additional worksheets to complement the existing teaching materials. The new teaching materials were to be of help to teachers with little experience (Laanpere *et al.*, 2007; Video).

> "We presume that we will use current schoolbooks and teaching materials as well. And when we are making those worksheets we will surely refer to the sources. ... Then each member in our group did some searching and found the teaching materials which have proved helpful in his work. Indeed, we have a number of different worksheets,…tests in our computers, games, and now we can see that they all prove useful." (Member of G5, Video)

The work produced by G5 is on the *MMM*-project's home page. However, it can be noticed that most of the teaching materials come from only one teacher.

## Community or pseudocommunity

It is a rather difficult task to detect whether any learning community characteristic features can be found in any group (see also McGraw *et al.*, 2001). As I did not have any focused video recordings of the groups when working together at the seminars, there are no direct sources of what the work inside the group was like. It can be decided only indirectly whether we consider a group a learning community or a pseudocommunity, although videos, division of task inside the group, written materials, and above all the teaching materials in the groups' folders on the *MMM*-project's home page can be of help. This sort of complex analysis allows drawing some conclusions of the developing degree of the groups.

I have some difficulties when judging the work of G1 because I was the member of this group. There is not much material in the members' folders on the *MMM*-project home page, but I know that all the members of the group sent their materials by e-mail to the member who created the groups' home page, on which rather intensive correspondence took place, especially during the last week before the fourth seminar. The address of G1's home page was sent to all the group members so that everyone could suggest any alterations to be made. Also, at the presentation all the group members were very active (Video). So the qualities of Q2, Q3 and Q4 appeared, and earlier we have referred to Q1 and Q6 in connection to G1. Due to the intensive interaction and the fact that all group members contributed, G1 can be considered to be a community of practice in the sense of Wenger (1998).

All the members of G2 worked hard, collecting teaching materials in their folders on the *MMM*-project home page (the biggest amount of materials compared with the other groups), however their processes did not converge towards a shared conclusion, and the group's folder is empty. On the video it can be seen that at the presentation at the fourth seminar the members of the group remain rather passive. Because of the passivity in producing their own material and in interaction, G2 can be considered to be in the developing phase as a group at the end of the first stage of the project. A weak developing degree of working communities at their first stage is also mentioned by Jaworski (2005).

G3 contained a very influential person and my understanding is that the other members in the group accepted his views about teaching percentages, without adding any or very little of their own. The analysis of the group members' folders on the *MMM*-project's home page affirms the assumption – their content was not in accordance with the group's explicated common aim as it was presented in a seminar meeting (*MMM*-project's home page; Video). Onward, when analysing the materials on the *MMM*-project's home page, it can be noticed that all the materials in the G5 folder mainly originated from only one group member, although at the initial phase all was planned differently (Video). In the work of G3 and G5 the qualities of a community of practice (in sense of Wenger, 1998) do not appear. Grossman *et al.* (2001) refer to the basic quality of a pseudocommunity is that the members of it "act as if they are already a community that shares values and common beliefs". In my opinion these groups (G3 and G5) are not pseudocommunities in this sense exactly. In both cases there is some inherent discordance between the group's public report and the group's actual work on the *MMM*-project's home page. Yet, one might call them pseudocommunities as most of the work seems to be done by a single (or a couple of) member(s) and other members remained rather passive.

In my opinion G4 compiled a very interesting, complete and novel collection of teaching materials (Q6). According to the recollections the work process was very intensive (Q3, Q4).

> "Common understanding developed among us on the grounds of everyday activities and experiences. We all had tried something new and we all could point out the benefits or weak facets of our experiments. As far as I know we all tried to put into practice most parts of other members' experiments in our schools. … I have a sad story to tell, I cannot be blamed for having a small ego, and as a vice-principal (at a school, R. R.) I have acquired an ability to force my views upon others and I tend to do it in every situation. Therefore, I claim that I influenced other members of the group – but it's no use crying over spilt milk." (member of G4, from e-mail to R. Reinup, Sept. 10th 2008).

Although this one member was concerned with having too much influence, the material produced did not originate from a single group member. Moreover, the material was produced in collaboration, not simply collected together. Therefore we can consider this group to have developed into a community of practice (Wenger, 1998).

## SUMMARY

Every group of a learning community consists of different people and that's why every different group develops its individual face. One of the main aims with the communities is to gain a new quality through the cooperation of different members with different experiences (Wenger, 1998). In the first phase of the *MMM*-project the groups had to make new proposals and give their solutions to some problems that might help to improve the quality of teaching percentages at Estonian schools. The task set by the tutors was the same for every group, yet every group had a different approach.

There were certain similarities as to the division of task: each group member was responsible for one specific sphere (G1, G2, G4 and G5). The most typical division of task was the thematic approach (G2, G4 and G5). In group G1, taking into account each members' abilities, the participants divided tasks according to the contents of the task. This is a more sophisticated approach.

The group members relied on their own experiences when finding solutions to the tasks given to them, although in some groups (G3 and G5) it can be seen that the whole group relied on the experience of a couple of its members. During the whole project the tutors commented on the work inside the groups. G1 received a concrete suggestion from the tutors and the group took it into account. From references of the articles written by the groups it can see that G4 & G5 gained ideas from the literature.

There were no concrete proofs of how the communities developed. In my opinion G1 and G4 were the most highly developed groups. In G1 the group members understood each other's abilities well (the tasks were given to the most able members), and there was a quick flow of information (e-mails, supporting each other at the presentation); they found suitable strategies and original solutions (they made their own home page). The work in G4 can be characterised as a fluent exchange of experiences (most of it was put into practice by various members). They found suitable strategies (the work was based on Merrill's taxonomy) and they found an original solution (a set of worksheets). In the work of both groups G1 and G4 appeared to contain most of the qualities of communities of practice (Wenger, 1998), so I think that these groups can be called communities of practice. In the other groups the progress is somewhat questionable at the end of the first stage. G2 could not give a unified original solution, although there were a lot of teaching materials in the group members' folders. Generally, only two members of G3 put their views and experiences together and one of them presented it (based on the analysis of the Video). In G5 there was some cooperation formally, but the main author of the whole report is a single member of the group (based on the analysis of the group's folder on the *MMM*-project's home page).

The project with Estonian mathematics teachers confirmed Jaworski's (2005) presumption that in the first phase of the work the community is still developing.

> "The reports we heard gave us lots of ideas to think over but they all did not have enough time to mature, and to put them into practice when teaching percentages at school. I am quite sure that the result here is rather a reflection of some former experiences than anything new, created in the course of the *MMM*-project." (member of G4, from e-mail to R. Reinup, Sept. 10th 2008).

Some of the groups in the *MMM*-project developed more than the others, but participation (either actively or passively in the community's work) was instructive for all its members.

> "In my opinion, cooperation was the major driving force. An idea emerged, then someone made it clearer and someone else explained something. We all brought some worksheets; I was discussing my plans on my worksheet, but ideas began to spring up and everyone contributed – some gave more, some gave less. I am convinced that this sort of cooperation gave us lots of ideas and added willingness to achieve better results with pupils at school." (member of G1, from e-mail to R. Reinup, Sept. 6th 2008).

Every idea needs time to mature. When comparing the teachers' views during the whole *MMM*-project (from the beginning to the final phase), it can be noticed that during the project the participants developed a much more positive attitude in the subject (Kaljas, Kislenko, Hannula & Lepik, in press).

All five groups also presented their concepts and ideas worked out during the *MMM*-project at the Estonian mathematics teachers' annual conference, which is one of the biggest mathematics teachers forums in Estonia. The large amount of teaching materials on the *MMM*-project's home page is available to all mathematics teachers all over Estonia. Today the *MMM*-project has ended. The researchers can make conclusions and also start planning other projects of a similar kind in the future.

## NOTES

1. In Estonian Meile Meeldib Matemaatika (*MMM*) – We Like the Mathematics

2. http://zope.eenet.ee/mmmprojekt

## REFERENCES

Gagne, R. M. & Merrill, M. D. (1990). In Conversation. No.3: An Overview of M. David Merrill's New Component Design Theory. *Educational Technology*, *30* (9), 36 - 42

Goodchild, S. & Jaworski, B. (2005). Using Contradictions in a Teaching and Learning Development Project. In H. L. Chick & J. L. Vincent (Eds.), *Proc. 29th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol 3, pp. 41 – 48). Melbourne, Australia: PME.

Grossman, P., Wineburg, S., & Woolworth, S. (2001). Toward a Theory of Teacher Community. *Teachers College Record, 103*(6), 942-1012.

Hannula, M. S., Lepik, M. & Kaljas, T. (2007). Mathematics Teachers' Desire to Develop. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of the 5th Congress of the European Society for Research in Mathematics Education* (Working Group 2, pp. 239 – 247). Larnaca, Cyprus: CERME.

Jaworski, B. (2005). Learning Communities in Mathematics: Research and Development in Mathematics Teaching and Learning. In C. Bergsten, B. Grevholm, H. Strømskag Måsøval & F. Rønning (Eds.), *Relating Practice and Research in Mathematics Education. Proceedings of NORMA 05, Fourth Nordic Conf. of Mathematics Education* (pp. 71 – 96). Trondheim: Tapir Academic Press, Norway.

Kaljas, T., Kislenko, K., Hannula, M. S. & Lepik, M. (In press). Some Tendencies in the Changes of Teachers' Beliefs about Mathematics Teaching. Submitted. To appear in the proceedings of NORMA 08 conference, April 21th – 25th, 2008.

Laanpere, E., Kattai, K. & Sasi, T. (2007). Abimaterjale algajale matemaatikaõpetajale protsentide õpetamiseks [Support Materials to the Beginning Mathematics Teacher for Teaching Percentage Calculation]. In E. Abel (Ed.), *Koolimatemaatika XXXIV [School Mathematics XXXIV]*. Tartu: Eesti Matemaatika Selts, 23 - 25.

Matiisen, K., Kalda, I., Kasendi, S., Tamm, K. & Vahtramäe, M (2007). Läbi eheda taipamise teadmiste poole [Though Genuine Comprehension to Deep Knowledge]. In E. Abel (Ed.), *Koolimatemaatika XXXIV [School Mathematics XXXIV]*. Tartu: Eesti Matemaatika Selts, 18 - 22.

McGraw, R. Arbaugh, F. & Lynch, K. (2001). Mathematics Teacher Professional Development as the Development of Communities of Practice. In M. van den Heuvel-Panhuizen (Ed.) *Proc. 25th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol 3, pp. 269 – 276). Utrecht, Netherlands: PME.

Ojasoo, A., Kaasik, H., Lahi, R. & Pärnamaa, U. (2007). Protsent koos osa ja terviku leidmisega [The Percentage Together with Finding a Part and the Whole]. In E. Abel (Ed.), *Koolimatemaatika XXXIV [School Mathematics XXXIV]*. Tartu: Eesti Matemaatika Selts, 16 - 18.

Olson, J. C. & Kirtley, K. (2003). The Transition of a Secondary Mathematics Teacher: From a Reform Listener to a Believer. In H. L. Chick & J. L. Vincent (Eds.), *Proc. 29th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol 4, pp. 25 - 32). Melbourne, Australia: PME.

Pihlap, S., Aluoja, L., Kopli, V., Koppel, I., Lepik, V. & Reinup, R. (2007). Lõbusad protsendid [Amusing Percentage]. In E. Abel (Ed.), *Koolimatemaatika XXXIV [School Mathematics XXXIV]*. Tartu: Eesti Matemaatika Selts, 8 - 11.

Wenger, E. (1998). *Communities of Practice: Learning, Meaning, and Identity*. Cambridge: Cambridge University Press.

# FOUNDATION KNOWLEDGE FOR TEACHING: CONTRASTING ELEMENTARY AND SECONDARY MATHEMATICS

Tim Rowland
University of Cambridge, UK

*This paper describes and analyses two mathematics lessons, one about subtraction for very young pupils, the other about gradients and graphs for lower secondary school pupils. The focus of the analysis is on teacher knowledge, and on the fundamental mathematical and mathematics-pedagogical requisites that underpin teaching these topics to these pupils. The claim is that, in the case of the elementary mathematics, the relevant 'foundation' knowledge is to teachers what Foundations of Mathematics is to mathematicians: invisible until it becomes necessary to know it: and that this very invisibility poses particular challenges to teachers of young children.*

Keywords: teacher knowledge, subtraction, gradient, foundations of mathematics

## INTRODUCTION

The complex and multi-dimensional character of mathematical knowledge for teaching is now better understood thanks to the seminal work of Lee Shulman (1986) and several subsequent studies. Mathematics teacher knowledge has also been analysed and discussed in several papers at earlier CERME meetings. Recurrent concepts in these discussions are subject matter knowledge (SMK) and pedagogical content knowledge (PCK). For mathematics educators, PCK is perhaps particularly interesting, in that it captures the notion of mathematical knowledge of a kind specific to the teaching profession. That is to say, it encompasses a large, and increasing, body of mathematical knowledge that would not be acquired in the process of learning mathematics for non-pedagogical purposes. The otherwise well-educated citizen does not need it, neither does the engineer, economist, biologist – or mathematician, for that matter. Instances of such knowledge include diverse representations of fractions, for example, or the Principles of Counting (in this latter case see, for example, Turner, 2007).

Another strand of CERME thinking on mathematical knowledge in and for teaching includes the examination of teaching episodes against different kinds of descriptive and analytical frameworks (see e.g. Ainley and Luntley, 2006; Huckstep et al., 2006, Potari et al., 2007). The Knowledge Quartet framework of Rowland et al. (2005) emphasises three ways in which 'Foundation Knowledge' becomes visible in the classroom, for example in the teacher's choice and pedagogical deployment of representations and examples. The underpinning Foundation Knowledge is rooted in the teacher's 'theoretical' background and in their system of beliefs.

> [Foundation Knowledge] concerns trainees' knowledge, understanding and ready recourse to their learning in the academy, in preparation (intentionally or otherwise) for their role in the classroom. It differs from the other three units [of the Knowledge Quartet] in the sense that it is about knowledge possessed, irrespective of whether it is being put to purposeful use. […] A key feature of this category is its propositional form (Shulman, 1986). It is what teachers learn in their 'personal' education and in their 'training' (pre-service in this instance). We take the view that the possession of such knowledge has the potential to inform pedagogical choices and strategies in a fundamental way. By 'fundamental' we have in mind a rational, reasoned approach to decision-making that rests on something other than imitation or habit. The key components of this theoretical background are: knowledge and understanding of mathematics per se; knowledge of significant tracts of the literature and thinking which has resulted from systematic enquiry into the teaching and learning of mathematics; and espoused beliefs about mathematics, including beliefs about why and how it is learnt. (Rowland 2005, p. 259)

The study of Potari et al. (2007) is unusual in this field (of teacher knowledge) in that it sets out "to explore teachers' mathematical and pedagogical awareness in higher secondary education and more specifically in calculus teaching." (p. 1955). The authors note the substantial body of work on teacher knowledge in primary or early secondary education, and assert that "teachers' knowledge in upper secondary or higher education has a special meaning as the mathematical knowledge becomes more multifaceted and the integration of mathematics and pedagogy is more difficult to be achieved." (p. 1955). The claim, then, is that the task of coordinating content and pedagogy becomes more complex as the mathematics becomes more advanced. This paper sidesteps that particular claim. Instead, I examine two lessons conducted with pupils whose ages differ by about seven years. One is at the beginning of compulsory schooling in England (Year 1, pupil age 5-6), the other in lower secondary school (Year 8, pupil age 12-13). The analytical framework is the Knowledge Quartet in both cases, and the focus is on Foundation Knowledge in particular. My claim will be as follows: that whereas from the mathematical point of view, the subject matter under consideration with the Year 8 class is significantly more complex than that in the Year 1 lesson, the PCK necessary to teach the latter well has something in common with Foundations of Mathematics in the mathematician's repertoire. Therefore it is difficult to conclude, in any straightforward way, which teacher has the more demanding task mathematically, where this ['mathematically'] is taken to encompass mathematical knowledge for teaching in the widest sense, as indicated by Shulman and made explicit by Ball et al. (2005).

The pattern in the following two sections will be to give a descriptive synopsis of the lesson first (i.e. to say what the lesson was about), followed by an account, necessarily selective, of the teacher Foundation Knowledge relevant to teaching this lesson.

## YEAR 1 LESSON: SUBTRACTION

The teacher, Naomi, was in preservice teacher education. The learning objectives stated in her lesson plan are as follows:

> To understand subtraction as ‘difference’.
> For more able pupils, to find small differences by counting on.
> Vocabulary - difference, how many more than, take away.

Naomi begins the lesson with a seven-minute Mental and Oral Starter[1] designed to practise number bonds to 10. In turn, the children are given a number between zero and ten, and required to state how many more are needed to make ten.

The Introduction to the Main Activity lasts nearly 20 minutes. Naomi sets up various ‘difference’ problems, initially in the context of frogs in two ponds. Her pond has four, her neighbour’s has two. Magnetic ‘frogs’ are lined up on a vertical board, in two neat rows. She asks first how many more frogs she has and then requests the difference between the numbers of frogs. Pairs of children are invited forward to place numbers of frogs (e.g. 5, 4) on the board, and the differences are explained and discussed. Before long, she asks how these differences could be written as a “take away sum”. With assistance, a girl, Zara, writes 5-4=1. Later, Naomi shows how the difference between two numbers can be found by counting on from the smaller.

The children are then assigned their group tasks. One group (‘Whales’), supported by a teaching assistant, is supplied with a worksheet in which various icons (such as cars and apples) are lined up to ‘show’ the difference, as Naomi had demonstrated with the frogs. Two further groups (‘Dolphins’ and ‘Octopuses’) have difference word problems (e.g. I have 8 sweets and you have 10 sweets) and are directed to use ‘multilink’ plastic cubes to solve them, following the ‘frogs’ pairing procedure. The remaining two groups have a similar problem sheet, but are directed to use the counting-on method to find the differences.

Nine minutes later, Naomi calls the class together on the carpet for an eight-minute Plenary, in which she uses two large, foam 1-6 dice to generate two numbers, asking the children for the difference each time. Their answers indicate that there is still widespread confusion among the children, in terms of her intended learning outcomes.

### Foundation knowledge: subtraction

Carpenter and Moser (1983) identify four broad types of subtraction problem structure, which they call *change, combine, compare, equalise*. Two of these problem types are particularly relevant to Naomi’s lesson. First, the change-separate problem, exemplified by Carpenter and Moser by: “Connie had 13 marbles. She gave 5

---

[1] The National Numeracy Strategy *Framework* (DfEE, 1999) guidance effectively segments each mathematics lesson into three distinctive and readily-identifiable phases: the *mental and oral starter*; the *main activity* (an *introduction* by the teacher, followed by *group work*, with tasks differentiated by pupil ability); and the concluding *plenary*.

marbles to Jim. How many marbles does she have left" (p. 16). The UK practitioner language for this is subtraction as 'take away' (DfEE, 1999, p. 5/28).

Secondly, the compare problem type, one version of which is: "Connie has 13 marbles and Jim has 5 marbles. How many more marbles does Connie have than Jim". (Carpenter and Moser, 1983, p. 16). This subtraction problem type has to do with situations in which two sets (Connie's marbles and Jim's) are considered simultaneously - what Carpenter and Moser describe as "static relationships", involving "the comparison of two distinct, disjoint sets"(p. 15). This contrasts with change problems, which involve an action on and transformation of a single set (Connie's marbles). Again, the National Numeracy Strategy *Framework* (DfEE, 1999) reflects the tradition of UK practitioners in referring to the compare structure as 'subtraction as *difference*'. We return to this point in a moment.

Carpenter and Moser go on to show that the semantics of problem structure, as discussed above, by no means determines the processes of solution adopted by individual children, although the structure might suggest a paradigm, or canonical, strategy. They describe six broad categories of subtraction strategy identified in the research literature. Some involve actions with concrete materials, others depend on forms of counting, yet others on known facts (such as 10-5) and derived facts (such as 11-5, derived from knowing e.g. 5+5). Most strategies with materials are associated with a parallel counting strategy. For example, *separating from*, the canonical strategy for the change-separate ('take-away') structure described above, involves constructing the larger set and then removing a number of objects corresponding to the subtrahend number. Counting the remaining objects yields the answer. The parallel counting strategy is called *counting down from*. The child counts backwards, beginning with the minuend. The number of iterations in the backward counting sequence is equal to the subtrahend. The last number uttered is the answer. Clearly, therefore, the child needs a suitable strategy for keeping track of the number of iterations; one way would be to tally them, typically with fingers. The *counting up* strategy involves a *forward* count beginning with the smaller number (subtrahend). The last number uttered is the minuend. This time, the number of iterations in the forward counting sequence is equal to the answer. Finally, Carpenter and Moser's taxonomy of strategies includes *matching*, which is unusual in that it has no purely 'mental' parallel in the absence of concrete objects. The child puts out two sets of objects with the appropriate cardinalities. The sets are then matched one-to-one. Counting (or subitising) the unmatched cubes gives the answer. It is relevant to note here Carpenter and Moser's finding with Grade 1 to 3 children that the matching strategy is very rarely used. The only exception to this rule was by Grade 1 children who had received no formal instruction in addition and subtraction. The majority of these children who successfully solved a compare-type problem did so by using a matching strategy. By Grade 2, matching had given way to counting up.

The National Numeracy Strategy *Framework* (DfEE, 1999) reflects typical Early Years education practice in recommending the introduction of subtraction, first as take-away, in Year R (pupil age 4-5), then as comparison in Year 1. One consequence of this Early Years initiation is the almost universal use of 'take away' as a synonym for subtraction (Haylock and Cockburn, 1997, p. 38). Another peculiarly-British complication is that the word 'difference' has come to be associated in rather a special way with the comparison structure for subtraction. It is not easy to be definite how and when this came about, but one useful reference is the teacher's manual for the highly-influential *Mathematics for Schools* (Fletcher, 1971) primary text book series. The series was 'new maths' in spirit, tempered with typically-British pragmatism. In a section entitled *Comparison and 'take away'*, Fletcher describes comparison in terms of matching the elements of two sets. Some elements of the larger set remain unmatched. Fletcher writes:

> The cardinal number of this unmatched subset denotes the *difference* between the cardinal number of Set A and Set B. In determining a *difference* we compare a set of objects by matching its members with another set of objects. (p. 9, emphasis in the original)

It is clear that Fletcher is associating the word 'difference' with comparison in order to *distinguish* it from take-away, although the grounds for doing so are not made explicit. The same association can be seen in recent UK teaching handbooks, for example:

> Story 2 introduces […] the *comparison* structure. […] When comparing two sets we may ask 'how many more in A?' or 'how many fewer in B?' or 'what is the difference between A and B?' (Haylock and Cockburn, 1997, p. 39).

Crucially, as we remarked earlier, the NNS itself refers to the compare structure as 'subtraction as *difference*'. However, at the same time, the term *difference* is the unique name of the outcome of *any* subtraction operation, on a par with *sum, product* and *quotient* in relation to the other three arithmetic operations. There is evidence that these complexities, and others, present obstacles to the pupils throughout the lesson (Rowland, 2006).

## YEAR 8: GRAPHS OF LINEAR FUNCTIONS[2]

The teacher, Suzie, had about 7 years' teaching experience. The lesson begins with 10 minutes' whole-class revision of fractions simplification e.g. $^{24}/_{6}$, $^{5}/_{25}$. Suzie then writes the lesson aims on a board:

> Find the gradient of straight lines.
> Use the gradient and the intercept on the y-axis to find the equation of straight lines.

[2] From the ESRC-funded T-media Project 2005-07, University of Cambridge. Principal Investigator Sara Hennessy

Suzie asks what ‘gradient’ means. She develops one response - “how steep” - in terms of steep hills. Other pupil ideas include: road, roof, a slide, swing frame, ski slope, stairs.

Suzie then writes on the board: “gradient = up/along”. She rolls the whiteboard to a squared section, and draws a line segment between two lattice points (4 along, 8 up). Suzie completes the triangle, using endpoints of the line segment, to show the horizontal and vertical increments. She says that the gradient is 8/4 = 2.

Suzie then draws another line segment alongside the first. Its gradient is 3/6. Some pupils say “2”. In response, Suzie asks: what is 3/6? One girl asks: is it 1/3? Susie says: It is ½. She asks which line (segment) has bigger gradient? She says that 2 is bigger than ½. One pupil refers to the two completed triangles that Suzie has drawn, and asks if it’s about area [i.e. does the first line have bigger gradient because the first triangle has greater area?]. This phase lasts 15 minutes.

There is then individual/paired work for 15 minutes. Pupils share laptop computers and load the graphing software Autograph. Suzie distributes a worksheet. The sheet asks them to draw $y=x$, $y=2x$, $y=3x$ and find the gradients (and generalise). Then it shows graphs of two lines through the origin and asks for their equations. Finally, it asks for a prediction of the graph of $2x+1$, with Autograph check. Suzie circulates to assist pairs.

The lesson concludes with a short plenary. Suzie projects $y=x$ (from her laptop) on a screen and asks about the gradient. Likewise $y=2x$, $y=3x$. In each case it is calculated using a segment with one point at the origin. A boy says “the number before $x$ is always the gradient”.

Then Suzie displays the graph of $y = 2x+3$. She picks the segment between (-1, 1) and (0, 3) to calculate the gradient. Suzie writes “$y=2x+3$” and annotates “gradient” near the symbol ‘2’, and “cross the $y$-axis (intercept)” alongside ‘3’. Finally Suzie displays another line on the large screen, and asks “What is its equation?” She finds the gradient starting from (0, 1). The intercept is 1. Suzie writes $y=3x+1$, and the lesson concludes.

## Foundation knowledge: gradient

Some reflections of a *mathematical* kind on the nature of the ‘gradient’, a concept which occupied much of the lesson time, is prompted by the examples that Suzie drew on the whiteboard when she introduced the concept quantitatively. Her examples were of line segments, whereas gradient is an attribute of (infinite) lines. Indeed, the graphing software (Autograph) that they used later draws lines, not line segments. Fundamental issues to be understood and considered by the teacher, therefore, include:

- the gradient of a line is found by isolating a segment of the line;

- any segment yields the same ratio (this could be tested empirically: theoretically, it relates to similar triangles).

There also exists knowledge of an explicitly pedagogical kind – more PCK than SMK – about the teaching and learning of the concept ‘gradient’. This is accessible in part by didactical reflections related to the mathematical observations already made:

- some segments facilitate identifying the increases in abscissa ($x$-coordinate) and ordinate ($y$-coordinate) better than others;
- the increase in abscissa should be ‘simple’ (ideally 1) to facilitate calculation of the ratio (unless one uses a calculator).

There were few problems with finding the gradient of $y=mx$ because (0, 0) could be taken to be one end of a line segment, and (1, $m$) the other. However, $y = 2x+3$ was much more problematic. So was $y = 3x+1$, and it seemed that few pupils followed Suzie’s demonstration at the end of the lesson.

Beyond pure reflection, there is knowledge to be gleaned from empirical research. The iconic *Concepts in Secondary Mathematics and Science* study found “a large gap between the relatively simple reading of information from a graph and the appreciation of an algebraic relationship” (Kerslake, 1981, p. 135). In particular, the notion that proportional linear relationships hold in all segments of a line, and that lines are parallel if and only if they have the same gradient, was understood by very few pupils aged 13-15. In another study, Bell and Janvier (1981) identified what they call “slope-height confusion”, whereby slope as a ratio is not distinguished from the linear dimensions of a line. This resonates with the pupil’s question about area, although it is not the same. More recently, Hadjidemetriou and Williams (2002) have found that teachers tend to underestimate the difficulties experienced by children in answering graphical test items, not least because they themselves had the misconception the item was designed to elicit.

## DISCUSSION

It is reasonable to claim that a particularly pithy concept (subtraction; gradient) lies at the heart of each of these lessons, and, from my observations, lies at the root of the pupils’ difficulty in learning what had been explicitly stated as the objectives of each lesson. This remark is not intended as a criticism of the two teachers involved, both of whom were committed to developing their teaching, and to the cause of mathematics teacher education. The complexity of the concepts would remain whoever was teaching them, and for other learners of similar ages. In both cases, there exists research evidence to suggest what can be expected of pupils (at the relevant ages) who have experienced instruction in these topics. This is useful in terms of anticipating the complexity of the material to be taught, and in terms of having realistic expectations of what will be learned, both because of and despite one’s best efforts.

What I find particularly interesting is the analysis of the concepts themselves. Some of this kind of analysis is achievable by ‘deep thought’, as it were, but in some cases it needs particularly insightful observational research (such as that cited on counting) to prise apart, or unpack, processes and skills that inevitably become automated, and therefore trivial, to adult users of those competences. The complexity of such skills necessarily becomes invisible to the educated citizen, yet it needs to be laid bare if they set out to teach them. My proposal here is that much elementary mathematics teaching is ‘difficult’, compared with teaching in the secondary grades and beyond, because the very concepts being taught, such as subtraction, lie somewhere beneath our conscious awareness, and our ability to analyse in pedagogically useful ways. Secondary and tertiary mathematics teaching is ‘difficult’ for different reasons, where teacher knowledge is concerned. In the case of Suzie’s lesson, for example, the teacher needs a good understanding of the defining characteristics of functions (e.g. Freudenthal, 1983; Even, 1999), which is ‘advanced’ knowledge in that it comes within the scope of undergraduate mathematics study. They also need a thought-out, connected understanding of the different ways in which functions can be *represented* symbolically and graphically, and how to navigate both within and between these two semiotic systems (Presmeg, 2006). Even (op cit.) found that this understanding could not be taken for granted in her prospective secondary teacher participants.

I liken much of the Foundation knowledge that underpins the teaching of elementary mathematics concepts – and this is where I arrive at the claim set out earlier – to the place of Foundations of Mathematics in mathematics itself, and in the world of the practising, so-called ‘working’, mathematician. Most mathematicians can get on with their work without the need to ask “But what is a set, a number, a line, a sentence, a theorem, …” and so on. From time to time, particular individuals are motivated to ask, and to attempt to answer, such questions, for various reasons: out of curiosity, or in order to resolve paradoxes, or to explain why a proof cannot be accomplished. In some ways, it is easier to continue building up the edifice of mathematics than to dig down beneath it, to establish the foundations. In the same way, engaging with the foundations of mathematical ideas that educated citizens take for granted, in order to make them accessible to young learners, poses its own distinctive challenges. For more advanced mathematical topics, the challenge to teachers lies more in the complexity of the concepts, the extent of the prerequisite concepts, and the sophistication of the semiotic systems with which they are represented in mainstream mathematical practice.

## REFERENCES

Ainley, J. & Luntley, M. (2006). What teachers know: the knowledge bases of classroom practice. In M. Bosch (Ed.), *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (pp. 1410-1419). Barcelona, Spain: FUNDEMI IQS, Universitat Ramon Llull. (Compact Disk)

Ball, D.L., Hill, H.C. & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, Fall 2005, 14-22.

Bell, A. & Janvier, C. (1981). The interpretation of graphs representing situations. *For the Learning of Mathematics*, 2(1), 34-42.

Carpenter, T. P., & Moser, J. M. (1983). The acquisition of addition and subtraction concepts. In R. Lesh and M. Landau (Eds.), *The acquisition of mathematical concepts and processes* (pp. 7–44). New York: Academic Press.

DfEE (1999). *The National Numeracy Strategy: framework for teaching mathematics from Reception to Year 6*. Sudbury: DfEE Publications.

Even, R. (1990). Subject matter knowledge for teaching and the case of functions. *Educational Studies in Mathematics*, 21, 521-544.

Fletcher, H. (1971). *Mathematics for Schools. Level II Books 1 and 2, Teacher's Resource Book*. London: Addison-Wesley

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Dordrecht: D. Reidel Publishing Company.

Hadjidemetriou, C. & Williams, J. (2002). Teachers' pedagogical content knowledge, from a cognitivist to a situated perspective. In A. D. Cockburn and E. Nardi (Eds.), *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education*, (Volume 3 pp. 57-64). Norwich: University of East Anglia.

Haylock, D. & Cockburn, A. (1997). *Understanding mathematics in the lower primary years*. London: Paul Chapman.

Huckstep, P., Rowland, T. & Thwaites, A. (2006). The knowledge quartet: considering Chloe. In M. Bosch (Ed.), *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (pp. 1568-1578). Barcelona, Spain: FUNDEMI IQS, Universitat Ramon Llull. (Compact Disk)

Kerslake, D. (1981). Graphs. In K. M. Hart (Ed.), *Children's understanding of mathematics 11-16* (pp. 120-136). London: John Murray.

Potari, D., Zachariades, T., Christou, C., Kyriazis, G. & Pitta-Pantazi, D. (2007). Teachers' mathematical knowledge and pedagogical practices in the teaching of derivative. In D. Pitta-Pantazi and G. Philippou (Eds.) *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 1955-2004). Nicosia, Cyprus: Department of Education, University of Cyprus. (Compact Disk).

Presmeg, N. C. (2006). A semiotic view of the role of imagery and inscriptions in mathematics teaching and learning. In J. Navotna, H. Moraova, M. Kratna, & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education*, (Volume 1 pp. 19-34). Prague, Czech Republic: Charles University.

Rowland, T., Huckstep, P. & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: the knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, 8(3), 255-281.

Rowland, T. (2006). Subtraction – difference or comparison?. *Mathematics in School*, 35(2), 32-35.

Shulman, L. S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher*, 15 (2), 4-14

Turner, F. (2007). The mathematics content knowledge of beginning teachers: the case of Amy. In D. Pitta-Pantazi and G. Philippou (Eds.) *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 2004-2013). Nicosia, Cyprus: Department of Education, University of Cyprus. (Compact Disk).

# RESULTS OF A COMPARATIVE STUDY OF FUTURE TEACHERS FROM AUSTRALIA, GERMANY AND HONG KONG WITH REGARD TO COMPETENCIES IN ARGUMENTATION AND PROOF

Björn Schwarz, Gabriele Kaiser

University of Hamburg

*The article describes the conceptions and first results of an enrichment study to the international comparative study on the efficacy of teacher education, Mathematics Teaching in the 21st Century (MT21). The study focuses on the professional knowledge of future teacher students in three countries – Australia, Germany and Hong Kong – with regard to the mathematical areas of modelling and argumentation and proof. After describing the theoretical framework and the applied methodological approach some selected results with regard to argumentation and proof are presented.*

Keywords: Education, Mathematical content knowledge, Pedagogical content knowledge, Proof.

## Background of the study

Although teacher education has already been criticised for a long time, only rarely systematic evaluation and studies concerning the efficiency of teacher education and how future teachers perform during and at the end of their education can be found (for an overview on the debate see Blömeke *et al*., 2008). Even in the field that is covered by most of the existing studies – the education of mathematics teachers – research deficits have to be stated: the research is often short term, of a non-cumulative nature, and conducted within the researcher's own training institution. Only recently more empirical studies on mathematics teacher education have been developed (cf. Chick *et al*., 2006, Adler *et al*., 2005).

In order to overcome this deficit the IEA (International Association for the Evaluation of Educational Achievement) currently carries out an international comparative study focusing on the efficiency of teacher education and the professional knowledge of future teachers called TEDS-M (Teacher Education and Development Study – Learning to Teach Mathematics). This study concentrates on future mathematics teachers and is conducted in 20 countries worldwide. We also refer to the COACTIV – study, another study on teacher education using similar conceptualisations of professional knowledge of mathematics teachers (see among others Krauss, Baumert & Blum 2008). Furthermore in order not only to develop a theoretical framework and adequate instruments for the TEDS-M study but also to offer a first research attempt to fill existing research gaps, a pilot study for TEDS-M was conducted called Mathematics Teaching in the 21st Century (MT211 [1]). This study also aimed to shed light on the important field of mathematics teacher education from a comparative perspective. For this among others the knowledge and beliefs of future lower secondary teachers were investigated (for results see e.g. Blömeke, Kaiser, Lehmann, 2008, Schmidt *et al*., 2008).

The study described in this paper is a complementary study to MT21 with the aim of gaining supplementary results basing on qualitative data as an addition to the quantitative data of MT21. This study is a collaborative study between researchers at universities in Germany, Hong Kong and Australia, using the theoretical framework and theoretical conceptualisation from MT21, but carrying out qualitatively oriented detailed in-depth studies on selected topics of the professional knowledge of future teachers, namely modelling and argumentation and proof, the latter being the theme of this paper. The study is only focussing on future teachers and their first phase of teacher education (for details see Schwarz *et al*., 2008).

## THEORETICAL FRAMEWORK OF THE STUDY

The initial ideas of MT21 are considerations about the central aspects of teachers' professional competencies and by this the related theoretical framework is also the theoretical basis of the supplementary study. Concerning the professional knowledge of teachers the study follows the ideas basically defined by Shulman (1986). He fundamentally distinguishes two domains, namely general pedagogical knowledge and content knowledge. The latter is further divided into three parts:

- subject matter content knowledge
- pedagogical content knowledge
- curricular knowledge

For the study these areas of content knowledge are further sub-divided. In the area of subject matter content knowledge for example with regard to Bromme (1995) mathematics as a school subject and mathematics as a scientific discipline are differentiated.

Beside these cognitive components furthermore also an affective and value-orientated component is taken into consideration. This component especially accounts for the epistemological beliefs, more precisely the beliefs towards mathematics itself and the beliefs towards teaching and learning mathematics. Again in accordance with the theoretical conceptualisations of MT21 (see Blömeke, Kaiser, Lehmann, 2008) the differentiation of different beliefs towards mathematics of Grigutsch, Raatz and Törner (1998) is basis of the study. Here four kinds of beliefs are distinguished with relation to mathematics:

- formalism-aspect of mathematics
- scheme-aspect of mathematics
- process-aspect of mathematics

- application-aspect of mathematics

Based on these theoretical distinctions concerning professional knowledge of future teachers the overall aim of our study is to answer the following questions:

- What kind of knowledge with regard to the described domains of teachers' professional knowledge do future teachers acquire during their university study?
- Which connections between the described domains of knowledge and the beliefs can be reconstructed within these future teachers?

In this paper from a mathematical content related perspective we concentrate on the area of argumentation and proof. Furthermore because of the limited space we only focus on the first question and describe some selected results. For a more detailed description of results related to the area argumentation and proof see Schwarz et al. (2008). For first results related to the second question with regard to the mathematical area of modelling see Schwarz, Kaiser, Buchholtz (2008).

Concerning the area of argumentation and proof we refer to specific European traditions, in which various kinds of reasoning and proofs are distinguished, especially "pre-formal proofs" and "formal proofs". These notions were elaborated by Blum and Kirsch (1991): pre-formal proof means "a chain of correct, but not formally represented conclusions which refer to valid, non-formal premises" (Blum & Kirsch, 1991, p. 187).

Concerning the role of proof in mathematics teaching, Holland (1996) details the plea of Blum and Kirsch (1991) for pre-formal proofs besides formal proofs as follows: For him pre-formal proofs may be sufficient in mathematics lessons with cognitively weaker students, in other classes both kinds of proofs should be conducted. Pre-formal proofs have many advantages due to their illustrative style. In addition, pre-formal proofs contribute substantially to a deeper understanding of the discussed theorems and they place emphasis on the application-oriented, experimental and pictorial aspects of mathematics. However, their disadvantage is their incompleteness, their reference to visualisations, which require formal proofs in order to convey an appropriate image of mathematics as science to the students. The scientific advantage of formal proofs, namely their completeness, is often accompanied by a certain complexity, which may cause barriers for the students' understanding and might be time-consuming. However, there is no doubt, that treating proofs in mathematics lessons is meaningful with the aim of developing general abilities, such as heuristic abilities. The teaching of these two different kinds of proofs leads to high demands on teachers and future teachers. Teachers must possess mathematical content knowledge at a higher level of school mathematics and university level knowledge on mathematics on proof. This comprises the ability to identify different proof structures (pre-formal – formal), the ability to execute proofs on different levels and to know alternative specific mathematical proofs.

Additionally, teachers should be able to recognise or to establish connections between different topic areas. To sum up: Teachers should have adequate knowledge of the above-described didactical considerations on proving as well (for details see Holland, 1996, pp. 51-58). It can be expected that in addition to being able to construct proofs, teachers will need to draw on their mathematical knowledge about the different structures of proving such as special cases or experimental 'proofs', pre-formal proofs, and formal proofs and pedagogical content knowledge when planning teaching experiences and when judging the adequacy or correctness of their, and their students' proofs in various mathematical content domains.

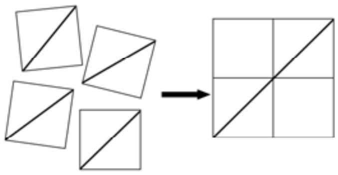
## METHODICAL APPROACH

Based on the methodological approach of triangulation questionnaires with open questions and in-depth thematically oriented interviews were developed. This offers the opportunity to deepen the quantitative results of MT21 by means of this qualitative orientated data. The instruments are, as described above, restricted to the areas of modelling and argumentation and proof. The questionnaire consists of seven items that are domain-overlapping designed – as so-called 'Bridging Items'. Each of the items captures several areas of knowledge and related beliefs on the base of the distinctions described above. In detail three items deal with modelling and real world examples, three with argumentation and proof and one is about how to handle heterogeneity when teaching mathematics. Furthermore, demographic information like number of semesters, second subject and attended seminars and teaching experiences – including extra-university teaching experiences - are collected. This questioning has been conducted with 79 future mathematics teachers on a voluntary base within the scope of pro-seminars and advanced seminars for future teachers at a German university. In Australia, 46 future teachers from two universities participated and in Hong Kong 84 future teachers from one institution.

Complimentary to this questionnaire an interview guide for a problem-centred guided interview was developed, which contains pre-structured and open questions (i.e., elaborating questions) on modelling and argumentation and proof. The questions are linked to the items in the questionnaire in the sense that the have the same theoretical base and cover the same sub-domains of teachers' professional knowledge. The selection of the interviewees follows theoretical considerations and takes the achievements in the questionnaire into account. That means interviewees were selected according to an interesting answering pattern in the questionnaire or extraordinary high or low knowledge in one or more domains.

The evaluation of the questionnaires as well as of the interviews is carried out by means of the qualitative content analysis method by Mayring (2000). More detailed we apply a method of analysis that aims at extracting a specific structure from the material by referring to predefined criteria (deductive application of categories). From there, by means of formulation of definitions, identification of typical passages from the responses as so-called anchor examples and development of coding rules, a coding manual has been constructed to be used to analyse and to code the material. For this, coding means the assignment of the material according to the evaluation categories. More precisely the method of structuring scaling (ibid.) is applied by which the material is evaluated by using scales (predominantly ordinal scales). Subsequently, quantitative analyses according to frequency or contingency can be carried out.

In the following one exemplary item of the questionnaire is described, which shows, how the different facets of professional knowledge – pedagogical content knowledge, mathematical knowledge and beliefs - are linked. A similar item is included in the interview, so that it is possible to connect the evaluation of the data on a rich data base.

> Read the following statement:
>
> **If you double the side length of a square, the length of each diagonal will be doubled as well.**
>
> The following pre-formal proof is given:
>
> *You use squared tiles of the same size. If you use four tiles to make one square, you will get a square with a side length twice the length of the squared tiles.*
>
> *You can see immediately, that each diagonal has twice the length of the ones of the squared tiles because the two diagonals of two tiles are put directly together.*
>
> 
>
> a) Is this argumentation a sufficient proof for you? Please give a short explanation.
>
> b) Please formulate a formal proof for the statement above about diagonals and squares.
>
> c) What proof would you use in your mathematics lessons? Please explain your position.
>
> d) Can a pre-formal proof be sufficient as the only kind of proof in mathematics lessons? Please explain your position.

e) Please name the advantages and disadvantages of a formal and pre-formal proof.

f) Can the pre-formal and the formal proof for the statement about the length of diagonals in squares be generalised for any rectangle? Please give a short explanation.

g) What do you think about the meaning of proofs for mathematics lessons in the secondary school?

**Figure 1: Task from the questionnaire concerning argumentation and proof**

## SELECTED RESULTS

Both, part b) and part f) of the task described above lay their focus on the future teachers' mathematical content knowledge. Part b) does especially not require any mathematics at a university level but only knowledge about fundamental geometrical theorems (e.g. Pythagoras theorem) and abilities concerning elementary algebraic transformations and abilities in formulation proofs. The items was coded on a five-point-scale while both codes, +1 and +2, means a right solution (answers coded with +2 in addition have a comprehensible structure) and -2 means serious mistakes like circular arguments or just a rephrasing of the pre-formal proof while a formal one is required. Examples of future teachers' responses and a more detailed description of the different coding of different answers are not presented here because of the limited space. Related descriptions can be found in Schwarz et al. (2008).

The results are the following:

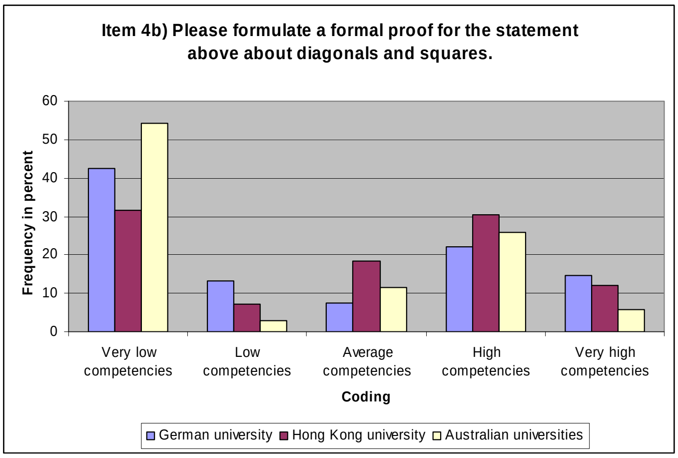



**Figure 2: Results of item 4b)**

One can see that for almost all institutions, the majority, in most instances, of future teachers in this case study were not able to execute formal proofs, requiring only lower secondary mathematical content, in an adequate and mathematically correct way.

Very similar results can be seen with regard to item f). Here also no university mathematics is needed but just an understanding of a proof suitable for lower secondary mathematics teaching. Again answers were coded on a five-point-scale with +1 and +2 meaning right solutions and -1 and -2 meaning wrong solutions. Then the results are the following:

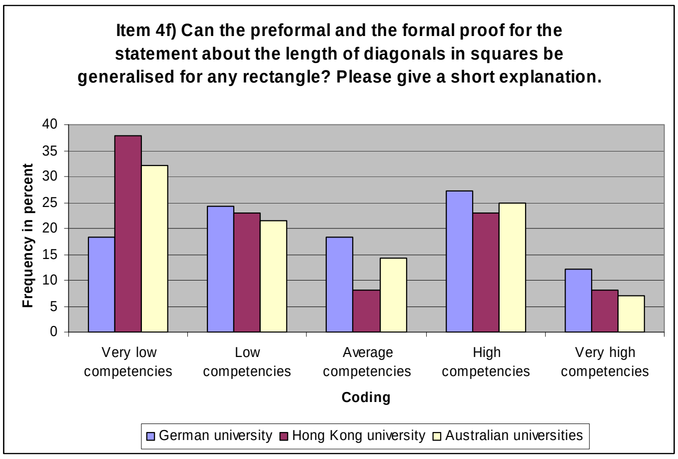



**Figure 3: Results of item 4f)**

Again, in most cases, the majority is not able to recognise and satisfactorily generalise a given mathematical proof.

In contrast, in all samples there was evidence of at least average competencies of pedagogical content reflection about formal and pre-formal proving in mathematics teaching with the exception of the Australian sample with respect to the sufficiency of pre-formal proof as the only type of proof in mathematics lessons. The related results are presented in a more qualitative way in the following paragraphs.

Preferences for pre-formal proving are evident, both with respect to mathematical content knowledge and pedagogical content knowledge. In contrast to the Hong Kong and Australian samples, there was a strong tendency in the German data for pre-formal proving to be incorporated into the pedagogical content-based discussion particularly with respect to problems of using proof with students of different abilities. In both the Hong Kong and Australian data, future teachers indicated a broad open-mindedness to various didactical conceptions but the pre-formal proof was perceived as an atypical part of mathematics teaching, possibly reflecting the use of alternate terms and conceptions for argumentation and proving that is not formal proof in the teacher education courses in these contexts. In both samples, mathematical content considerations tended to be the basis for didactical reflections.

With regards to affinity towards proving in lower secondary mathematics lessons Australian, Hong Kong and German students indicated a high to very high affinity to proving. It was assumed a higher affinity to proving would be expressed in more distinct pedagogical content reflection; however, the nature of these reflections differed with the samples. Future teachers in the German sample assumed dealing with proofs helped develop students' argumentation abilities especially with respect to their own hypotheses rather than their completeness of mathematical theorems. The difficulties students might have with proving in the classroom also came to the fore. In contrast, the Hong Kong and Australian future teachers rarely mentioned difficulties students might have with proving. The responses of future teachers from both Hong Kong and Australia reflected a formal image of mathematics being reinforced through use of formal proofs in teaching and the practice of proving leading to the comprehension of mathematical theorems.

## SUMMARY AND OUTLOOK

The paper describes first results of an additional study to the international comparative study on the efficiency of teacher education MT21. With regard to a theoretical framework distinguishing between different areas of teachers' professional competence results concerning future teachers' knowledge in different areas are presented restricted to the mathematical field of argumentation and proof.

As the presented additional study only focuses on future teachers, which means university students, no statements concerning the professional knowledge of practicing teachers can be made.

With regard to the further work to be done one of the next steps of the evaluation will be a more detailed distinction between different subgroups of the sample and the particular characteristics of their professional competence. For this evaluation the sample will be divided twice. On the one hand different school types the future teachers are studying for can be differentiated. On the other hand future teachers in different phases of their university studies, which means beginners or students at the end of their studies, can be distinguished. Besides that the results of the analyses of the interviews are to be linked to the results of the questionnaires. First results of these analyses can be found in Corleis et al. (2008). Finally the results of the additional study are to be related to the results of the main study MT21.

## NOTES

1. The previous name of this study was PTEDS.

## REFERENCES

Adler, J., Ball, D.L., Krainer, K., Lin, F.-L., & Novotna, J. (2005). Reflections on an emerging field: Researching mathematics teacher education. *Educational Studies in Mathematics, 60*, 359–381.

Blömeke, S., Felbrich, A., Müller, C., Kaiser, G. & Lehmann, R. (2008). Effectiveness of teacher education. State of research, measurement issues and consequences for future studies. *ZDM – The International Journal on Mathematics Education*, 40(5), 719 - 734.

Blömeke, S., Kaiser, G. & Lehmann, R. (Eds.) (2008). *Professionelle Kompetenz angehender Lehrerinnen und Lehrer. Wissen, Überzeugungen und*

*Lerngelegenheiten deutscher Mathematikstudierender und -referendare – Erste Ergebnisse zur Wirksamkeit der Lehrerausbildung.* Münster: Waxmann.

Blum, W., & Kirsch, A. (1991). Pre-formal proving: Examples and reflections. *Educational Studies in Mathematics, 22*(2), 183 – 203.

Bromme, R. (1995). What Exactly is ‘Pedagogical Content Knowledge’? – Critical Remarks Regarding a Fruitful Research Program. In Hopmann, S. & Riquarts, K. (Eds.). *Didaktik and/or Curriculum.* IPN-Schriftenreihe 147 (pp. 205 - 216). Kiel: IPN.

Chick, H. L., Baker, M., Pham, T., & Cheng, H. (2006). Aspects of teachers' pedagogical content knowledge for decimals. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings of the 30th annual conference of the International Group for the Psychology of Mathematics Education* (vol. 2, pp. 297-304). Prague: PME.

Corleis, A., Schwarz, B., Kaiser, G. & Leung, I. K. C. (2008). Content and pedagogical content knowledge in argumentation and proof of future teachers: a comparative case study in Germany and Hong Kong. *ZDM – The International Journal on Mathematics Education*, 40(5), 813 - 832.

Grigutsch, S., Raatz, U. & Törner, G. (1998). Einstellungen gegenüber Mathematik bei Mathematiklehrern. *Journal für Mathematik-Didaktik*, 19 (1), 3 - 45.

Holland, G. (1996). *Geometrie in der Sekundarstufe*. Heidelberg: Spektrum.

Krauss, S.; Baumert, J. & Blum, W. (2008). Secondary Mathematics Teachers' Pedagogical Content Knowledge and Content Knowledge: Validation of the COACTIV Constructs. *ZDM – The International Journal on Mathematics Education*, 40(5), 873 - 892.

Mayring, P. (2000). *Qualitative Inhaltsanalyse: Grundlagen und Techniken.* Weinheim: Deutscher Studien Verlag.

Shulman, L.S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*, 15(2), 4-14.

Schmidt, W. H., Houang, R. T., Cogan, L., Blömeke, S., Tatto, M. T., Hsieh, F. J., Santillan, M., Bankov, K., Han, S. I., Cedillo, T., Schwille, J. & Paine, L. (2008). Opportunity to learn in the preparation of mathematics teachers: its structure and how it varies across six countries. *ZDM – The International Journal on Mathematics Education*, 40(5), 735 - 747.

Schwarz, B., Kaiser, G., & Buchholtz, N. (2008). Vertiefende qualitative Analysen zur professionellen Komptenz angehender Mathematiklehrkräfte am Beispiel von Modellierung und Realitätsbezügen. In Blömeke, S., Kaiser, G. & Lehmann, R. (Eds). *Professionelle Kompetenz angehender Lehrerinnen und Lehrer. Wissen, Überzeugungen und Lerngelegenheiten deutscher Mathematik-*

*Studierender und –referendare – Erste Ergebnisse zur Wirksamkeit der Lehrerausbildung*. Münster: Waxmann, 391-424.

Schwarz, B., Leung, I. K. C., Buchholtz, N., Kaiser, G., Stillman, G., Brown, J. & Vale, C. (2008). Future Teachers' Professional Knowledge on Argumentation and Proof: A Case Study from Three Countries. *ZDM – The International Journal on Mathematics Education,* 40(5), 791 - 811.

# KATE'S CONCEPTIONS OF MATHEMATICS TEACHING: INFLUENCES IN THE FIRST THREE YEARS

Fay Turner

University of Cambridge

*In this paper I report on findings from a four year study of beginning teachers. The findings presented concern the conceptions of mathematics teaching for one of four case-study teachers and the influences on these conceptions. I present data from observations of lessons, interviews and written accounts that suggest Kate's conceptions of teaching became increasingly more consistent with a 'content-focused with an emphasis on conceptual development' view of teaching. Data are also presented which suggest that 'reflection' was the main influence on the development of Kate's conceptions both as an independent factor and in conjunction with the factors of 'experience' and 'working with others'.*

## INTRODUCTION

There is evidence that the conception of mathematics teaching held by individual teachers will contribute to the effectiveness of their teaching (Thompson, 1992; Askew, Brown, Rhodes, Johnson and Wiliam, 1997). The term conceptions is used here in the way suggested by Thompson (1992), as an inclusive term to include beliefs as well as other ideas such as mental images, concepts, meanings and preferences. Conceptions of mathematics teaching is clearly an area that needs to be addressed in any work which attempts to describe or influence the development of beginning teachers in relation to the teaching of mathematics. Assessing teachers' conceptions and the promotion of such conceptions that are believed to be positively influential in children's learning were seen as integral to my PhD study, an aspect of which I report on here.

Khus and Ball (1986) proposed four models of teachers' views about mathematics teaching, *a classroom-focused view*, a *content-focused with an emphasis on performance view*, a *content-focused with an emphasis on conceptual understanding view* and a *learner-focused view*. I used these models as a theoretical framework for the analysis of data collected in my study. Though I have analysed the data in relation to all four of Khus and Ball's models of conceptions of mathematics teaching, restrictions of space here only allow discussion in relation to the *content-focused with an emphasis on performance* and the *content-focused with an emphasis on conceptual understanding views.*

The aim of my study was to investigate the way in which beginning teachers' understanding of mathematics content knowledge needed for teaching might be developed through reflection using the *Knowledge Quartet framework*. This framework was used as a tool for identification and discussion of the teachers' mathematics content knowledge as evidenced in their teaching. The Knowledge

Quartet framework consists of four dimensions, *Foundation*, *Transformation*, *Connection* and *Contingency*. Details of this framework, and an account of how it was developed, may be found in the paper presented by Tim Rowland at the CERME meeting in Spain (Rowland, Huckstep and Thwaites, 2005).

Teacher's beliefs about mathematics and mathematics teaching were considered to be a component of mathematics content knowledge and are incorporated in the Foundation dimension of the Knowledge Quartet framework. Findings relating to the development of the Foundation aspect of one teacher's mathematical content knowledge were presented in a paper at the CERME meeting in Cyprus (Turner, 2007). The focus of the 2007 paper was on Amy, and drew on data from the first two years of the study. This paper focuses on the aspect of conceptions about mathematics teaching from within the Foundation dimension and presents findings relating to Kate over the full four years of the study.

## THE STUDY

The study began with 12 student teachers from the 2004-5 cohort of primary (5-11 years) postgraduate pre-service teacher education course at the University of Cambridge. The numbers reduced, as anticipated, to 9 in the second year, then 6 in the third year and finally 4 in the fourth and last year of the study. All participants were observed teaching during the final placement of their training year, twice during the first year, three times during the second year and once in the third year of their teaching. These lessons were all video-taped. In the training year the video-tapes were the basis for stimulated recall discussions using the Knowledge Quartet framework to focus on the mathematical content of the lesson. During the first year of teaching, feedback using the Knowledge Quartet framework was given following the two observed lessons. Participants were then sent a DVD with a recording of their lesson, and a request to observe the lesson and write their reflections on it. In the second year of their teaching only minimal feedback was given following the lesson as I wanted to see how the teachers would independently make use of the Knowledge Quartet in their reflections. They were sent DVDs of their three lessons and wrote reflections on each of these, drawing on their previous training in using the Knowledge Quartet framework. Participants also wrote regular reflections on their mathematics teaching which they sent to me. Group meetings were held to discuss the mathematics teaching and participation in the project of participants. These happened at the end of the training year and the first year of teaching, and at the end of each term in the second year of teaching. In their third year of teaching each teacher was interviewed individually in the Autumn and Spring terms and a group meeting was held in the Spring term.

Case studies were built from observations of teaching, discussions following observed lessons, contributions to group meetings, written reflections and individual interviews. Data from transcripts of discussions following observed lessons and group interviews as well as from written reflections was all analysed using the qualitative data analysis software NVivo. A grounded theory approach (Glaser and Strauss, 1967) was used which led to the emergence of a hierarchical organisation of codes into a number of themes. Analysis of data attributed to codes under the NVivo theme 'beliefs', and the Knowledge Quartet analysis of observed lessons were used to build a description of the participants conceptions of mathematics and mathematics teaching over the four years of the study. Analysis of data attributed to codes under the themes of 'experience', 'reflection' and 'working with others' allowed inferences to be made about the factors associated with changes to participants' conceptions. Though data from all four case studies have been analysed in relation to changes in their conceptions of mathematics teaching, there is only room to report on Kate here.

Since in this discussion I hope to build a picture of the way in which the participants' conceptions developed over time, it is necessary to refer to times at which different data were collected. To aid clarity, and achieve brevity in this, I will use the date of the year and a number only to identify the timescale. Table 1 is intended to help the reader place the data within this timescale.

**Table 1: Notation used to indicate the timescale of data collected in the study**

| Notation used | | Place in career |
|---|---|---|
| 2004 | Autumn term | Trainee teacher |
| 2005/6(1) | Autumn / Spring term | First year in teaching post |
| 2005/6(2) | Spring / Summer term | |
| 2006/7(1) | Autumn term | Second year in teaching post |
| 2006/7(2) | Spring term | |
| 2006/7(3) | Summer Term | |
| 2008(1) | Autumn Term | Third year in teaching post |
| 2008(2) | Spring Term | |
| 2008 (3) | Summer term | |

## FINDINGS

Analysis of teaching and of data coded under the heading 'beliefs', provided an account of Kate's conceptions of teaching over the first three years of her career. In Kate's lesson observed in 2004, the Knowledge Quartet code 'reliance on procedures' featured strongly and suggested a view which *emphasised performance*. Kate was teaching a lesson about doubling single digit numbers and demonstrated recording the doubling process by writing an addition in a witch's cauldron with the answer in a bubble above e.g. '3 + 3' in the cauldron and '6' in the bubble. To record doubling of two digit numbers an extra bubble was added for the 'tens numbers' e.g. '23 + 23' in the cauldron, '4' in the tens bubble and '6' in the units bubble. When questioned about this in the post-lesson reflective interview Kate suggested an amendment,

> If I was going to do the tens and units, I should have asked for the units first 'cus that's what they know they have to start with, the most significant number which is tens.

Kate focused on a procedure reflecting the standard algorithm which suggested an emphasis on performance view of teaching. However in this same lesson, there was an indication that Kate was concerned to develop conceptual understanding. The Knowledge Quartet code 'making connections between concepts' was attributed when Kate made connections between doubles and near doubles and even and odd numbers and used pictorial representation to demonstrate why doubles must be even and near doubles odd.

In the lesson observed in 2005/6(1), Kate introduced the concept of multiplication by making the connection with repeated addition. She used a number of different representations modelling repeated addition to develop understanding of the concept of multiplication. However, when they came to do some problems themselves, the children were given specific procedures for calculating and recording. This lesson seemed to reflect a mixture of *content-focused views* of teaching with both an *emphasis on performance* and an *emphasis on conceptual development*.

The second lesson observed in 2005/6 did not feature 'reliance on procedures' and Kate made use of demonstrations to develop the children's understanding of capacity and conservation. However, at the group interview in 2005/6(2), Kate suggested that she thought the children preferred an approach which emphasised procedures.

> They really like doing boring things, they like doing number sentence things, they don't like the other [problem solving] it's more difficult, but they really like number sentences.

The three lessons observed in 2006/7 all demonstrated a concern for developing conceptual understanding rather than focusing on performance. In the lesson observed in 2006/7(1), there were no instances of 'reliance on procedures' and Kate used a number of demonstrations to build the children's understanding of measuring using appropriate non-standard and standard units. In the lesson observed in 2006/7(2), Kate made use of a number of different representations to develop the difference conception of subtraction and also asked the children to explain their own strategies for completing the calculations. In the warm up part of the lesson observed in 2006/7(3), Kate set problems involving making the largest and smallest numbers on a spiked abacus using specified numbers of beads. This was designed to develop their conceptual understanding of place value. The main part of this lesson involved shading of different fractions on various grids. The way in which Kate introduced this, and the activities set for the children seemed to be aimed at developing a conceptual understanding of vulgar fractions.

The suggestion that Kate's emphasis was on conceptual understanding in 2006/7, was supported by analysis of the NVivo coding of data. Kate had five instances of the code 'conceptual understanding' attributed to her data from 2006/7. In her reflective account written in 2006/7(3), Kate wrote,

> Following the quite broad objectives of the new strategy, we have been trying to teach about data handling in quite a conceptual way and get children to think about the advantages and disadvantages of different ways to represent data.

Though observations of teaching and analysis of the NVivo data suggested Kate had moved towards an *emphasis on conceptual understanding view* of teaching, there were also a number of instances of her data from 2006/7 and 2007/8 which suggested she still held an *emphasis on performance view*. Two codes considered to reflect an *emphasis on performance view*, which featured strongly in her data, were ‘teaching different strategies’ and ‘need for structured work’. Five instances from reflective accounts written in 2006/7 were coded as ‘teaching different strategies’.

> We have been looking at different addition strategies … We had specific teaching sessions on some of these areas, then had some activities in which children were encouraged to choose a method for themselves. 2006/7(1)

This instance, and others like it, indicated that Kate felt she needed to give children a ‘toolbox’ of strategies from which to choose in order to perform calculations. Later in the year she seemed to have moved her position towards one in which she felt it was more helpful to focus on just some specific methods.

> The week was structured around teaching a few particular methods, which is a little different from the approach we have often taken before when we have given the children opportunities to choose their own methods. 2006/7(2)

Kate’s move towards an approach involving teaching specific methods with which children can be successful seemed to reflect an emphasis on performance.

Data coded as ‘need for structured work’, suggested that Kate seemed more concerned that children achieved success in solving problems than that they developed a conceptual understanding. During the interview in 2007/8(2), we discussed the teaching of ‘word problems’. Kate indicated that she focused on getting the children to look for specific words in order to decide what sort of calculation was involved.

> So, rather than understanding the concept behind the problem, it was … we wanted the children to know what they could do, and that’s why I repeated the same lesson again. This time we approached it a bit differently and said ‘if you can spot one of these words, then you can work out for yourselves what it means and you will be able to do it’.

During the interview in 2007/8(2) Kate suggested that she recognised her teaching focused on achievement or performance rather than on developing conceptual understanding through exploration.

> I don’t think that we do much open-ended, and that is perhaps a bit of a weakness in the way that I teach at the moment, because quite often, quite often in lessons I tell them what I want them to achieve.

Though Kate sometimes focused on performance in 2007/8, there was evidence from the lesson observed in 2007/8(2) that she continued to emphasise conceptual understanding. In this lesson Kate demonstrated the commutativity rule for addition before introducing the strategy of putting the bigger number first. She showed this by pinning two sets of differently coloured clothes pegs on a coat hanger to illustrate an addition e.g. 2 + 3, and then turned the coat hanger around to show the addition 3 + 2. Kate did not simply tell the children the rule but demonstrated why it was the case. Later in the lesson, Kate demonstrated adding ten by moving down one row on a hundred grid. She asked the children why adding 10 to 23 gave the answer 33. Kate tried without success to get a response which showed an understanding of place value in relation to the layout of the grid. In the post-lesson reflective interview, Kate stated that she was unhappy that pupils had responded in this procedural way, and said that she would work on an approach directed at understanding why this procedure works.

Kate's data suggest that over the first three years of teaching her conceptions of mathematics teaching had encompassed elements of a *content-focused view with an emphasis on performance* and a *content-focused view with an emphasis on conceptual understanding*. All of Kate's lessons observed over the three years indicated that Kate was trying to develop conceptual understanding in her pupils, and this was supported by analysis of the NVivo coding of her data. Kate's later comments suggest that she was consciously trying to focus more on developing conceptual understanding. However these comments also suggest that she continued to be concerned that children were taught specific strategies, suggesting a view which emphasises performance.

The data discussed above presented a picture of Kate's conceptions of mathematics teaching over the first three years of her career. An analysis of data under the NVivo coding headings, 'experience', 'working with others' and 'reflection', gave some insight into the influences on these conceptions. Three instances of data under the heading 'experience' suggest that this was an influence on Kate's conceptions of mathematics teaching as *content-focused with an emphasis on conceptual understanding*. In her reflective account 2006/7(1) Kate wrote,

> From last time we covered place value I realised that the majority of my year ones were not very clear on this concept. I wanted to make sure they understood the importance of tens and units on how we write our numbers.

During the interview in 2007/8(1), I asked what Kate thought had influenced the way in which her teaching had changed.

> I think having done it before and knowing it works and sometimes I think when I have been teaching things, I have thought 'do I really understand this', or I have thought, ' I think I might be giving a misconception here or something', and then the next time I am really careful not to.

I would argue however that 'experience' alone did not influence Kate's conceptions of mathematics teaching. Rather, an examination of the three instances, demonstrate that it was Kate's reflection on her experience that influenced her conceptions of mathematics teaching. Phrases such as 'I realised', 'I have thought' and 'extrapolating in my head', all suggest active reflection.

There were several instances of data attributed to codes under the heading 'working with others' that suggested this too influenced Kate's conceptions of mathematics teaching. Some such instances seemed to suggest that her colleagues had a view which emphasised on performance, while Kate's view was more one which emphasised conceptual understanding. In her reflective account 2006/7(2), Kate wrote,

> Various materials suggest you should use them [empty number lines] in a 'come and show me how you are going to use this in your own way' kind of approach. However my colleague believes that we should only be teaching counting on along the empty numberline because that is what the children will be taught in year three.

Kate seemed to be in a dilemma because she was concerned with conceptual understanding while her colleague seemed to focus on content of the school curriculum. Two instances from the interview in 2007/8(1) suggest that Kate's 'enculturation into a community of practice' (Lave, 1988) involved exposure to views which emphasised performance. In the first of these, Kate's use of the term 'we', suggested that an emphasis on performance had resulted from shared planning.

> We are trying to work on getting them to have skills of the physical, and the sort of organisational skills of recording their maths and they sort of need a structure to do it in.

In the second instance Kate was replying to my question about whether she ever talked to other people about reflections on her teaching.

> Yes, occasionally. I think I would say, 'they found that really difficult, the numbers were too high and they didn't get a chance to work on the process because they were using those numbers', or 'that was really quick and they could have done another'.

This suggested that Kate saw her conversations about mathematics teaching with colleagues as being focused on the performance of the children rather than their conceptual understanding.

There were a number of instances of data under the heading 'working with others' that suggested Kate had an *emphasis on conceptual understanding* view of teaching. However, these did not necessarily suggest that Kate's colleagues had been influential in developing this view. In her reflective account 2006/7(1), Kate discussed a difference of opinion about a planned investigation.

> The person planning for our team had planned for the children to investigate the question 'do all rectangles have four sides'. When this was first suggested it struck me as a rather trivial question, but as I continued to think about it I thought it was not a very good question at all because it suggested there was something intrinsically 'rectangular' about the examples they would be spotting which would allow them to recognise them as rectangles without taking into account their four-sideness.
>
> I haven't discussed this with my colleagues as I didn't want to be awkward, but I made a note to myself to keep my eyes open at planning meetings so I can politely say something straight away if I am uncomfortable with the mathematical ideas behind our planning in any other cases!

Kate focused on the conceptual appropriateness of the task despite the influence of her colleague, rather than because of it. During the interview in 2007/8(1), I asked Kate whether she ever talked to her colleagues about issues such as the use of representations in her teaching.

> Not as often as we should because nobody wants to do the planning again. Um, I guess I would just use the other representation rather than discussing it with anybody.

This instance suggested that Kate did not automatically take on the ideas of her colleagues, but considered their conceptual appropriateness and changed them in her own teaching if she thought it necessary. Kate's 'enculturation into her community of practice' seemed to have been mediated by critical reflection. Kate engaged in the process Wenger (1998) referred to as *critical alignment* in such a way that she developed a view of teaching that continued to be strongly *content-focused with an emphasis on conceptual development*, despite this not seeming to be the general view of her community of practice.

The factors of 'experience' and 'working with others,' seemed to have had some influence on Kate's conceptions of mathematics teaching. However, both these factors also involved the mediation of reflection. Reflection also emerged as a separate heading in the NVivo coding process and Kate had a greater number of instances of her data attributed to codes under the heading of 'reflection' than to 'experience' and 'working with others' taken together. Codes under the heading 'reflection' which related to conceptions about mathematics teaching included, 'changed thinking', 'justification of teaching', 'questioning own teaching', 'suggested improvements' and 'judgements about effectiveness'.

Some of the instances of Kate's data coded under the heading 'reflection' suggested a view of teaching that emphasised performance. In her reflective account written in 2006/7(1), she focused on how well the children had performed on the tasks.

They seemed much more prone to making mistakes [in subtraction than addition] such as being one out because of counting the one they started on. They found taking away using number lines really tricky and were quite unreliable at taking away using objects.

Though such comments focused on the children's performance of tasks there were also suggestions in them that Kate was thinking about why they had difficulties. Similarly, some comments made during the interviews in 2007/8, focused on children's performance on tasks but also mentioned understanding. For example,

> The year ones did a sheet of number sentences … that was a bad choice of sheet because it was an 'empty box' sheet and we hadn't been doing any empty boxes … they still got it wrong because they didn't understand what it was asking them … but I understood why they did it. So, it was OKish because they were quite purposefully engaged …

Though this instance suggested Kate focused on engagement rather than learning, it also indicated that she had given some thought to children's conceptual difficulties.

There were few instances of data that suggested Kate focused only on children's performance without in some way considering their conceptual understanding.

In her reflective accounts Kate made several comments which explicitly demonstrated her concern with the conceptual understanding that had, or had not been achieved through her teaching. For example,

> In the first lesson we did several activities which involved putting numbers into order and then went on to positioning numbers on a numberline for their independent activity, but I think this activity had more to do with place value than ordering numbers as they had to work out how many tens marks to count along and then think about the units. 2006/7(2)

Kate also made a number of comments during the interview in 2007/8(1) which suggested she held a view of teaching which emphasised conceptual development.

> The children thought that triangles would have a line of symmetry but the one we tried didn't. In retrospect I wish that we had discussed that a bit more because it would have been interesting to get all the triangles out of the box and compare them.

Data from the heading 'reflection', suggested that Kate's had a strong view of mathematics teaching as *content-focused with an emphasis on conceptual understanding*. Though, this does not necessarily suggest a causal link between reflection and her view, it can be argued that reflection did influence Kate's conceptions. Kate wrote these reflective accounts because of her involvement in the study. The kind of thinking she engaged in was therefore prompted by the requirement to reflect on her teaching using the Knowledge Quartet. During the interview in 2007/8(1), Kate confirmed that this framework had influenced her thinking,

> The first few things I would be thinking of are the organisational things, and then I try to think 'did they learn anything' and 'was the learning alright even if the organisation wasn't' kind of thing. So, I think it is useful to have some kind of structure to help you know what you need to know and what they need to know and how to learn it.

Later in the interview, Kate reiterated that the structure provided by the Knowledge Quartet helped her reflect on whether or not her teaching had been effective in promoting understanding.

> I think what I have said and how I have explained things, I am more aware than I would be if I didn't have such a clear idea of what I was looking for.

## Summary and implications

Analysis of longitudinal data from one case study of a beginning teacher has given some insight into the conceptions of mathematics teaching held by that teacher, as well as insight into the influences on those conceptions. Though finding about Kate's conceptions and the influences on them are inferential, the use of the Knowledge Quartet framework for the analysis of lessons, and the systematic analysis of all data from interviews and reflective accounts, gives a strong basis for these inferences. It is reasonable to suggest that Kate has developed a view of mathematics teaching that is increasingly *content-focused with an emphasis on conceptual understanding* and that the development of this view has been influenced by reflections on her teaching supported by the Knowledge Quartet framework. 'Experience' and 'working with others', have also been influential in developing Kate's conceptions of mathematics teaching. However, reflection was an important mediator in these two factors. There is evidence, not discussed here, that Kate had also moved towards a *learner-focused view* of mathematics teaching. The direction of development of Kate's conceptions is one which we might wish to replicate in other beginning teachers. If so, it would seem that finding ways of encouraging the sort of reflection on mathematics teaching that Kate has undertaken over the first years of her career, is an idea worth pursuing.

## References

Askew, M., Brown, M., Rhodes, V., Johnson, D., & Wiliam, D. (1997). *Effective Teachers of Numeracy.* Report of a study carried out for the Teacher training Agency 1995-96 by the School of Education King's College London

Kuhs, T. M., and Ball, D. L. (1986). *Approaches to mathematics: Mapping the domains of knowledge, skills and dispositions.* East Lancing:Michigan State University, Center on Teacher Education.

Lave, J. (1988). *Cognition in Practice*. Cambridge: Cambridge University Press.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). *'The Knowledge Quartet: Considering Chloe'*. CERME 4: Fourth Conference of the European Society for Research in Mathematics Education February 2005. Sant Felue de Guixols, Spain.

Thompson, A. G. (1992). Teachers' Beliefs and Conceptions: A synthesis of the research. In D. Grouws (Ed), *A Handbook of Research on Mathematics Teaching and Learning* (pp. 127-146). National Council of Teachers of Mathematics.

Turner, F. (2007). *'The mathematics content knowledge of beginning teachers: the case of Amy'*. CERME 5: Fifth Conference of the European Society for Research in Mathematics Education, February 2007. Larnaca, Cyprus.

Wenger, E. (1998). *Communities of practice: Learning meaning and identity*. Cambridge: Cambridge University Press.

# PRE-SERVICE TEACHER-GENERATED ANALOGIES FOR FUNCTION CONCEPTS

Behiye Ubuz[1], Ayşegül Eryılmaz[2], Utkun Aydın[3] , Ibrahim Bayazıt[4]

[1] Middle East Technical University, Ankara, TR

[2] Anadolu University, Eskişehir, TR

[3] Muğla University, Muğla, TR

[4] Erciyes Universitesi, Kayseri, TR

*The purpose of the present study was to determine pre-service teacher-generated analogies in teaching function concepts and then to discuss them in terms of the content validity – whether analogies used are epistemologically appropriate to illustrate the essence and the properties of the functions as well as the structural relations between the analogues and the targeted concepts. The videotaped data of five pre-service teachers' were collected from their microteaching during "Practice Teaching in Secondary Education" course. Results revealed that pre-service teachers did not consider too much on their analogical models. So they generally failed to make effective transformations between the analogies and the target concepts.*

*Keywords: Function, analogy, pre-service teacher, content validity, teacher training*

## INTRODUCTION

What distinguishes a mathematics teacher from mathematics major is "the capacity of a teacher to transform the content knowledge he or she posses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (Shulman, 1987, p. 15). In order to move from the personal comprehension to preparing comprehension of others, some combination of the following processes: preparation, representation, instructional selections, adaptation and tailoring to students' characteristics are proposed (Shulman, 1987). For representation of the selected sequence, teacher makes use of appropriate analogies, metaphors, examples, demonstrations, explanations, etc.

Analogies constitute one crucial component of the teachers' pedagogical content knowledge that they need most to transform subject matter into forms that could be grasped by the students of different ability and social background. Analogies are heuristic tools that enhance imagination and creativity in terms of making causal relations between the unknown and the well-known concepts (Gentner, 1998). By developing mental models students have the opportunity to access to a wide range of conceptual explanations and transformations that facilitate capturing similarities and making parallels between the concepts in areas other than mathematics and the concepts in different contexts within mathematics itself. Therefore, this article focuses on pre-service teacher-generated analogies in teaching function concepts. Function concept is central for secondary school curriculum and advanced mathematical topics taught at school and university level. Further, the function concept is considered to have a unifying role in mathematics that provides meaningful representations of real-life situations (Lloyd & Wilson, 1998). Hence, the use of analogies is very common in the teaching of functions.

Pedagogical content knowledge (PCK) refers to “the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction” (Shulman, 1987, p. 8). That is, PCK is a key aspect to address in the study of teaching. To use an example in our context, pedagogical content knowledge refers not only to knowledge about functions, but also to knowledge about the teaching of functions with analogies. To teach functions with analogies teachers should transform the subject matter for the purpose of teaching and give arguments about it. That is, they should consider the characteristics of the function concept, choose or construct well constructed analogies, and consider the similarities and differences between the different aspects of the function concepts and the analog concepts. Therefore, the study reported here is related to pre-service teacher pedagogical content knowledge. Since the process of learning is influenced by the teacher, it is therefore important to understand how teachers explain what a function is to students, what they emphasize and what they do not; and what ways they choose to help students understand.

The present study contributes to a growing body of research in the field of function by examining pre-service teacher generated analogies to determine the analogies and the target concepts and then to discuss them in terms of the content validity – whether source analogies used are epistemologically appropriate to illustrate the essence and the properties of the functions as well as the structural relations between the analogues and the targeted concepts. More specifically, we posed two main research questions for this study: (1) How do the pre-service teachers manage with the analogies they introduce? and (2) Are these analogies relevant?

Task analysis of the lessons of the pre-service teachers provides less experienced mathematics textbook authors and teachers with guidelines on how to form and use analogies effectively in teaching functions. A careful examination of an analogy is a prerequisite to using it effectively in instruction. When teachers and authors use an analogy, they should anticipate analogy-caused misconceptions and eliminate them by forming epistemologically appropriate analogies and by mapping the similarities and differences between the different aspects of the function concepts and the analogies constructed. The present study directly responds to a need among mathematics educators for insight into the nature of analogies in function concepts and guidance on how to construct ones that are pedagogically effective.

## THE STUDY

### Context and Participants

The study was conducted with all pre-service teachers (PT1, PT2, PT3, PT4 and PT5) taking "Practice Teaching in Secondary Education" course that was offered in Master of Science without Thesis Program at Middle East Technical University during 2005-2006 fall semester. One was male and four were female. Three of the participants (PT2, PT3, and PT5) had experience in teaching mathematics at an institution where additional courses out of school were offered and other two had experience in teaching mathematics as a private tutor. Three graduated from mathematics department (PT2, PT3, and PT4), and attending to the Master of Science without Thesis Program and rest were continuing previous mathematics teacher education program to get their bachelor degree. Master of Science without Thesis Program is a certificate program to teach mathematics at secondary school level (grades 9-12). All these students were the total number of the students in their second term.

"Practice Teaching in Secondary Education" course involves practice teaching in classroom environment for acquiring required skills in becoming an effective mathematics teacher. In this course pre-service teachers spend their six class hours in real classroom environment at an arranged public secondary school, and two class hours at the university. In that two hours period at the university, pre-service teachers presented sample lessons one by one to their colleagues and the instructor.

At the beginning of the course, function topics covered at the 9th grade and triangles topics covered at the 10th grade were assigned to each participant to be presented in a 30 minutes period at the university, to provide an effective flow of lesson and to cover all topics relevant to functions and triangles. Each participant prepared three lesson plans about assigned topics to be presented at the classroom. Two of those presentations were on functions and one on triangles. Additionally, they also did teaching two times at the school with presence of the instructor (the first researcher) and the classroom teacher. At other times they did teaching at the school when the classroom teacher allowed them to do. Teaching at the university and the school constituted 30 percent of the course grade. Lesson plans constituted 15 percent of the course grade.

While preparing the lesson plans, they mainly focused on objectives, materials, teaching techniques and the development process in the lesson.

### The Design and the Analysis

The study used a case study approach with naturalistic observation. The data were drawn from the observation of five pre-service teachers' microteaching on functions conducted in two hours period at the University Class. Topics about functions involved function concepts, operation on functions, composite functions, and types of functions (constant, identity, greatest value, partial, and signum functions). In order to provide flexibility, they were not restricted to use any specific method in their presentations. During some presentations, the use of analogy method aroused. The use of analogy, however, mostly did not appear in the lesson plans. The courses were presented in three different sequences: 1) analogies, definition or rules, and solving examples, 2) definition or rules, analogies, solving examples, and 3) definition or rules, and solving examples. This indicates that analogies appeared either while exemplifying definition or rules or making introductions to the topics. In the Methods of Science and Mathematics Teaching courses the history of and some misconceptions about functions had been included but not theories and applications of analogy. All presentations and discussions were video-taped and transcribed.

Literature about epistemology of the functions (e.g. Cooney & Wilson, 1993; & Harel & Dubinsky, 1992) and the guidelines in the Teaching with Analogies Model developed from task analyses (Glynn, Duit, & Thiele, 1995) provided a conceptual base for the data analysis. Content analysis (Philips & Hardy, 2002) was conducted to discern meaning in the teacher's written and spoken expressions. Lessons were fully transcribed and considered line by line whilst annotated field notes were used as supplementary sources. The first phase of data analysis included detecting analogy-based teaching instances and identifying source analogies and the target concepts. The subsequent phases embraced in-depth examinations of spotted cases in accord with 'content validity – whether analogies used are epistemologically appropriate to illustrate the essence and the properties of the functions as well as the structural relations between the analogues and the targeted concepts. The validity of the analysis was achieved by utilizing multiple classifiers to arrive at an agreed upon classification of analogies and their target concepts as well as their epistemologically appropriateness.

## FINDINGS

Data indicate the key analogical models used in teaching function, composite function and types of function concepts particularly while defining or explaining them. The analog and target concept matching was summarized in Table 1.

| Analog (Familiar Situation) | Target (Mathematics Concept) |
|---|---|
| 1. Function machine | Function concept, Composite function |
| 2. Posting a letter | Composite function |
| 3. Packing-Unpacking a present to a friend | Inverse function |
| 4. A perforated pail | Identity function |
| 5. Age | Partial functions, Greatest value function |
| 6. Watering a tree | Greatest value function |
| 7. The shelters in the apartment | Greatest value function |
| 8. Eating a cake | Greatest value function |

**Table 1: Analog and the target concept relations**

Here three analogies are presented and discussed because of the space restriction.

## Posting a Letter Analogy

“Posting a letter” analogy was given by one of the participants [PT5] during the composite function lesson provided by [PT2]. This analogy was provided to make clear the definition and the explanations. As seen in the dialog, [PT5], however, did not focus on what the inputs and outputs for f and g are. As a result of that, one of the participants [PT1] got confused and then asked “But I can write letters to two different people?”. This question reveals the importance of developing relationships among analogies and target concepts. Thereupon, the instructor posed questions as such “What is the domain in each case?”, Is it people or letters?, etc. If we consider the “writing a letter” analogy, then the function $f: A \rightarrow B$ is composed by f (writing) to an argument x (people) with an output (letters). This analogy could be given for not being a function because the univalence or single-valued requirement, that for each element in the domain there be only one element in the range, is not supplied in this analogy.

> I think about posting a letter example. Let’s take the action of taking the letter to the post office as f function and the letter to be posted as x. Different people’s letters may arrive to the same address. For example my siblings’ letters would arrive to my family’s address too. There occur two actions here. The first operation is “I take the letter to the post office.” And the second operation is “The postman takes the letter to my family.” We name the first action as f and the second action as g. The composite of the actions is g o f. In the end the arrival of the letter requires the composite of two actions. [PT5]

“Posting a letter” analogy could be an example for composite function provided that the functions $f: A \rightarrow B$ and $g: B \rightarrow C$ are composed by first applying f (posting a letter to the post office) to an argument x (letters) and then applying g (posting letters from the post office to their arrival points) to the result (letters at the post office). Thus $g \circ f$ is the arrival of the letters to their addresses. It must, however, be mentioned that every letter written must have been posted as for each x in A, there exists some y in B such that x is related to y. Otherwise, a binary relation could not be met.

## A Perforated Pail Analogy

“A Perforated Pail” analogy was constructed to remind identity function. When someone put something into the bore pail, it will fall dawn as it is. For all input, the output will be the same again. As seen below, [PT2] brought up some examples such as putting a pencil or shoe in the bore pail. She mentioned that the pail does not make any operation on the material. However, the size of the hole on the pail must be big enough for the materials to pass through. If it is not, then this could violate the total condition of being function. Furthermore, the hole on the pail should not give any damage to the material while passing through since identity function is a function that always returns the same things used as its argument. She, however, did not mention the breakdown point of this analogy.

> Think about a bore pail…. We put a pencil in it and then we get a pencil again. Or, we put a shoe in it and then we get the same shoe. The pail does not make any operations on them. You get what you put. Then what we called that function: The identity function. [PT2]

The identity function of f on A is defined to be that function with domain and range A which satisfies f(x) = x for all elements x in A. In the case of "Perforated Pail" analogy, while the function f: A→ A is composed by applying f (putting materials to the bore pail) to an argument x (materials) with an output f(x) (materials).

**Function Machine Analogy**

[PT2] used "Function Machine" Analogy to remind function concept and to introduce Composite function. First, she drew a function machine figure together with the explanation as such "You have a raw material named x *[began to draw Figure 1]* and you have a machine that gives output. You put x to this machine and this machine gives you the output as *f(x)"*.

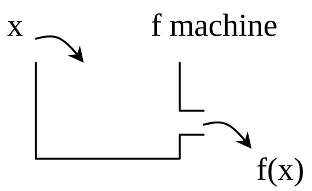



**Figure 1: Pictorial analogy for function concept**

To exemplify this further "Mixer" analogy - where banana and milk are input and the milkshake is output - was constructed. This, however, is not an appropriate analogy for functions of one variable. "Mixer" analogy can be an example of functions of several variables. When she was asked to make clear what the domain of the function mentioned in the analogy is, she could not make a connection to the function with two variables. One possible explanation for this inappropriate analogy is not considering the function as *mixer(milk, banana) = milkshake*. Further, the instructor expressed that "washing machine" analogy is appropriate for functions of one variable. In this analogy, inputs are dirty clothes, process is cleaning and the outputs are clean clothes.

While introducing the composite function, she first stated that "composite" is a kind of operation like addition and subtraction but operation with different rules. Taking into account the previous function machine figure, she extended the figure to be a pictorial analogy (see Figure 2) for composite function by pointing out that "In the f machine x turns out to be f(x) and then we put f(x) in the g machine. So we get (gof)(x) composite function".

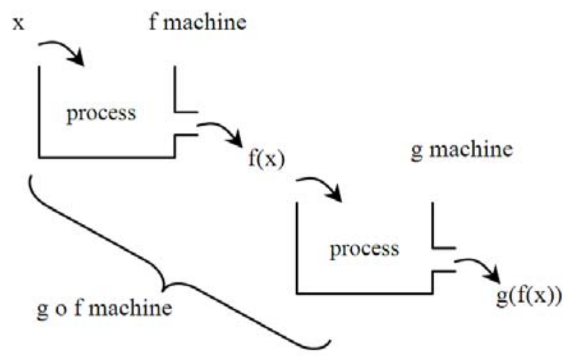



**Figure 2: Pictorial analogy for composite function**

However, the "washing machine" analogy that was given for functions could have been extended to composite functions. In the case of "washing machine" analogy the functions $f: A \rightarrow B$ and $g: B \rightarrow C$ can be composed by first applying $f$ (washing process in washing machine) to an argument $x$ (dirty clothes) and then applying $g$ (drying process in a dryer) to the result. Thus one obtains a function $g \circ f: A \rightarrow C$ defined by $(g \circ f)(x) = g(f(x))$ for all $x$.

## CONCLUSION

The present findings suggest that analogies need to be carefully thought out to be effective in order not to cause any confusion. The analogical models constructed by the pre-service teachers in the present study were analyzed in terms of whether the analogies constructed are epistemologically appropriate to illustrate the essence and the properties of the functions as well as the structural relations between the analogues and the targeted concepts. While mapping the analogies to the target concepts, the important things are the similarities as well as the break down points between them. The way the pre-service teachers used analogies could fall short of contributing to the students to develop epistemologically correct and conceptually rich knowledge of function due to two reasons. First, the source analogues were epistemologically inappropriate to illustrate the essence and the properties of the functions. Second, the analogies were epistemologically appropriate to illuminate the function concept, yet the teacher did not establish the mappings between the two.

In general they spontaneously followed the three steps: i) selecting an analogy (ii) mapping the analogy to the target (iii) evaluating the analogical inferences. Even the analogical models help students to visualize the newly learned symbols, concepts, and procedures, pre-service teachers need to know and show where the analogy breaks down and carefully negotiate the conceptual outcome. PTs should articulate the similarities and differences between the analogy and the target concept while they are presenting an analogy, and also should be aware of the limitations of the constructed analogy.

In the sense of these findings, it can be concluded that pre-service teachers' knowledge about the use of analogies were insufficient, and participants of the study were weak in transforming knowledge and developing sophisticated ideas in the process of teaching functions. In line with that, pre-service teachers did not consider too much on their analogical mappings and they were not able to construct the adequate relationships between the analogies and the target concepts along with the processes of mapping the analogical features onto target concept features. The difficulty appeared while developing sophisticated ideas in the process of teaching did not occur in giving mathematical definitions, rules, and procedures. For example, function was defined correctly as "f is a relation from set A to set B. If each element in set A correspond only one element in set B, then this relation is a function."

One of the limitations of the present study was that pre-service teachers were restricted to present function concept. May be if they were more flexible in the topic selection they would choose another mathematics topic in which they are more capable, thus they would generate more productive analogical models.

## IMPLICATIONS

In teacher preparation courses, student teachers should be asked to generate their own analogies in different contexts of mathematics. This kind of courses could provide them an opportunity to constitute an available repertoire of analogies (Thiele & Treagust, 1994) and to create analogy-enhanced teaching materials. In addition, this array of experiences could allow them to discuss, model, and justify their interpretations of the concepts and to provide different approaches to the teaching of the concepts. The analogies discussed here will help pre-service and in-service teachers develop a sound relational knowledge of the function concepts as well as consider carefully on their analogical mappings to construct epistemologically appropriate ones and to map the similarities and differences between the analogies and target concepts. Discussing the analogies reported here with pre-service and in-service teachers could deepen their understanding of function concept as well as functions pedagogy to offer perspectives on a sound generation of analogies.

In the light of the discussions of the teacher generated analogies, mathematics textbook authors and teachers can develop productive analogies for various mathematical concepts. Carefully crafted analogies can serve as initial mental models for the introduction and presentation of newly learned mathematical concepts.

As a result of this investigation, a further study was planned to describe the multiple analogical models used to introduce and teach grade 9 function concepts. We examine the pre-service teacher's reasons for using models, explain each model's development during the lessons, and analyze the understandings they derived from the models.

Teachers should engage their students in a discussion in which the limitations of the analogy are identified.

## REFERENCES


Cooney, T. J., & Wilson, M. R. (1993). Teachers' thinking about functions: Historical and research perspectives. In T. Romberg, E. Fennema, & T. Carpenter (Eds.), *Integrating research on the graphical representation of function*, (pp.131-158). Hillsdale, NJ: Lawrence Erlbaum.

Gentner, D. (1998). Analogy. In W. Bechtel & G. Graham (Eds.), *A companion to cognitive science* (pp. 107-113). Oxford: Blackwell.

Glynn, S., Duit, R., & Thiele, R. (1995). Teaching with analogies: A strategy for constructing knowledge. In S.M. Glynn & R. Duit (Eds.), *Learning science in the schools: Research reforming practice* (pp. 247–273). Mahwah, NJ: Erlbaum.

Harel, G., & Dubinsky, E. (1992). *The concept of function aspects of epistemology and pedagogy*. Washington, D.C.: Mathematical Association of America. 85–106.

Lloyd, G. M., & Wilson, M. (1998). Supporting innovation: The impact of a teacher’s conceptions of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education*, 29(3), 248-274.

Phillips, N., & Hardy, C. (2002). *Discourse Analysis: Investigating Processes of Social Construction*. United Kingdom: Sage Publications Inc.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57, 114-1 35.

Thiele, R. B. & Treagust, D. F. (1994). An interpretive examination of high school chemistry teachers’ analogical explanations. *Journal of Research in Science Teaching*, 31, 227-242.

# TECHNOLOGY AND MATHEMATICS TEACHING PRACTICES: ABOUT IN-SERVICE AND PRE-SERVICE TEACHERS

Maha ABBOUD-BLANCHARD

DIDIREM, Research team in the didactics of mathematics, University Paris Diderot

*Abstract: This article examines the practices of in-service and pre-service teachers in technology based lessons by exploring three dimensions: the students' tasks, the students groups' management and the discourse of the teacher. Regularities emerging from two case studies about in-service teachers are compared to results of a larger study about pre-service teachers. The article shows that what characterize teachers' practices in technology environments is not the same in the two populations of teachers and thus suggests some propositions for the design of training strategies seeking to improve the practices of novice teachers.*

Key-words: technology, teaching practices, ordinary teachers, pre-service teachers

## INTRODUCTION

For the last decade constraints and difficulties encountered by mathematics teachers integrating technologies has been an ongoing issue. Indeed the contrast between the technological development and the weakness of the integration of computer technologies in classrooms despite the abundance of governmental funding, questions necessarily researchers (Artigue, 2000), (Ruthven, 2007). Some researches have considered the role of teachers in the classroom use of technology throughout a holistic approach examining thus the influence of key factors on their activity (Monaghan, 2004); others have investigated teachers' ideas about their own experience of successful classroom use of computer-based tools and resources (Ruthven & Hennessy, 2002); others have shown discrepancies and variability in the ways teachers use technology in their mathematics classrooms (Kendal & Stacey, 2002). Research about student teachers' practices and their determinants in technological environment is nevertheless rather rare. It stresses particularly the problems that student teachers have to overcome such as their lack of familiarity and confidence with technology or their need to make explicit the connections between technological and paper-and-pencil work (O'Reilly, 2006). Furthermore, it stresses the growing awareness that technology-based lessons require extra time for planning and for teaching.

In this paper, I want to contribute to the research on issues related to teaching practices in technology environments and issues related to teacher education in these environments. I will do so by relying on the results of three research projects that I have carried out in the last four years. I will firstly present two case studies about teachers' practices in technology-based lessons taken from the two first researches. Secondly, I will highlight regularities that emerge from these studies. I will finally try to cross these findings with results of a third research about pre-service mathematics teachers using computer technologies and conclude by issues about teacher education arising from this synthetic view on the three studies.

## CASE-STUDIES

The two case studies that I am presenting here have been carried out in two researches: the first one on the characterization of the practices of 'ordinary' teachers using dynamic geometry (Abboud-Blanchard, 2008); the second one on the analysis of the activity of volunteer teachers using exercise-bases (Artigue *et al.*, 2006). In these two studies the main issue is to characterize teacher's activity in a technology-based lesson according to three polarities in complete interaction: the tasks proposed for students' learning (cognitive pole), the management of the students' groups (pragmatic pole), and the discourse and the interaction with students (relational pole).

### Framework and Method

These studies use methods and concepts developed within the general framework of the two-fold approach which combines both a didactical and an ergonomical perspective in analysing the factors that determine the teacher's activity as well as that of students prompted by the teacher in class (Robert & Rogalski, 2005). Within this framework, analyzing lessons takes into consideration the fact that there are two main types of channels used by the teacher in classroom management: the organization of tasks prescribed to students (cognitive-epistemological dimension) and the direct interactions through verbal communication (mediative-interactive dimension). Furthermore, the authors (*ibid*) differentiate task from activity: task is what is to be carried out; activity is what a person develops when realising the task.

For each of the two case-studies I will first report on the *a priori* analysis of the students' tasks and what they are supposed to undertake in terms of initiative and use of knowledge already acquired and actually needed to execute the tasks. Secondly, I will present the lesson in progress, that is to say, what really happened in the classroom by underlining the teacher's aids and by studying the features of his/her discourse. The teacher intervenes often to provide assistance to the students sometimes modifying their activities. Robert (2007) defined two types of aids, depending on whether they modify the activities scheduled or they add something to the students' action. The first, "procedural help", deals with the prescribed tasks by modifying activities with regard to those planned from the presentation of the task. It corresponds to indications that the teacher supplies to the students before or during their work. The second, "constructive help", adds something between the strict activity of the student and the (expected) construction of the knowledge that could result from this activity. The analysis of the teacher's discourse provides more information about how he/she contributes to model students' activities. This analysis has been undertaken by using a methodology constructed by Paries (2004) who adapted tools used in psychology, notably the functions of scaffolding defined by Bruner (1983) who regarded interaction as the major form of assistance provided by adults for cognitive development. Thus, Paries studied the role of discourse in the mediation of cognitive development and defined functions of the mathematics teacher's discourse by specifying the manner in which he/she intervenes gradually in details in the students' work. Paries distinguishes two groups of functions:

- The “cognitive functions” linked to the task, to the realisation of the task and to the mathematical content. These functions are: introduction of a task or dividing a task into sub-tasks, assessment, justification and structuring.

- The “functions of enrolment” apparently independent from the task, at least in their formulation, but can have an impact on its realisation. They allow the teacher to maintain communication. These functions are: engagement, mobilization of the student's attention and encouragement.

**William's case**

William has volunteered to participate in a government project to use exercise-bases with his grade 10 students (first year of the upper secondary level - aged 15/16 years). I chose to present here this study because William's case could be considered as representative of those of the other teachers engaged in the project.

He is a regular user of technology in both his personal and professional activities. He sees the use of exercises-bases in the classroom as facilitator without neither change in the approach of mathematics contents nor change in teaching practices: this software is just an additional mean that will be added to (and not replace) usual practices.

*Students' tasks*

The observed session is a training one and it took place in the computer room; students were assigned by groups of two to a computer. William's discourse was recorded; a remote cordless microphone was attached to the teacher. An observer was present in the classroom. William has chosen in the exercise-base a module of exercises-generator concerning two tasks: (1) To find the reduced equation of a straight line. The straight line is drawn on the screen with two of its points A and B in an orthonormal cartesian system; students have to find the values of $m$ and $p$ in the equation $y = mx + p$. (2) To solve systems of two linear equations (first degree equations with two unknowns)

In both cases, students must make their calculations on paper and give the two numbers solutions to the software that validates them in terms of true / false.

These two tasks are similar to paper-and-pencil tasks; the only difference is that each student can train at his/her own pace.

*The development of the lesson and the teacher's help*

During the lesson William tries to check up the work of every group of students with some regularity, and even when he moves at the request of a student, he quickly control the work of other students along his path. Despite this, students put a lot more time than what William had planned (half an hour per task). This gap between the planned and actual time resolution leads William to ask students to move to the second set of exercises, although a few students have still some difficulties with the first exercises.

Among the interactions with students, I note only four collective ones which concern particularly the management of the session; the rest are individual (per group to a computer) interactions. Some aids are related to the handling of the software: they consist primarily to explain how to switch from one exercise to another or to resolve a technical problem. They are usually brief, local, and allow the student to continue the resolution. The individual help concern mostly mathematical resolution; they are of various kinds and are often procedural help: - controlling the resolution and calculations; - validating an answer or helping find the error (often at the request of students); - structuring the resolution or asking students to do it.

The frequency and variety of these mathematics aids show that the execution of the mathematical tasks seems to require a strong mobilization of the teacher.

To sum up, I notice that William, who is at ease in a technology environment, succeeds in providing students effective aids for handling computers and exercise-bases software. The class gives the impression of "functioning" in a satisfactory manner, all students work and progress. Nevertheless, the teacher is highly mobilized on the mathematical level; the majority of students cannot progress in the resolution without his help. So, despite an "illusion" of autonomy of students, the presence of the teacher seems indispensable.

*The functions of discourse*

I will not detail here the study of the teacher's discourse, because of the restricted length of this paper; I will rather give some significant percentages of the functions of discourse. I note first a small percentage (9%) of the functions of enrolment. Everything indicates that students are "supported" by the technology environment and work without needing to be constantly motivated by the teacher. The function of structuring occupies 21% of the total, because when helping students, William first begins by helping them bring "order" in their calculations. This is also due to the desire that students work more quickly because the time doesn't progress as William has planned (see above). The function of assessment occupies a high percentage (47%) because the software provides validation only in terms of true/false for the solution given by the student. The students are therefore responsible for the control of calculations but they seek constantly the teacher's help for this assessment. This requires the teacher to take over the function of accompanying the resolution and control of progress, and interpretation of the results not validated.

In addition to these results on the functions of discourse, I note that the functions succeed in a similar order with each group of students. Indeed when the teacher comes to see a group: he assesses or takes a stock of the situation of resolution, sometimes he structures it, and then he gives a sub-task to the students to execute until he comes back. This phenomenon of repeating the same succession of action in each group with aid substantially similar implies a strong mobilization of the teacher which is 'non-economic' in terms of classroom management.

**Anna's case**

Anna is an 'ordinary' teacher not engaged in any innovation or research project. She has an episodic use of technological tools with her students that one wouldn't qualify as significant use. I present here her case because she corresponds to what we, in the research project, consider to be an average teacher representative of ordinary teachers. The lesson studied here is about space geometry in a grade 9 class (fourth year of middle school - aged 14/15 years). It takes place in the computer room with the use of dynamic geometry software; students are assigned by groups of two or three to a computer. The lesson observation was videotaped. The camera was at a rear corner of the classroom. A remote cordless microphone was attached to the teacher. No observer was present in the classroom. The topic is the section of a pyramid by a plane parallel to the basis, and Anna uses a ready-to-use session designed by the software developers.

*Students' tasks*

The figure downloaded by the students is a given cube ABCDEFGH in which they have drawn in a previous session: I, middle of [EF] and J, middle of [AB] and have also found the lengths JC and JD. First, the students have to draw the section of the pyramid IJCD by a plane passing by M, the middle of [IJ], and parallel to the basis JCD, getting thus two points N (middle of [IC]) and Q (middle of [ID]). This technological-task (t-task) is entirely guided by a set of manipulation commands and students only need to follow the instructions given in the worksheet provided by Anna. Secondly, they have to examine, with the software commands, the triangles JCD and MNQ. The aim here is that students get to see MNQ as the 1/2 reduction of JCD. Once done, tasks that follow are mathematical-tasks (m-tasks): to calculate the areas of triangles MNQ and JCD, to calculate the volume of IMNQ and IJCD to compare these two volumes. These m-tasks are complex and require a certain number of adjustments such as taking initiatives (to construct a height in a triangle in order to calculate its area) or operating a change of frames (when comparing the two volumes) that consists in introducing the comparison of two numbers in a geometrical frame. Therefore, t-tasks are designed to be simple, guided and quickly executed in order to get a stronger focus from the students on m-tasks. The latter are more complex and require time to be carried out.

*The development of the lesson and the teacher's help*

Globally, I note that students are often in an autonomy-mode and for very long moments. When she is present, Anna divides the task into sub-tasks to be immediately executed by students, in a bid to allow them to pursue quickly their work. The teacher's collective interactions are rare and mostly concern the management of the session.

The assistance of the teacher consists almost exclusively in procedural help, simplifying the students' activities. The division of tasks into simple sub-tasks is clear: sometimes Anna nearly dictates the work to do and at times she even takes herself the mouse to accomplish some sub-tasks. Often, when the teacher is interacting with a group, students only follow her instructions, or even finish a sentence that she begins. I might here underline that the teacher stays with every group a very short time and thus her assistance allows the students to pursue their work on their own. One can wonder if dividing the task is some how a way for Anna to be efficient. Still, Anna did not succeed to meet her objective; students were too slow in the construction of the section of the pyramid. She had prepared simple t-tasks in order to help the students to start quickly the mathematical activity. Perceiving during the lesson that these tasks took more of time than expected, she tried to accelerate their execution by doing the work herself or by coaching students step by step in the execution.

*The functions of discourse*

As in William's case I only give here some significant percentages of the functions of discourse. I first observe that the functions of enrolment have a low percentage (7%) which might be explained by the fact that the mobilization of the students' attention and the engagement in tasks is supported by the technology-environment itself. I notice also that structuring accounts for an important rate among cognitive functions (28%). As stated above, Anna is aware of the slow execution of the tasks and tries, by this mean, to accelerate the pace. As for the cognitive function of the introduction of sub-tasks, the high percentage (21%) is coherent with the analysis of the m-tasks. These tasks are complex, need adjustments, and on top of that, students' work progresses slowly. Assessment stands at 35% and corresponds to interactions with groups of students and not to collective interactions. Actually, after the start (collective phase), the class splits into several 'mini-classes' (groups of two or three students per computer) which function separately and to which the teacher talks independently from the remainder of the class. Besides, certain functions of the discourse apparently succeeded in these 'mini-classes' in this same order: assessment, structuring and introduction of a sub-task.

## Regularities emerging from the two case-studies

Despite of the different contexts and profiles of the two teachers and also the different nature of the software used, a number of regularities emerge from the two studies, I want to emphasize these in this section. I will do so in order to highlight what actually is characteristic of a technology-based lesson led by in-service teachers. I will also illustrate continuities between these findings and those of some researches mentioned above, to suggest that a number of results may be more widely transferable.

On the cognitive level, in the two cases the exercises chosen by the teachers, in technology environment, are similar to the ones that would be proposed in pencil-and-paper environment; the resolution of mathematics tasks is identical to what could be proposed in non-technology environment. This result is close to what Kendal and Stacey (*ibid*) underline about CAS (Computer Algebra Systems).the mathematical knowledge and skills stay globally within the range of those expected in non-technological environment. Indeed, the teacher has, on the cognitive level, a practically similar activity as in a non-technology environment. In the open environment of dynamic geometry we see that Anna has chosen a ready-to-use sequence where all the questions of the exercise except one, are feasible in a pencil-and-paper environment. In the environment of exercise-bases, William has also chosen training exercises used in pencil-and-paper environment. The content of the interventions of the two teachers when it comes to mathematical tasks is therefore identical to what they would have said or done in non-technology environments since there is no reference to the specificity of technology environment in these interventions. This can be traced to some indications provided by Ruthven and Hennessy (2002) about teachers who initially view technology through the lens of their established practice, and employ it accordingly. This fact certainly favours the connection of these sessions with the rest of learning process and helps to explain why for these teachers this connection is not perceived as problematic.

On the pragmatic and relational levels, firstly I note that the work in computer room generally entails that students must be in groups of two or three per machine. Consequently, there is a 'class split' in several 'mini-classes' working relatively independently, and a quasi disappearance of collective phases except the collective time management. The teacher is not able, in certain cases, to generalize the supply of certain indications given only to some students whereas they could be useful to all the others. Artigue *et al.* (ibid) encountered the same features notably the fact that individual interactions substitute for collective interactions and that institutionalisation phases are nonexistent because of the different 'trajectories' of students. Besides it, for each of the mini-classes, the teacher adapts to what students are doing and to their current reasoning, whereas in pencil-and-paper lessons, it is more often that the students have to adjust themselves to the teacher's project (Abboud-Blanchard & Paries, 2008). This appears to be an important element of the management of a technology-based lesson which differentiates it from a non-technology one. Moreover, the analysis of the interactions showed similarities in the successions of the functions of the discourse among the mini-classes. Secondly, as to the aid provided to students, I observe that the teacher focuses on local mathematical aid without decontextualization. There is a clear majority of cognitive functions of the discourse that operate as help, but mainly procedural help. This type of support is partly motivated by the teacher's concern about the progress of the students' work, in order to have all the tasks prepared for the session completed. This echoes a strong trend of teaching practices in the computer room underscored by several researches (Monaghan, 2004). Other characteristics seem to be related to specificities of the environment and enhance the previous difficulties. Indeed, not all the students handle the software with ease, thus the teacher has to provide technical help which is not common in a mathematics course. Thirdly, in individual interventions that predominate, the rate of interventions of enrolment is much weaker than what is generally observed in non-technology class sessions (Paries, 2004). The functions of enrolment are rarely present in the discourse of the teacher; they seem to be taken in charge by the software. The teacher has also to 'share' with the computer certain functions of enrolment, which disturbs the usual management of the class.

Thus, the teacher's role in technology based-lessons seems to be essential according to the pragmatic and the relational poles. Indeed in the two case studies students' tasks were enough guided, one could *a priori* expect to see the teachers a bit observers ( rather than actors) of their students' learning. The analysis shows that this is not the case; teachers are very present and very engaged in the students' work.

## ISSUES ABOUT TEACHER EDUCATION

As member of a research team investigating the uses of technology by pre-service teachers, I studied the professional dissertations made by these teachers in which they report about technology-based lessons that they prepared and carried out in their classes (Abboud-Blanchard & Lagrange, 2006). The data come then only from what the teachers themselves reported and not from class observations.

The main result that I want to highlight in this paper is the focus of these dissertations on the preparation of students' mathematical tasks, while the teacher's activity is overlooked. Aspects of the teacher's role are very rarely questioned; they are rather mentioned as “events” in the body of the reports and in the conclusions. Indeed, the learning activities are often document-based, students being assigned tasks based on a written document that teachers deliver at the beginning of the session. In such classroom documents, tasks are organised as a series of subject-based questions, with instructions on how to use the software. Furthermore, in the development of lessons reported in the dissertations, it seems that the teacher has a marginal role in the technology-based lessons carried out and reported by pre-service teachers. For example, at the beginning of a typical lesson, the pre-service teacher provides guidance to the students on manipulating the software and makes sure that they understand the assignment. Then the students work on their own in the computer room and the teacher’s activity is limited to individual help to manipulate the software. My hypothesis is that the teacher’s marginal intervention can be explained – at least partially - by the prescriptive nature of the tasks. Another reason may be that pre-service teachers transfer part of their role to the computer, a kind of ‘joint partnership’.

### Comparing results about pre-service and in-service teachers

My aim in this section isn't to make a detailed comparison of the two first case studies and the study of pre-service teachers. A direct comparison wouldn't be relevant notably because of the differences of the methodologies used. I'm rather presenting here a synthetic approach of the three studies focusing on the results relative to the three poles developed above: cognitive, pragmatic and relational.

In the studies on the activity of in-service teachers I showed that the cognitive pole isn't what seems to be problematic for these teachers in technology-based lessons. What differentiate the teacher's activity in these lessons with the same in non-technology ones are mainly the management of students (pragmatic pole) and the interactions with students (relational pole). Thus what makes a technology-based lesson 'works' with experienced teachers seems likely more related to the pragmatic and relational poles than to the cognitive one. Whereas the study of the practices of pre-service teachers shows on the one hand that they focus on the cognitive pole and they neglect the two other poles, and on the other hand that they report their non satisfaction of how technology-based lessons took place. Moreover, when we ask pre-service teachers about their experiences of technology-based lessons they most frequently reflect on difficulties related to time management of the session and also to preparation work to set up the tasks of students. They also underline that the teacher is no longer the only holder of knowledge. However such reflections tend to remain at a general level and do not seem to provoke pre-service teachers into making propositions for a more suitable integration of technologies in mathematics teaching. This also reveals that despite of their increasing awareness of the specificity of technology environments in preparation work and class work; it does not necessarily lead to a wider reflection about real integration of technology in their practices.

Can we take advantage of this awareness to develop an approach of teacher education programs? During discussions within the WG12 of CERME 5 (Carillo *et al.*, 2007) it seems that there was a consensus among participants on the fact that awareness is necessary for reflection and on promoting reflection as a means of professional development. Seeking to improve the practices of novice teachers, this last pattern can be used for the design of training strategies such as the analysis of video episodes of experienced teachers using technologies with a special focus on the role of the teacher and his/her interactions with the students. Such analysis would help pre-service teachers to bridge between a focus on the preparation of students' mathematical tasks and another on their own activity during the lesson in order to help them overcome the state of didactic tinkering and go further to a successful integration of technologies in mathematics teaching and learning.

## REFERENCES

Abboud-Blanchard, M. & Paries, M. (2008). Etude de l'activité de l'enseignant dans une séance de géométrie dynamique au collège. In F. Vandebrouck (ed.), *La classe de mathématiques: activités des élèves et pratiques des enseignants* (pp. 261-291). Toulouse: Ed. Octarès.

Abboud-Blanchard, M. (2008). How mathematics teachers handle lessons in technology environments. Proceedings of the *5th Nordic Conference on Research in Mathematics Education*. Copenhagen, Denmark.

Abboud-Blanchard M. & Lagrange, J-B. (2006). Uses of ICT by pre-service teachers: towards a professional instrumentation? *The International Journal for Technology in Mathematics Education, volume 13, issue4,* 183-191.

Artigue, M. (dir), Abboud-Blanchard, M., Cazes, C. & Vandebrouck, F. (2006). *Suivi de l'expérimentation de la région Ile de France: ressources en ligne pour l'enseignement des mathématiques.* Rapport final. IREM de Paris 7.

Artigue, M. (2000). Instrumentation issues and the integration of computer technologies into secondary mathematics teaching. Developments in mathematics education in German-speaking Countries: *Selected Papers from the Annual Conference on Didactics of Mathematics*, Potsdam, 2000. Available at: http://webdoc.sub.gwdg.de/ebook/e/gdm/2000/ (accessed July 2008).

Bruner, J. (1983). *Le développement de l'enfant: savoir faire, savoir dire*. Paris: Presses Universitaires de France.

Carillo, J., Santos, L., Bills, L. & Marchive, A. (2007). Introduction to Working Groupe 12. Proceedings of the *Fifth Congress of the European Society for Research in Mathematics Education*. (CD-ROM) (pp. 1821-1826). Cyprus: University of Cyprus.

Kendal, M. & Stacey, K. (2002). Teachers in transition: Moving towards CSA-supported classrooms, *ZDM,* 34(5), 196-203.

Monaghan, J. (2004). Teachers' activities in technology-based mathematics lessons. *International Journal of Computers for Mathematical Learning,* 9, 327-357.

O'Reilly, D. (2006). Learning together: student teachers, children and graphics calculators. *The International Journal for Technology in Mathematics Education, volume 13(4),* 191-204.

Paries, M. (2004). Comparaison de pratiques d'enseignants de mathématiques-Relations entre discours des professeurs et activités potentielles des élèves, *Recherches en didactique des mathématiques,* 24, (2.3).

Robert, A. & Rogalski, J. (2005) A cross analysis of the mathematics teacher's activity. An example in a French 10$^{th}$ grade class, *Educational Studies in Mathematics, 59*, 269-298.

Robert, A. (2007). Contributions dans la partie 0, partie 1 et conclusion. In F. Vandebrouck (ed.), *La classe de mathématiques: activités des élèves et pratiques des* enseignants (pp. 9- 68). Toulouse: Ed. Octarès.

Ruthven, K. & Hennessy, S. (2002). A practitioner model of the use of computer-based tools and resources to support mathematics teaching and learning class, *Educational Studies in Mathematics, 49,* 47-88.

Ruthven, K. (2007). Teachers, technologies and the structures of schooling. Proceedings of the *Fifth Congress of the European Society for Research in Mathematics Education*. (CD-ROM) (pp. 52-67). Cyprus: University of Cyprus.

# TEACHERS AND TRIANGLES

Silvia Alatorre and Mariana Sáiz

National Pedagogical University, Mexico City

*During a workshop about triangles designed for in- and pre-service basic-school teachers, a diagnostic test was applied. The results are analysed in terms of several variables: the teachers' sex, the level at which they work, their occupation (namely, in- or pre-service teachers), and their professional experience. An important impact of the latter was found in the decrease of incorrect answers obtained.*

## FRAMEWORK

Shulman (1986) characterised the types of knowledge that he considered enabled teachers to carry out their practice. He proposed three categories: mathematical content knowledge (MCK), curriculum knowledge (CK), and pedagogical content knowledge (PCK).

There have been several discussions about Shulman's categories. We want to mention two in particular. The first one is a discussion both about the exact meaning of MCK. Some researchers stress that within MCK there is a difference between the knowledge of the formal academical discipline and the scholar subject (see e.g. Bromme, 1994). The former is the knowledge that professional mathematicians develop, and the latter is the mathematics that teachers must teach.

The second discussion is about how much MCK is a valid variable in understanding teachers' practices and designing teachers' education. There has been a variety of researches that show that "teachers' mathematics knowledge is generally problematic in terms of what teachers know, and how they hold this knowledge of mathematics concepts or processes, including fundamental concepts from the school mathematics curriculum. They do not always possess a deep, broad, and thorough understanding of the content they are to teach" (da Ponte & Chapman, 2006, p. 484). According to some authors, these researches are important because of several reasons. On the one hand, they allow to understand how "elementary teachers' understanding of subject matter influences presentation and formulation as well as the instructional representations that the teacher uses" (Sánchez & llinares, 1992, quoted by da Ponte & Chapman, 2006, p. 434). On the second hand, they have prompted "studies centred on describing student teachers' beliefs and knowledge as determining factors in their learning processes [... and have also] provided information used to prepare research-based material for use in teacher education and to develop research-based teacher education programmes." (Llinares and Krainer, 2006, p. 430). On the other extreme, some authors question MCK's importance, because "the academic mathematical knowledge may not be 'naturally' a helpful instrument for the teacher in the school practice, since some of its values and forms of conceptualizing objects conflict with the demands of that practice". (Moreira & David, 2007, p. 38). They stress that to help students to think mathematically, teachers need to understand student thinking, and thus the comprehension about the cognitive processes of the students becomes more important than MCK itself

While we are aware that many variables may qualify the importance of MCK, such as teachers' beliefs and practices, the cognitive processes of students, etc., we sustain that teachers should at least have a solid understanding of the contents they must teach. This does not always happen in Mexico, and in order to explain why, we must make a brief exposition of the Mexican situation about teacher training. Teachers receive their training not in universities but in "Escuelas Normales", which they attend after 6+3+3 years of regular schooling. There are "Escuelas Normales" (ENP) for student teachers who will become Primary school teachers (i.e, grades 1-6), and other "Escuelas Normales" (ENS) for those who will teach at the Secondary level (grades 7-9). At the ENP it is taken for granted that during those 12 years of previous schooling they have learnt all the mathematics they will ever need to teach, and that all they need to know about teaching mathematics is PCK; at the ENS student teachers have some courses focused on MCK. (Another situation in Mexico is the fact that there is not an assessment or a diagnostic about teachers' MCK with results widely spread). Thus, if teachers enter the ENP with misconceptions or deficiencies, these are not solved there, and the dragging of misconceptions and deficiencies becomes, through teachers' practice, a vicious circle. One of the well-known consequences of this process is that Mexico is always among the countries that obtain the lowest results in international assessments of students' performance, like PISA and TIMSS.

While other countries do not share the extremely low results in PISA and TIMSS, teachers' misconceptions and deficiencies are not exclusive of ours. For example, Hershkowitz & Vinner (1984, quoted in da Ponte & Chapman) investigated the processes of concept formation in children, through the comparison of students' learning and elementary teachers' knowledge of the same concepts; they found that one of the factors that affects the students' learning is the teachers' conceptions.

With respect to MCK, Llinares and Krainer (2006) acknowledge the importance of detecting student teachers' misconceptions but propose that it be done within the frame of student teacher's learning. They suggest that it is important to study the relationship between student teachers' conceptual and procedural knowledge, and for this teachers should know about children's mathematical thinking. One method they propose for the study of the mentioned relationship is the use of open-ended questions based on vignettes describing hypothetical classroom situations where students propose alternative solutions to some mathematical problems. This kind of tasks have also been used by Empson & Junk (2004), who suggest that some of the teachers' answers are influenced by a disconnection between teachers' MCK and their understanding of children's thought, with the consequence that they precipitate to correct mistakes without establishing a contact with what the student is thinking.

Presently, there is not a unified theoretical perspective on the researches about MCK and its relation to teachers' training and professional development. It has been suggested that "future work should include a focus on understanding the knowledge the teachers hold in terms of their sense making and in relation to practice [… and that there is a] need to pursue the theorization of teachers' mathematical knowledge, framing appropriate concepts to describe its features and processes, and to establish clear criteria of levels of proficiency of mathematics teachers and instruments to assess it." (da Ponte & Chapman, 2006, p. 467).

The work we are presenting here fits da Ponte and Chapman (2006) and Llinares and Krainer (2006) characterisations, a difference with the last ones being that we investigate not only pre-service teachers but in-service teachers as well. Our principal goal is to study in- and pre-service teachers' mathematical content knowledge, but not in an isolated manner. As other researchers (see for example Prestage and Perks, 2001), we are also interested in understanding how teachers obtain, maintain and organise their mathematical content knowledge. It is worth mentioning that we are aware that mathematical content knowledge should not be separated from the other two kinds of knowledge. With this in mind, we designed some workshops that will be described below.

## METHODOLOGY

### TAMBA: Workshops on Basic Mathematics for in- and pre-service teachers

Within a broader project that combines research with professional development, we designed a set of workshops called TAMBA (Talleres de Matemáticas Básicas). The workshops are offered as modules that can work independently or as a set. Each one is centred on one specific mathematical content linked to the elementary school curriculum in mathematics. They all have a duration of 2-4 hours, and a common structure: they start with a short paper-and-pencil diagnosis, which is immediately commented with the participants, followed by an activity designed to raise a cognitive conflict, which takes most of the workshop's time. After it, several issues are discussed in the group: the mathematical topics and the pedagogical difficulties, including the children's most frequent misconceptions. The workshops are video taped. The design of both the diagnosis and the activity is based on our previous knowledge of the population to which each workshop is directed, and on the specialised literature.

### Geometry in TAMBA

One of TAMBA's workshops is called "coloured triangles". After the diagnosis, which will be described below, the activity is centred on the unicity of the triangle's area whatever the side used as "base" (this topic follows from the item 3 of the diagnosis). Depending on the teachers' cognitive level on the subject, a demonstration is presented, and then the MCK and PCK issues of item 3 are discussed in the group.

The diagnostic evaluation has three items. In Item 1, four sets of three measures are given, and the participants are asked to say if a triangle can be built with them and, if not, why (two are possible and in the remaining two the triangle inequality is not accomplished). In Item 2, three triangles are given with measures for the sides and heights, and the participants are asked to say if the measures are possible or not, and why (two of the figures are not possible, because some heights are larger than a side from the same vertex). In Item 3, a hypothetical conversation between three girls who must calculate a triangle’s area is presented, where they all make different mistakes and do not agree on the calculation, and the teacher is asked to write what s/he would say to the girls.

The teachers’ answers to the written evaluation were analysed and classified according to their correctness and the kind of geometrical criteria used. The results, focused from a geometrical point of view, are being presented elsewhere. Here only the broad categories are briefly described. Teachers’ ideas were classified as correct or incorrect; in the second case, several misconceptions were identified: about the triangle inequality, the base and/or height, the Pythagorean theorem, or other geometrical misconceptions. Within each of these broad categories, some finer subcategories were identified. In addition, the amount of items answered by each of the participants was registered, as well as the amount of ideas that s/he expressed clearly.

## Implementation

The described workshop has been given twice. In 2007 it was offered to 36 teachers at the Conference of the Mexican Mathematical Society in the city of Monterrey (MR), and in 2008 it was offered to 31 teachers in a Teachers’ Centre in Mexico City (MC). Table 1 summarises the characteristics of the participants in both workshops:

| | SEX | | | LEVEL | | | OCCUPATION | | | | EXPERIENCE | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | F | M | N/A | Primary | Secondary | N/A | In-service | Pre-service | Other* | N/A | n | Mean ± SD in years | N/A |
| MR | 22 | 9 | 5 | 20 | 9 | 7 | 16 | 7 | 4 | 9 | 22 | 17.9 ± 10.6 | 0 |
| MC | 29 | 2 | 0 | 20 | 3 | 8 | 5 | 24 | 1 | 1 | 7 | 17.7 ± 9.6 | 7 |
| Total | 51 | 11 | 5 | 40 | 12 | 15 | 21 | 31 | 5 | 10 | 29 | 17.9±10.2 | 7 |

* “Other” occupations are pedagogical consultants (PC) and experts in Special-Education Teachers (SET).

**Table 1**

The main difference between both groups is that there were more in-service teachers in Monterrey and more pre-service ones in Mexico City. In addition, all of the pre-service teachers in Monterrey were of the secondary level, whereas in Mexico City 15 of the pre-service were of the primary level and 2 of the secondary level (7 more did not answer that question). Another difference is that in Monterrey the participant teachers were highly interested in Mathematics Education, and had applied for and obtained funding to participate in the Conference (which was given for teachers with high scores in a national assessment), whereas in Mexico City the participants were regular attendants to a Teachers Centre located in a low-income zone.

## RESULTS

For each participant, the percentage of items answered was calculated, as well as the percentage of those that had clear arguments. Then the total amount of ideas expressed was figured, each idea was classified according to one or several of the categories above mentioned, and the quantity thus obtained for each participant in each category was expressed as a percentage of the total amount of ideas expressed. Finally, for each category averages were calculated taking all of the participants (see Table 2) or diverse groups of them.

| | Items answered | With argument | Correct ideas | Incorrect ideas | Misconceptions | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Triangle inequality | Base | Height | Pythagorean th. | Other |
| All participants | 80.0% | 71.6% | 27.8% | 62.0% | 15.0% | 6.5% | 8.6% | 5.5% | 8.1% |

**Table 2 [1]**

As Table 2 shows, the average participants answered most of the items, and, when they did, mostly expressed their ideas with clear arguments. However, only a small percentage of these ideas were correct. Among the misconceptions, those about the triangle inequality were the most frequent.

In the following sections, these results will be analysed according to the recorded experimental variables: venue, sex, level, occupation, and teaching experience. Each time the arithmetic means are reported and analysed, although no statistical inferential analysis is carried out, the samples being neither representative nor large enough.

### Venue

The 36 participants of the workshop held in Monterrey (MR) and the 31 of Mexico City (MC) differed in all of the variables considered. Table 3 shows the results obtained by teachers in both venues.

| | Items answered | With argument | Correct ideas | Incorrect ideas | Misconceptions | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Triangle inequality | Base | Height | Pythago-rean th. | Other |
| MR | 91.9% | 77.3% | 45.2% | 42.2% | 9.2% | 6.0% | 5.8% | 8.4% | 2.7% |
| MC | 66.2% | 64.9% | 7.6% | 85.1% | 21.8% | 7.0% | 12.0% | 2.2% | 14.3% |

**Table 3**

The teachers in MR obtained better results from all points of view: they answered more items, and expressed better their reasoning (more answers with argument). They had six times as many correct ideas and about half of the incorrect ideas expressed by their counterparts in MC; also, MR teachers had fewer responses classified in all but one of the different detected misconceptions. The largest differences were in the misconceptions about the triangle inequality, where MC teachers more than doubled their MR counterparts, and "other" geometrical misconceptions, where MC teachers made five times as many mistakes as MR participants. The one exception is the incorrect uses of the Pythagorean theorem, where MR teachers had in average 8.4% answers as opposed to only 2.2% of MC teachers. All this, as will be shown later, is related to the different characteristics of the participants in both venues.

## Gender

There were also differences among the 62 teachers who reported their sex: In general, the 11 male respondents had better results than the 51 female participants did. Table 4 shows this.

| | Items answered | With argument | Correct ideas | Incorrect ideas | Misconceptions | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Triangle inequality | Base | Height | Pythago-rean th. | Other |
| F | 77.6% | 66.5% | 8.4% | 84.1% | 21.4% | 7.8% | 11.8% | 2.2% | 14.8% |
| M | 86.9% | 75.0% | 41.7% | 46.7% | 10.4% | 5.0% | 5.9% | 9.5% | 2.5% |

**Table 4**

The male teachers answered more questions in average than the female, and were slightly better in expressing their reasoning. Men had more of the correct ideas and fewer incorrect ones, and scored lower in all of the misconceptions, again with the exception of misuses of the Pythagorean theorem. This apparent gender effect will be commented later on.

## Level

Only 52 of the 67 participants declared in which level they work or study. Their results are shown in Table 5.

| | Items answered | With argument | Correct ideas | Incorrect ideas | Misconceptions | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Triangle inequality | Base | Height | Pythago-rean th. | Other |
| P | 78.1% | 74.6% | 25.3% | 63.8% | 20.2% | 5.5% | 9.4% | 6.9% | 7.3% |
| S | 86.2% | 77.4% | 40.1% | 52.7% | 11.6% | 8.1% | 5.0% | 8.0% | 4.6% |

**Table 5**

Generally speaking, the 12 teachers of the Secondary level had results that were only slightly better than those of the 40 of the Primary level: more items answered as an average, more responses with argument, more correct ideas, and fewer incorrect ones. However, it is noticeable that the distribution of misconceptions found is not homogenous: Secondary level teachers have fewer answers with misconceptions about the triangle inequality, the height and other errors, but have more answers with misconceptions about the triangle's base and the Pythagorean theorem.

## Occupation

Of the 67 participants, 57 declared if they were in-service teachers (21), pre-service teachers (31), or if they had other occupation (5 were PC or SET). Table 6 shows the results for the first two categories.

| | Items answered | With argument | Correct ideas | Incorrect ideas | Misconceptions | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Triangle inequality | Base | Height | Pythago-rean th. | Other |
| In- | 86.9% | 78.8% | 31.5% | 58.9% | 15.4% | 7.2% | 10.1% | 13.1% | 1.5% |
| Pre- | 68.3% | 60.9% | 17.1% | 72.8% | 17.9% | 5.0% | 5.7% | 3.1% | 12.1% |

**Table 6**

In-service teachers had better results than the pre-service ones: more items answered, more answers with argument, more of the correct ideas, and fewer incorrect ones. However, in-service teachers scored higher than pre-service ones in three of the identified misconceptions: about the triangle's base and height, and about the Pythagorean theorem.

## Experience

Of the 36 participants who were in-service teachers, PC, or SET, 22 declared their teaching experience.  Their results are shown in Table 7.

| | Items answered | With argument | Correct ideas | Incorrect ideas | Misconceptions | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Triangle inequality | Base | Height | Pythago-rean th. | Other |
| 1-10 yrs | 81.9% | 80.3% | 16.2% | 76.2% | 18.2% | 6.1% | 11.7% | 22.0% | 6.2% |
| 11-20 yrs | 98.3% | 86.5% | 31.3% | 60.3% | 13.1% | 13.1% | 18.3% | 5.6% | 5.2% |
| >20 yrs | 90.9% | 78.1% | 53.2% | 34.1% | 9.0% | 5.1% | 6.5% | 0.0% | 4.2% |

**Table 7**

Teachers with more years of experience have a tendency towards better results, and teachers with less experience towards worse results, in almost all aspects. However, teachers with between 11 and 20 years of teaching experience have more answers classified as misconceptions on base and height than the other two groups.

Overall, the teaching experience does have a marked influence on a decrease in incorrect ideas, as the graph of Figure 1 shows (in it the value for 0 years is the average for all student teachers). The correlation coefficient between teaching experience and percentage of incorrect ideas is r = –0.51.

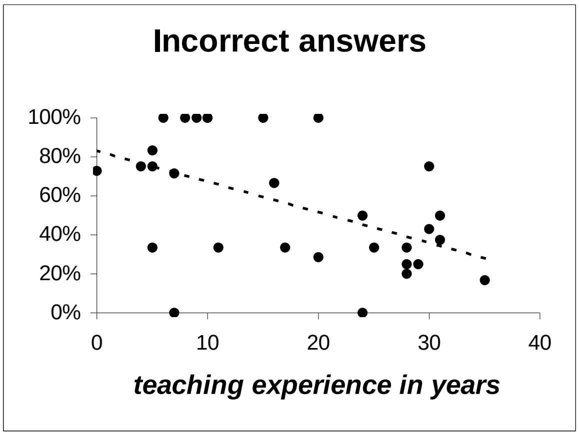



**Figure 1**

## Language and didactical competence

Another characteristic of the responses to the diagnosis given by the participants is the quality of the language used and of the didactical explanations provided in the hypothetical situation of Item 3. Although we do not have here the space to show the analysis that we carried out, we want to state some of the findings. Many answers are based on orders or assessment, which reflect the disconnection described by Empson & Junk (2004) between MCK and the understanding of children's thought. It is also evident, as was stressed by Boero et al. (2002), that the natural language can provoke difficulties in the acquisition of concepts. Finally, some teachers, particularly of the Secondary level, have an attitude that could be expressed as "*I know so much that you cannot understand me*".

## ANALYSIS AND CONCLUSIONS

Two considerations must be taken into account. Firstly, we must stress that if a teacher does not manifest a misconception, this does not necessarily mean that s/he does not have it; it could also be that in his/her expression the misconception just did not show. Secondly, although no hard facts can be deduced of the information obtained from this study, the results we have shown can be interpreted in terms of possible tendencies that could be investigated in a next step of the research.

It would seem that, with respect to MCK relating triangles, male teachers, secondary school teachers, in-service teachers and highly experienced teachers obtain better results than their counterparts do.

The gender effect that we found in these results could make sexists happy. However, in the group of teachers that participated in the two workshops, 62% of the female teachers were pre-service ones, and among the male teachers the percentage was 20%; thus, the gender effect could be confounded with the variable "occupation". The other groups with better results were to be expected: teachers of the Secondary level receive more mathematical training in ENS, and in-service teachers have dealt with the teaching (and are thus more in contact with the students' way of thinking, in accordance with the findings of Empson and Junk's, 2004), and even more so as their teaching experience increases.

As for the differences between the obtained results in the two venues, the better results of MR can be related to two factors. The first factor is that, as Table 1 shows, in MR there were more Secondary level teachers (25% vs 10%), and more in-service teachers (44% vs 16%): two of the three "better" groups (with no differences on the fourth variable, the teaching experience). The second factor, which could be of even more importance, is the difference in the ways that teachers arrived to the workshops. MR teachers were highly interested in mathematics and its teaching, and also had good scores in a national assessment, whereas MC teachers did not share this characteristics and were regular attendants of a teachers' centre in a low-income part of the city.

It can be interesting to comment on the cases that stray from the reported tendencies, which relate to misconceptions about the triangle's base and/or height, and about the Pythagorean theorem. We carried out an analysis using the fine-categories in addition to the broad ones about base and height described and used in this paper, which we do not have here the space to present. However, this analysis shows that some of the misconceptions can be linked to didactical strategies (where the informal and potentially incorrect use of mathematics serves a didactical purpose), and that modern teacher training is slowly (and partly!) fighting some misconceptions about base and height, through fewer prototypical examples in the textbooks for student teachers. As for the misuses of the Pythagorean theorem, there are more answers with this classification in two of the three "better" groups (Secondary, in-service). One possible interpretation of this is that the groups with a higher level in general also have some idea about the existence of the Pythagorean theorem and, approximately, what it is about. (It could also be that more recently trained teachers have heard about the theorem). However, all of the teachers who pretended to use this result did it in one of several incorrect ways; this relates to Hershkowitz (1990) characterisation of misconceptions that increase as the students advance throughout their schooling.

The effect that the teaching experience has in decreasing (but not nullifying!) the amount of incorrect answers is something that must be valued in professional development programs. When the teacher (and particularly the Primary school one) starts her/his practice, s/he must deal not only with the students' difficulties in the learning of mathematics, but also with her/his own deficiencies in MCK, which in turn have the effect of not only perpetuating but also aggravating their students' misconceptions. The professional practice can help in dealing with both the students' learning difficulties and the teacher difficulties in MCK, but if s/he had more support with MCK, the pedagogical difficulties would be easier to handle. Therefore, we coincide with Bromme (1994) in that MCK must be understood as the scholar subject, and we assert that it is something that must be attended to, diagnosed and solved, both in initial training and in professional development.

## NOTE

1. The 71.6% of ideas with argument is 100% minus the answers without clear argument: 10.1% that were potentially correct and 18.3% that were incorrect. The 100% of ideas is formed by correct ones, plus those that were potentially correct but without clear argument, plus the incorrect ones, including those without argument. The same calculations were carried out for the other tables.

## REFERENCES


Boero, P., Douek, N., & Ferrari, P.L. (2002). Developing mastery of natural language: approaches to theoretical aspects of mathematics. In L. English (ed.), *Handbook of International Research in Mathematics Education*, (pp. 241-270). New Jersey: Lawrence Erlbaum Associates Publishers and NCTM.

Bromme, R. (1994). Beyond subject matter: a psychological topology of teachers' professional knowledge. In R. Biehler, R. Scholz, R. Strässer, & B. Winkelmann (eds.), *Didactics of mathematics as a scientific discipline* (pp. 73–88). Dordrecht: Kluwer.

Empson, S. & Junk, D. (2004). Teachers' knowledge of children's mathematics after implementing a student-centered curriculum. *Journal of Mathematics Teacher Education*, *7*( 2), 121-144.

Hershkowitz, R. (1990). Psychological aspects of learning geometry. In Nesher, P. & Kilpatrick, J. (Eds) *Mathematics and cognition: A research & synthesis by the International Group for the Psychology of Mathematics Education* (pp. 70-95). Great Britain: Cambridge University Press.

Llinares, S. & Krainer, K. (2006). Teachers and Teacher Educators as Learners. In A. Gutiérrez & P. Boero (eds.), *Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future* (pp. 429-459). The Netherlands: Sense Publishers.

Moreira, P. & David, M. (2008). Academic mathematics and mathematical knowledge needed in school teaching practice: some conflicting elements. *Journal of Mathematics Teacher Education*, 11(1), 23-40.

Prestage, S. & Perks, P. (2001). Models and super-models: ways of thinking about professional knowledge in mathematics teaching. In C. Morgan & K. Jones (eds.), *Research in Mathematics Education,* Volume 3, pp. 101-114. London: British Society for Research into Learning Mathematics.

da Ponte, J.P. & Chapman, O. (2006). Mathematics teachers' knowledge and practices. In A. Gutierrez & P. Boero (eds.), *Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future* (pp. 461-494). The Netherlands: Sense Publishers.

Shulman, L. S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher, 15* (2), 4-14.

# MATHEMATICS TEACHER EDUCATION RESEARCH AND PRACTICE: RESEARCHING INSIDE THE *MICA* PROGRAM

**Joyce Mgombelo**[1] **& Chantal Buteau** [2]

[1]*Teacher Education Department &* [2]*Department of Mathematics,*

*Brock University, St. Catharines (CANADA)*

*The paper describes an ongoing collaborative work between department of mathematics and department of pre-service teacher education, aimed at connecting research and practice in the development and study of mathematics teacher education. The work draws from learning experiences of future teachers through the designing and implementing Learning Objects in department of mathematics. The focus of research is to address the need for a better understanding of how future teachers of secondary school mathematics are shaped by didactic-sensitive activities during their undergraduate mathematics education.*

Keywords: *future teachers; mathematics needed for teaching; research and practice; innovative undergraduate mathematics program, mathematics teacher education*

## Introduction

In their preface to a special issue of *Educational Studies in Mathematics*, titled "Connecting Research, Practice and Theory in the Development and Study of Mathematics Education," Even and Ball (2003) highlighted the need for addressing the gap between theory and practice, the divide between mathematics and mathematics education, and the divide between mathematicians and mathematics educators in the study of mathematics education. As they noted, there are emerging efforts to build collaborations and connections focused on the issues of practice in order to develop and study mathematics education. It is this sort of sensitivity to building connections and collaboration in addressing issues of practice and research that underpins our research. The central focus of our research is to address the need for a better understanding of how future teachers of secondary school mathematics are shaped by didactic-sensitive learning experiences during their undergraduate mathematics education (Mgombelo & Buteau, 2008a, 2008b). The research draws from learning experiences of future teachers in a non-traditional core undergraduate mathematics program called "Mathematics Integrated with Computers and Applications" (MICA) (Ben-el-Mechaiekh, Buteau, & Ralph, 2007; Ralph 2001). Among other things, MICA, launched at our institution in 2001, integrates computer, applications and modeling where students make extensive use of technology in ways that support their growth in mathematics (Ralph & Pead, 2006). Previous work describing MICA student learning experiences is reported in Muller and Buteau (2006); Buteau and Muller (2006); and Muller et al. (in press). Our focus in this paper is to describe our ongoing collaborative work aimed at connecting research and practice in the development and study of mathematics teacher education.

The rationale for our research is based on epistemological and practical grounds.

Mathematics teacher education is premised on the assumption that one has to be educated in mathematics in order to be able to teach it. This assumption highlights the well know problem of divide in mathematics teacher education between mathematics and teaching. From an epistemological perspective, the question is how mathematics and teaching could be integrated in mathematics teacher education. An initial characterization of this integration comes from Shulman's (1986) work on pedagogical content knowledge. Recently, Ball and Bass (2002) elaborated on pedagogical content knowledge and used the term *mathematics knowledge for teaching* to capture the complex relationship between mathematics content knowledge and teaching. This is the epistemological ground for our research.

In practice, any mathematics teacher education program has to contend with questions of how much mathematics and how much method or educational study should comprise such programs, and then whether and how these programs should integrate or separate out opportunities to learn mathematics and teaching (Adler & Davis, 2006). Answers to these questions are reflected in a wide spectrum of variations of programs, opportunities, and learning activities for future teachers (Mgombelo et al. 2006). In addition, there are also lessons from mathematics teacher education research and practice. With regard to secondary school teacher education, many teachers still struggle with teaching school mathematics for understanding even though their knowledge of mathematics may be adequate (Kinach, 2002). This points to mathematics needed for teaching.

Following Ball and Bass's (2002) work on mathematics for teaching there has been recognition that mathematics teacher education is an important area of study in departments of mathematics (Conference Board of Mathematical Sciences [CBMS], 2001; Davis & Simmt, 2005). For example, the 2001 report from the CBMS on "The Mathematical Education of Teachers" has two main recommendations for ways in which mathematics departments can attain these goals:

> First, the content and teaching of core mathematics major courses can be redesigned to help future teachers make insightful connections between the advanced mathematics they are learning and the high school mathematics they will be teaching. Second, mathematics departments can support the design, development, and offering of a capstone course sequence for teachers in which conceptual difficulties, fundamental ideas, and techniques of high school mathematics are examined from an advanced standpoint (p.123).

It is with this sort of understanding that some departments of mathematics have made ongoing and emerging attempts to reform their programs to provide meaningful experiences for future teachers (Bednarz 2001; CMS 2003; Muller & Buteau 2006; Pesonen & Malvera 2000). This points to the need for research to investigate whether and how these attempts impact future teachers' learning of mathematics needed for teaching (Bednarz 2001). More importantly, as we noted earlier, for this research to be meaningful and productive, collaboration among mathematicians and mathematics educators is crucial (Even & Ball, 2003; Mgombelo & Buteau, 2006). We are addressing this need for research and collaboration in our research. We, a mathematician and a mathematics educator, are interested in collaboratively extending our understanding of how future teachers of secondary school mathematics are shaped by their experience of designing so-called *Learning Objects* in the MICA program. In the following section we describe the MICA program and what we learned from reflections on practice regarding the students' learning experiences.

## Learning from Practice: The MICA experience

In 2001, our institution launched its innovative core undergraduate MICA program based on guiding principles (a) to encourage student's creativity and intellectual independence, and (b) to develop mathematical concepts hand in hand with computers and applications. MICA also strives to strengthen the concurrent mathematics teacher education program. It exposes future teachers to a broad range of mathematical experiences rather than to a deep concentration in one or two areas. Future teachers also make extensive use of different software programs such as Maple, *Journey Through Calculus* (Ralph, 1999), Geometer's SketchPad, and Minitab, all of which nurture the understanding of mathematics.

In addition to a revision of all the traditional courses under the above-mentioned guiding principles, three innovative, core project-based courses, called MICA I - III, were introduced in which all students learn to investigate mathematics concepts by designing and implementing interactive computer programs, so-called *Exploratory Objects* (Muller et al., in press), from year one. As their final projects in MICA courses, students individually (or in groups of two) complete an original interactive computer program on a topic *of their own choosing*. These projects can be (a) exploratory (e.g., testing his/her own conjecture; see *Structure of the Hailstone Sequence* Exploratory Object, (MICA Student Projects, n.d.); (b) an application (e.g., modeling or simulation; see *Running in the Rain* Exploratory Object, MICA Student Projects); or (c) didactic, i.e., so-called *Learning Object*s (LO). The latter, generally designed by future teachers, are innovative, interactive, highly engaging, and user-friendly computer environments that teach one or two mathematical concepts at the school level. For example, a 9-task adventure with Herculus covering (Grade 4) perimeter and area; a journey through MathVille for learning the (Grade 9) exponent laws; or a fourfold Pythagorean Theorem plate-form offering (i) a review of right angles and triangles, (ii) an exploration of the theorem, (iii) a game to practice, and (iv) a five question test with applications, are all projects designed by first-year future teachers (see respectively *Hercules and Area* LO, *Exponent Laws* LO, and *Exploring the Pythagorean Theorem* LO, MICA Student Projects).

Overall, observations and reflections on students' experiences of designing LOs and Exploratory Objects indicated that the experiences promoted positive student learning experiences. Muller et al. (2008) summarize these experiences:

> We suggest that the students develop the following skills: (a) to express their mathematical ideas in an exact way; (b) to self-assess their mathematics; (c) to

> realize their creativity in mathematics and in communicating their understanding of mathematics; and (d) to become independent in mathematical thinking. We also suggest that students are exposed to the opportunity (a) to concretize personalized original mathematics work, and (b) to identify with their future profession. Finally, our observations lead us to suggest that students develop a personal relationship with the activity of designing and implementing an ELO; indeed, students seem to demonstrate a strong engagement and ownership in the activity, and exhibit much pride of their ELO (p.4).

These reflections prompted a pragmatic collaborative project between the Department of Mathematics and the Department of Pre-Service Education which involved LOs designed by MICA students and teacher candidates enrolled in pre-service education elementary mathematics methods course (Grades 4 to 8) (Muller et al., in press). Pre-service teacher candidates were asked to use LOs to learn or review the involved mathematics in the Object and to write their reflections on their experience. Their overall experience was positive as they appreciated the LOs and commented on their high regard for the first-year MICA student LO designers. Some teacher candidates who self-identified as having math anxiety, thought that the LOs provided a safe environment for them to re-learn mathematics.

Reflecting on MICA student learning experiences as well as pre-service teacher candidates' experiences of using the LOs, we started to focus on the MICA future teachers' experiences of designing and implementing LOs. It was clear to us designing and implementing LOs involves mathematical didactics work. Interesting empirical questions started to emerge: In what ways do future teachers experiences of designing and implementing LOs promote their learning of mathematics needed for teaching? What aspects of designing and implementing LOs prompt such a positive experience? How do these future teachers' learning experiences through designing and implementing LOs differ from their learning experiences in other traditional activities? These questions led us to focus on the suggested future teachers' development of a "personal relationship with the activity of designing and implementing [a] Learning Object" (Muller et al. 2008). We postulated that future teachers' behaviour, in terms of dedication, pride, ownership, and engagement with the activity could be a key to the future teachers' positive experiences and their learning of mathematics needed for teaching. This pointed to an in-depth investigation to explore the impact of future teachers experiences of designing and implementing LOs on their learning (Mgombelo & Buteau, 2008a).

**Researching inside MICA: Learning Mathematics Needed for Teaching through the Designing and Implementing of LOs**

The purpose of our research is to explore how future teachers of secondary school mathematics are shaped by their didactic-sensitive learning experiences during their undergraduate mathematics education. Our research is guided by the following questions: (a) Does the experience of designing and implementing LOs promote future teachers' learning of mathematics needed for teaching? (b) In what ways do designing and implementing LOs provoke future teachers' awareness of their own learning of mathematics as well as what does it mean for students to learn mathematics? Guided by previously mentioned postulate (that ownership, dedication, engagement of the activity, and pride are key for the positive learning experience) we are interested in probing deeper into these future teachers' experiences in order to capture the qualitative aspects of their learning of the mathematics needed for teaching. The goal in our research is not to measure this impact in terms of how much do future teachers know mathematics needed for teaching. Our focus in the research is on future teachers' "knowing." Given the complexity of this kind of research we initially conducted a pilot –small scale study (2006-07). The goal of the pilot study was to gather first evidence of future teachers' experiences as well as to inform the design of a large scale study.

Guided by the above postulate our pilot study was framed by Mason and Spence's (1999) work on "knowing-to act" as a kind of knowing that requires awareness. Building on Gattegno's (1970) work on awareness, Mason (1998) further elaborates on the relationship of "knowing-to act" and awareness in mathematics teacher education. Mason developed three forms of awareness: "awareness-in-action," which involves a human being's powers of construal and of acting in the material world; "awareness-in-discipline," which is awareness of awareness-in-action emerging when awareness-in-action is brought into explicit awareness and formalized; and finally, "awareness in counsel," which is awareness of awareness-in-discipline involving becoming able to let others work on their awareness-in-discipline. To put this into a mathematics perspective, awareness-in-action might be exemplified by an act of counting numbers (1, 2, 3) without being aware of the underlying notions such as one to one correspondence. Awareness-in-discipline emerges when one becomes aware of this one to one correspondence in counting. Finally, awareness-in-counsel emerges when one is able to support others develop their awareness of counting as well as develop their awareness of the notion of one to one correspondence. Mason's levels of awareness served as analytical/interpretive tool for analyzing data

Data were collected from detailed questionnaires, journals, and focus group discussions that involved 4 future teachers enrolled in the MICA program, 4 teacher candidates in the Department of Pre-Service, and 1 practicing teacher. In order to probe MICA future teachers' experiences deeply in terms of awareness, questions and prompts in the questionnaires and journals were open-ended. The roles of the Pre-service teacher candidates and the practicing teacher in the research were to facilitate data collection through focus group discussion and not to act as research subjects.

All data from questionnaires, LOs, and transcripts from videos were analysed according to the interpretation of themes guided by the postulate that ownership, engagement in the activity and pride were key for positive learning experiences and by using Mason's three forms of awareness as outlined in the conceptual framework..

Using Mason's levels of awareness we identified which levels were engaged as well as ways in which they related to experiences of ownership, engagement and pride. Our analysis of data further elaborated on three prospective teacher behaviour aspects, ownership, engagement, and pride. We briefly elaborate these aspects.

*Ownership*

As noted earlier in this paper, prospective secondary school teachers can perform a number of school mathematics tasks without problem. Using Mason's (1998) forms of awareness, we could say these future teachers have awareness-in-action of mathematics needed for the tasks. Yet (as noted) if you ask future teachers how they would explain a mathematics concept or skill to someone who is learning for the first time, most of them would respond by rule-based explanation (e.g., negative times negative is positive in case of integers multiplication). These future teachers would be attending to content of their awareness-in-action and not their awareness of their awareness-in-action. As Mason notes, the behaviours to which awareness-in-action play a role can somewhat be trained without explicit allusion to awareness. We found a different scenario with the experience of designing and implementing LOs. This experience seems to prompt future teachers to take into account their own experience of learning the mathematics in order to generate ideas on how to design their LOs in ways that will make sense for the user's learning of mathematics in question. It is this future teachers' attention to their learning in order to bring to awareness their awareness-in-action that we refer to as *ownership*. This is exemplified by the following prospective teacher's response to the questionnaire question on why she chose the topic for her LO.

> My MICA I Learning Object [...] dealt with explaining and practicing multiplication…. I chose this topic because in Grade four I was very, very behind on my multiplication. I could not do the calculations in my head, and I was stuck on the first sheet of questions my teacher would give us… Since it is something I struggled with and something that I have to overcome to become a Math major, I thought it would be a great idea to develop a program that could allow students to practice without just doing the same questions over and over. I also included different ways of thinking about what multiplication means (Mgombelo & Buteau, 2008a)

It underlines that this prospective teacher attended to her own learning of multiplication or own awareness in action of multiplication. The prospective teacher in the above response did not want to design a program based on multiplication routines and rules but instead wanted to include the different ways of thinking about what multiplication means – this involves awareness.

*Engagement*

Awareness-in-discipline arises when we become aware of awareness-in-action. According to Mason (1998), the term "discipline" means encountering both facts and techniques as well as habits of thought, types of meaningful questions, and methods of resolving those questions. Our analysis of the data indicates that through the designing and implementation of LOs, future teachers engage with mathematics in terms of both aspects outlined above by Mason. Our analysis further indicated that future teachers' experiences of designing and implementing LOs tend to elicit the need to explain and attend to different representations and meanings of mathematics concepts, a very important aspect of mathematics for teaching (Ball & Bass, 2002; Davis & Simmt, 2005). We distinguish *engagement* as another aspect of learning mathematics needed for teaching. Engagement with mathematics is recognized in the way future teachers use games, graphics, and colors in their LOs in order to engage students in a meaningful way. These future teachers attended to different representations or meanings of mathematics concepts such as grid or area models of multiplication as revealed in a response from a prospective teacher questionnaire below.

> I learned how to keep instructions short and simple, and how to gear a lesson towards your audience. I learned to think about the audience I was trying to reach and what would be engaging to them. I added in Bart Simpson and made it as bright and colorful as I could. I learned multiple ways of explaining multiplication. (Mgombelo & Buteau, 2008a)

We see from the above response from the prospective teacher questionnaire, that she "learned to think about the audience …and what would be engaging to them." It is through this experience that she learned multiple ways of explaining multiplication. It is worth to note that this experience involves both future teachers' own engagement with mathematics as well as their audience's (students') engagement as revealed in the above response.

*Pride*

In order to sustain ownership and engagement in mathematics activities in the way we have described here, future teachers have to invest themselves in the activity (in terms of energy, emotion, interest, etc.). In addition to investing themselves, they need to have a sense of purpose and accomplishment. We have identified this investment as *pride*, the third aspect of future teachers' learning of mathematics needed for teaching. Here is an example from a prospective teacher's response that supports our claim.

> You're always thinking about ideas and ways to improve your project while you are in class, watching television [...] (Mgombelo & Buteau, 2008a)

We can see clearly from the above quote how much personal energy, or in other words, dedication, this prospective teacher invested in the project. Our small scale study addressed the need to know about the impact of designing and implementing LOs on the learning of mathematics needed for teaching. It strongly suggests that the experience of designing and implementing LOs promotes future teachers' learning.

## Conclusions: Further Research and Practice Collaborations

Our work underscores the importance of collaboration between mathematicians and mathematics educators in connecting practice and research in mathematics teacher education. From our pilot study further empirical questions emerged: What aspects of the designing and implementing LOs prompt such a positive experience? In what ways do prospective teachers' learning provoked by designing and implementing LOs differ from other traditional learning tasks? These questions have led to a larger-scale, collaborative research project (involving some 30 MICA future teachers candidates each followed over two years) that will thoroughly investigate the students' "repositioning" in terms of engagement, ownership, and pride, with respect to mathematics and mathematics didactics when realizing their MICA final projects (the LOs) compared to more traditional mathematics activities. We are also interested in exploring the characteristics or features of the learning activity (of designing and implementing a LO on a topic of their own choosing) that promote learning. A theoretical framework has been thereafter developed to guide this comprehensive study (Mgombelo & Buteau, 2008b). It mainly relies on Brousseau's (1997) work on theory of didactic situations; Mason's (1998) work on knowing-to act as previously discussed; and on positioning theory.

Our work has been extending on the connection between research and practice in many different ways. First, a collaborative Learning Object project building on Grade 5 students' ideas from a local school (Buteau et al. 2008) has been completed. The project involved the principal, 2 teachers, and Grade 5 students from the elementary school, as well as a mathematics student, pre-service teacher candidates, and both co-authors from our institution. The principal commented,

> From day one, our Grade 5 students were extremely motivated and engaged in developing this tool that will be used by students from other schools. (Buteau et al., 2008, p.28)

A second connection yielded in the ongoing integration of MICA Learning Object use for didactical assignments in the Methods course at our institution. In addition, Mgombelo's informal observations about MICA pre-service students with stronger dispositions towards learning versus non-MICA pre-service students led her to reflect on the design of the course. This naturally leads to asking what is it exactly in the MICA education program that seems to promote this disposition - a question that points to our long-term research program. Thirdly, the research has been guiding Buteau's reflections on her teaching practices of the MICA I course and on the MICA activities (e.g., the description of the student development process of designing and implementing Exploratory and Learning Objects, (Buteau & Muller 2008), thus pointing back to the LO activity attributes that might promote learning mathematics for teaching.

## References

Adler, J. & Davis, Z. (2006, January). Studying mathematics for teaching inside teacher education. *Proceedings of the SAARMSTE Conference, Pretoria, South Africa*.

Ball, D., & Bass, H. (2002, May). Toward a practice-based theory of mathematical knowledge for teaching. Plenary lecture at the 26th annual meeting of the Canadian Mathematics Educators Study Group, Kingston, Ontario, Canada.

Bednarz, N. (2001). Didactique des mathématiques et formation des enseignants : le cas de l'université du Québec à Montréal. *Revue canadienne de l'enseignement des sciences, des mathématiques et de la technologie*, 1(1), 61-80.

Ben-El-Mechaiekh, H., Buteau, C., & Ralph, B. (2007). MICA: A novel direction in undergraduate mathematics teaching. *Canadian Mathematics Society Notes*, *29*(6)

Brousseau, G. (1997). *Theory of didactical situations in mathematics 1970-1990*. Dordrecht, The Netherlands: Kluwer Academic.

Buteau, C., Camilleri, S., Fodil, K., Lacroix, M.-E., & Mgombelo, J. (2008). Fantasy fractions: When a grade 5 class creates computer mathematics games. *The Ontario Mathematics Gazette, 46*(3), 26-30.

Buteau, C., & Muller, E. (2006). Evolving technologies integrated into undergraduate mathematics education. In L. H. Son, N. Sinclair, J. B. Lagrange, & C. Hoyles (Eds.), *Proceedings of the ICMI Study 17 Conference: Digital technologies in mathematics education - Rethinking the terrain*, Hanoi University of Technology, Hanoi, Vietnam.

Buteau, C. & Muller, E. (2008). Student development process of designing and implementing exploratory and learning objects. Paper submitted to CERME

Canadian Mathematical Society. (n.d.) Retrieved September 25, 2008, from http://www.math.ca/Events/CSMF2003/proceedings/grp_9e.pdf

Conference Board of Mathematical Sciences. (2001). *The mathematical education of teachers*. Washington, DC: Mathematical Association of America.

Davis, B., & Simmt, E. (2005). Mathematics-for-teaching: An ongoing investigation of mathematics that teachers (need to) know. *Educational Studies in Mathematics, 30*, 1-27.

Even, R., & Ball, D. L. (2003). Preface: Connecting research, practice and theory in the development and study of mathematics education. *Educational Studies in Mathematics*, *54*, 139-146.

Gattegno, C. (1970). *What we owe children: The subordination of teaching to learning*. London: Routledge and Kegan Paul.

Kinach, B. M. (2002). Understanding and learning-to-explain by representing mathematics: Epistemological dilemmas facing teacher educators in the secondary mathematics "methods" course. *Journal of Mathematics Teacher Education, 5*, 153-186.

Mason, J. (1998). Enabling teachers to be real teachers: Necessary levels of awareness and structure of attention. *Journal of Mathematics Teacher Education*, *1*, 243-267.

Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics*, *38*, 135-161.

Mgombelo J., & Buteau, C. (2006). Establishing a mathematics learning community in the study of mathematics for teaching. In J. Novotna, H. Moraova, M. Kratka, & N. Stehlikova (Eds.), *Proceedings of the 30$^{th}$ Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, p. 301). Prague, Czech Republic: PME.

Mgombelo, J., & Buteau, C. (2008a). *Learning mathematics for teaching through designing and implementing learning objects*. Manuscript submitted for publication.

Mgombelo, J., & Buteau, C. (2008b). *Prospective secondary mathematics teachers repositiong by designing and implementing learning objects: A conceptual framework.* Manuscript submitted for publication.

Mgombelo, J., Orzech, M., Poole, D., & René de Cotret, S. (2006). Report of the CMESG working group: Secondary mathematics teacher development (La formation des enseignants de mathématiques du secondaire). In L. Peter (Ed.), *Proceedings of the annual meeting of Canadian Mathematics Education Study Group (CMESG)*, University of Calgary, Calgary, Alberta, Canada.

MICA Student Projects. (n.d.). Retrieved September 25, 2008, from http://www.brocku.ca/mathematics/resources/learningtools/learningobjects/students.php

Muller, E. & Buteau, C. (2006). Un nouveau rôle de l'informatique dans la formation initiale des enseignants. In Bednarz, N., Mary, C. (Eds.), "*L'enseignement des mathématiques face aux défis de l'école et des communautés*", Actes du colloque EMF 2006 (cédérom), Sherbrooke: Éditions du CRP

Muller, E., Buteau, C., Ralph, B., & Mgombelo, J. (in press): Learning mathematics through the design and implementation of exploratory and learning objects. *International Journal for Technology in Mathematics Education.*

Muller, E., Buteau, C., Klincsik M., Perjési-Hámori, I., & Sárvári, C. *Systemic integration of evolving technologies in undergraduate mathematics education and its impact on student retention.* Submitted (Sept. 2008) to IJMEST.

Pesonen, M. E., & Malvera, T. (2000). A reform in undergraduate mathematics curriculum: More emphasis on social and pedagogical skills. *International Journal of Mathematical Education in Science and Technology*, *31*(1), 113-124

Ralph, B. (1999). *Journey through calculus*. Pacific Grove, CA: Brookes/Cole.

Ralph, B., & Pead, D. (with Muller, E.). (2006). Uses of technology in mathematical modelling. In W. Blum (Ed.), *Applications and modelling in mathematics education* (pages). Dordrecht, Netherlands: Kluwer Academic.

Ralph, B. (2001). Retrieved September 25, 2008, from http://www.brocku.ca/ctl/pdf/Brock_Teaching_1.pdf

Shulman, D. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*, 4-14.

# COGNITIVE TRANSFORMATION IN PROFESSIONAL DEVELOPMENT: SOME CASE STUDIES

**Jorge Soto-Andrade** [1]

*Department of Mathematics, Faculty of Sciences,*

*Centre for Advanced Research in Education,*

*University of Chile*

*sotoandr@uchile.cl*

*Professional development programmes for in-service teachers constitute a complex task. We intend here to shed some light on the conditions that may entail a cognitive transformation in the involved teachers, building on our personal experience in these programmes and some case studies.*

*Keywords: Professional development, in-service teachers, metaphors, cognitive modes.*

## INTRODUCTION

In this paper I report on some didactic phenomena (in the sense of Margolinas, 1998) arising in our work in professional development for in-service primary teachers, at the University of Chile. These phenomena are related to the cognitive transformations that emerge in the being of the involved teachers, as well as researchers, under favourable circumstances, depending on "the time, the place and the people" (see Mason, 1998). Our work could be described as "theory-guided bricolage" in developmental research (Gravemeijer, 1998; Freudenthal, 1991), with the caveat that a detailed theory is not put forth first, because it rather grows out of the ongoing process. This approach to professional development or enhancement for in-service teachers is inspired by my former research on the fundamental role of metaphors and cognitive modes in the teaching-learning process (Soto-Andrade 2006, 2007). It involves "researching from the inside" (Mason, 1998), and it requires an embodied first-person approach (Varela, Thomson & Rosch, 1991), in an enactive perspective (Masciotra, Roth & Morel, 2007).

After recalling the fundamental components of a tentative theoretical framework, I set down below my main research hypotheses and proceed to report on some concrete examples of activities and germs of didactical situations (Brousseau, 1998), involving metaphors and switches in cognitive modes, that we have worked out with teachers. Translated quotes of several teachers' testimonies and reports are also included, as case studies. These give preliminary experimental evidence to support our hypotheses

[1] Supported by PBCT- CONICYT, Project CIE-05 and FONDEF Project D06i1023.

and suggest further research along these lines.

# THEORETICAL FRAMEWORK

## Nature and Role of Metaphors

It has been progressively recognized during the last decade (English, 1997; Lakoff & Núñez, 2000; Presmeg, 1997; Sfard 1994, and many others) that metaphors in mathematics education are not just rhetorical devices, but powerful cognitive tools that help us to build or grasp new concepts, as well as to solve problems in an efficient and friendly way (Soto-Andrade, 2006). We use conceptual metaphors (Lakoff & Núñez, 2000), that appear as inference preserving mappings going "upwards" from a source domain into a target domain, enabling us to understand the latter, usually more abstract and opaque, taking a foothold in the former, more down-to-earth and transparent in terms of our previous cognitive history. Metaphors are "met-befores", as Tall (2005) says.

## Cognitive modes

A cognitive mode is defined nowadays as one's preferred way to think, perceive and recall, in short, to cognize. It shows up, for instance, when trying to solve problems. Flessas and Lussier (2005) gave a first operational description of what they call the 4 basic cognitive modes ("styles cognitifs" in French), combining 2 dichotomies: verbal – non verbal and sequential – non sequential (or simultaneous), closely related to the left – right brain hemisphere dichotomy and to the frontal – occipital dichotomy. We so obtain the *sequential-verbal, sequential-non verbal, non sequential-verbal* and *non sequential – non verbal* cognitive modes. They emphasize that effective teaching of a group of students, who may display a high degree of cognitive diversity, requires teachers supple enough to tune fluently to the different cognitive modes of the students.

An example: check that you have the same number of fingers in your hands by using the 4 basic cognitive modes (see Soto-Andrade, 2007, for more examples).

In what follows I adhere mainly to the framework laid by Lakoff & Núñez (2000), Presmeg (1997) and Sfard (1994, 1998) for metaphors, Flessas & Lussier (2005) for cognitive modes, Brousseau (1998) for didactical situations and to the research paradigms of Mason (1998), Varela et al., (1991) and Masciotra *et al*. (2007).

# PROBLEMATICS

Professional development and enhancement for in-service teachers is a complex issue. In Chile, significant funding and human resources have been invested by the Ministry of Education, for more than two decades, to address this issue, but results have been rather scanty. Our students continue to perform poorly in international assessment tests like TIMMS or PISA, and also in national assessment tests like SIMCE [1]. Increasing evidence shows that after a typical 2 week intensive summer workshop, where they learn some more mathematics and design a couple of teaching modules, most teachers revert to their former inadequate teaching practices.

Under closer scrutiny, we have observed that most of our in-service primary teachers are unfamiliar with metaphors and cognitive modes, or visualization, in their practice. They are “frozen” in the verbal - sequential cognitive mode, unaware of this and also of the fact that their teaching is shaped by unconscious and misleading metaphors, like the acquisition metaphor (Sfard, 1998) or the container-filling or gastronomic metaphor (Soto-Andrade, 2006). They have special trouble in creating “unlocking metaphors” for the not specially gifted.

The urgent question is: How to promote a real change in the teaching practices of in-service teachers, in the short or mid term?

## RESEARCH HYPOTHESES

Our main research hypothesis is that metaphors and cognitive modes are key ingredients in a meaningful teaching-learning process. Moreover the deepest impact on this process is usually attained by metaphors that involve a switch from one cognitive mode to another.

We claim that competences regarding multi-modal cognition and use and creation of metaphors and representations are trainable and that measurable progress can be achieved in a one semester course. This, in spite of the fact that most of our teachers report that their initial training included no use of metaphors and privileged just one cognitive mode: the usually dominant verbal-sequential one.

We hypothesize that explicit work on metaphors and transits between cognitive modes will foster teacher’s deep understanding of elementary mathematics. Furthermore, it will affect their professional practice in the classroom, in particular enabling average students to understand and handle mathematical objects and processes that would otherwise be within reach of only a happy few.

## RESEARCH BACKGROUND AND METHODOLOGY

The background for our experimental research consisted in 5 classes (called “generations” in what follows) of in-service primary school teachers, of 30 teachers each, enrolled in a professional development programme, implemented by the University of Chile, on behalf of the National Ministry of Education, stretching from 2006 to 2008. This programme aims at “general” primary teachers, who are interested in enhancing their mathematical training, and certifies their mathematical proficiency after a 15 months period, where they must complete the requirements for 4 modules (numbers and data processing, geometry, ICT in education and problem-solving, 450 hours in all). They must also complete a 75 hour Seminar Project, which includes experimenting and theory-driven practice in the classroom.

Teachers applied for admission to this programme, with the support of their schools, and were selected according to their performance in a TIMMS like test, based mainly on mathematical contents pertaining to the curriculum of primary school. Selected teachers are usually highly motivated; they come to the University after hours, typically from 6 PM to 9:15 PM, at least twice a week, plus an intensive 2 week summer workshop. Gender distribution is 90% female, 10% male, on the average. Ages range from 25 to 60, even 68 in one case (see below).

This sort of programme opens up hitherto unknown possibilities for deeper work with teachers. In particular, as coordinator for the Numbers Module (160 hrs approx.) and advisor to the seminar projects of 6 teachers in each generation on the average, I had the opportunity to test several activities and a-didactical situations in work sessions with the teachers. This module aims mainly at reviewing the mathematics as well as the didactics of numbers, specially elementary integer arithmetic, fractions, ratios, decimal and binary description of numbers. Work sessions were interactive, with teachers usually working in small groups of four on the average.

The underlying idea for this module was to open up the opportunity for the teachers to have a first hand experience of problematic and challenging situations to be tackled, eventually "bare handed", where important mathematical objects or processes could emerge. So their experience would be an antidote to the usual cookbook recipe approach. Methodology consisted in observing the teachers, as they carried out various activities, with non intrusive guidance and support, recording their reactions, in video in some cases, and asking them to write reports on their work, besides communicating it orally to the whole class. After completion of the programme, I asked them to write a short report in the first person on their cognitive and affective experience, in the spirit of "researching from the inside" (Mason, 1998).

My viewpoint was that just recording contents taught plus results of post-tests administered to teachers provides a rather shallow understanding of their learning process. Instead, I tried to foster group work, monitoring the course of their work during sessions, by circulating and interacting with the groups, as a means to fathom their cognitive profiles and processes. This was complemented with the results of tests and challenges. The first-person report mentioned above also provided further insights into the process they had undergone. So my approach relies mainly in case studies rather than hard statistical evidence, emphasizing qualitative rather than quantitative assessment (see below however quotes on SIMCE [1] scores)

## EXPERIMENTAL ACTIVITIES AND PRELIMINARY RESULTS

I comment here on some concrete albeit paradigmatic examples of the activities carried out, together with excerpts of the teacher's reactions to them.

### Example 0: Do you have an innate approximate number sense?

To make them feel the contrast between verbal-sequential and non-verbal non-sequential cognitive modes, we began with some experiments aiming at activating their innate approximate number sense or "numerosity" in the sense of Dehaene (1997), Lakoff and Núñez (2000), Pica *et al.* (2004), Halberda, Mazzoco and Feigenson (2008). For instance, they were asked to tell whether there were more yellow dots or blue dots in a random array of dots of both colours shown just for 200 ms (Testing your Approximate Number Sense, 2008). Our fifth generation of teachers scored here an impressive average of 95%, much higher than the statistical average success of only 75% (as it was the case in a class of average Master in Science students in our Faculty). This suggests that primary school teachers tend to have a significantly better approximate number sense than random adults.

**Example 1: How to keep track of your lamas?**

*An 8 year old aymara shepherd is in charge of a herd of lamas (more than 40, it seems) at some barren place in the highlands in the north of Chile. But he is tired and would like to take a nap… How could he check that when he wakes up there are no lamas missing? He has no palm device, no paper and pencil, not even small stones, or sticks or a knife; just his bare hands. Moreover he does not know how to count calling numbers by their name. How could he manage to register the number of lamas in sight before going asleep and to recover it when waking up?*

Every generation of teachers engaged in group work, in groups of 4 to 5, to discuss how to tackle the problem. As a supporting aid, we simulated the lamas with a bunch of coins on the plate of an overhead projector. Usually, after half an hour or so, in one or two groups, the idea emerged of using the phalanges of their fingers, thumb excluded. The idea spread quickly and finally all groups rediscovered the classical method of non-verbal counting by dozens still used in the Middle East and Far East, where you touch with your right thumb the 12 phalanges of your right hand, say, one by one, and fold one finger in your left hand to register each complete round of 12 (Ifrah, 2005, p. 74). Most did that from little finger to index, but some did it from proximal to distal phalanges (the classical way) and others, the other way around. So they learned how to count non-verbally up to 60, using their fingers and they applied this successfully to the simulated herd of lamas on the overhead projector. They also related this with the ubiquitous emergence of the dozen and 60 in human cultures.

This example may be looked upon as an implementation of realistic mathematics education (Gravemeijer, 2007; Freudenthal, 1991). The underlying hypothesis and motivation for this activity is that it is important to practice and get the feeling of non-verbal arithmetic before engaging into classical arithmetic. So our idea was to prompt the teachers to go back to the non-verbal sequential mode in the context of counting. Their reactions to this sort of activity were stronger than expected:

> My (programme) experience was totally significant in the most strict sense of the expression. It brought to me important changes in my way to approach lessons, in my professional practice and personal interests. But not everything was a “rose garden”… After the first lessons I was quite disappointed, because this course didn’t make any sense to me. My expectations were to learn “more mathematics”, fill in my gaps and not to debate endlessly about why, what for and how. I was even more disappointed with the Numbers Module, with metaphors! I didn’t understand anything: I expected to solve hard arithmetical problems, to design endless exercise lists to calculate with fractions or decimals, to learn more and better algorithms, and it turned out that we were exposed to questions I had never asked myself: How do indigenes in the Amazonas do arithmetic, although they have no language for numbers? How can a shepherd boy know how many

> lamas he has if he doesn't know how to count? How could you teach counting to a little child, in a clever way? There, I had a cognitive break: I asked our teacher for an explanation of the aim of his lessons (I am now ashamed about that) and he kindly explained to me what he was after… (Evelyn, 32, 8 years of practice, 1st generation).

**Example 2: Who has more marbles?**

*John and Mary have a bag of marbles each, all of the same size. How can they tell who has more marbles?*

I invited the teachers, organized in small groups (3 to 4 each), to figure out other approaches than the usual sequential-verbal one (counting the marbles in each bag). Usually in less than half an hour they found at least one procedure for each cognitive mode (Soto-Andrade, 2007). The two pan balance for the non-verbal non-sequential mode emerged easily; also the idea of pairing off the marbles, without counting them, for the non-verbal sequential mode. Verbal - non sequential approaches took longer to appear (weighing simultaneously both bags in digital scales and reading off…).

**Example 3. Registering quantities with dice.**

*The indigenes in an Amazonian village want to keep track of the quantities of seeds stocked for next year. How could they register quantities up to thousands if they have just a handful of dice at hand and they have not invented zero yet?*

After half an hour work on the average, in small groups, the teachers find out, and begin even to do arithmetic in dice-system! They report to understand now much better the decimal system and try this activity with their pupils, with encouraging results. Among others, Gina (49, 25 years of practice, 4th generation) reported:

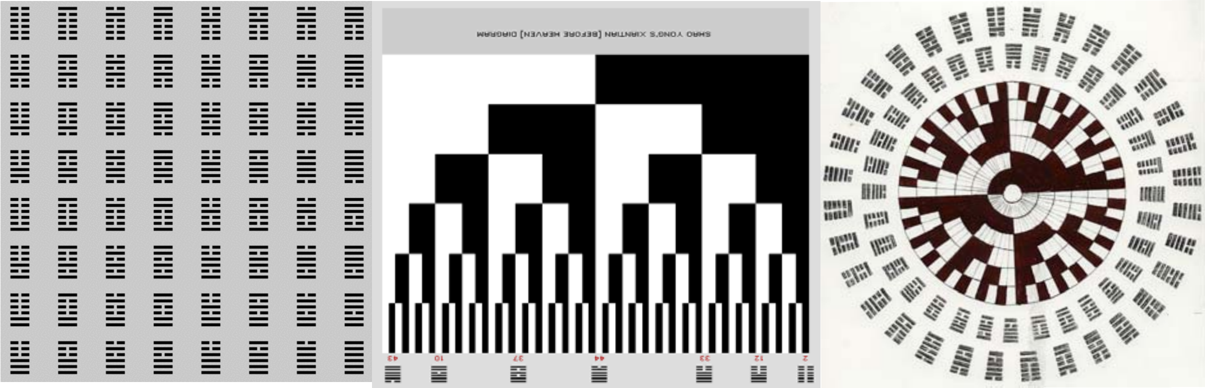
> This experience was very important to me, because you were able to "un-structure" my mind and take away my fear of numbers. Now I see that this fear came from a dull teaching, full of cookbook recipes, that never gave me the opportunity to enjoy discovering the way to solve problems all by myself. Numbers was my favourite subject in this programme, it allowed me to fly, to play, to err and not to feel silly…

**Example 4: The number sequence, otherwise…**

*Is it possible to represent the numerical sequence 0, 1, 2, 3, …. up to 63, let us say, in a non verbal and non sequential way?*

Teachers usually get to the point of discovering the given sequence, written in binary way, in Shao Yong's square (below left), and then of encapsulating it in a single image. (Soto-Andrade, 2007). In the first generation, 5 out of 30 teachers, after 30 minutes work in small groups, came up with diagrams equivalent to Shao Yong's Xiantian ("Before Heaven") or its inverted form (shown below, center, as in Marshall, 2006). Notice the underlying binary tree! In the 2nd generation, 6 out of 30 teachers, rediscovered Xiantian and, most remarkably, one of them, Ofelia (68, 50 years of practice), draw all by herself a circular version of Xiantian diagram (below right). In her own words:

> The Numbers Module shattered all my schemes. For the first time, my brain, archi-structured for algorithmic work, began to have a glimpse of a tiny light (showing the way) to working metaphorically, to solving a problem in different ways, to looking for different paths to reach the same target, not just be satisfied because I got there. I must confess that during the first weeks I was not able to fathom where we were heading to! When I first met a sequence of I Ching hexagrams, sincerely I was barely able to tell what I was looking at! So I never imagined that some weeks later I was going to be able to rediscover one the oldest binary trees, Shao-Yong's circular Xiantian. Later I spent hours trying to solve problems using different cognitive modes…



Here the teachers have the possibility of transiting from the usual verbal sequential mode (the given sequence) to the non-verbal sequential mode (iconic hexagram binary representation) and then to non-verbal non-sequential mode (Xiantian). When interviewed, they unanimously reported having understood, in this unexpected way, for the first time the binary description of numbers.

**Example 4. Brownie's walk**

Random walks provide a nice way to introduce probabilities. Instead of the well known drunkard's walk, we introduced to teachers with no previous training in probability a puppy called Brownie (a baby incarnation of Brownian motion), who escapes randomly from her home in the city when she smells the shampoo her master intends to give her. The stepwise description of her random walk can be tackled by rudimentary means, even by simulation, or with the help of efficient metaphors, like the Solomonic metaphor or the pedestrian metaphor (Soto-Andrade, 2006). In the first one, Brownie splits into 4 pieces, each going to each cardinal direction, and so on… In the second one, a pack of Brownies (a power of 4 preferably) runs away from home, dividing themselves equally into four packs at each corner, and so on… The latter has the virtue of allowing the teachers to work with natural frequencies, in the sense of Hoffrage, Gigerenzer, Krauss & Martignon (2002), avoiding fractions up to the last minute. We have here also an integrative problematic situation, involving geometry, arithmetic and algebra, besides randomness.

After engaging in activities of this sort, teachers reported:

> Cognitive metaphors simply surprised and fascinated me. I had learned with the traditional, mechanical system, and in that way I was teaching my students. Now, I learned about cognitive modes, how to reach every one of my students, and how, with the help of a metaphor, I succeeded in making mathematics closer, friendlier and more reachable. I got so convinced that I chose Numbers for my Seminar Project and I modified radically my professional practices. I wanted to prove that metaphors and these new approaches would give good results, not just for the emotional atmosphere in the classroom but also for "hard" tests. And indeed, my K-4 2007 class got the *first place in the country*, in the SIMCE assessment test [1], increasing by 25 points the previous score, up to 328 points, with no previous training for the test! (Evelyn, 32, 8 years of practice).

> I took advantage of this way of working to carry several activities to my classroom, using various metaphors, which made the students enjoy more my lessons, learning more easily. I transferred all this to my pupils. And this year 2007, our K-4 classes, taught by my colleague Lily (also a student in this programme) and myself increased dramatically their SIMCE score [1], from 281 to 304 points (former SIMCE scores for this grade, since 2002, were 287 and 282). This happened with no special training for the test, contrary to the case of many other schools; the students had just the regular lessons with us (Gina, 49, 25 years of practice).

> I had certain expectations: this program would deliver knowledge to me, besides methodologies to apply to my pupils. But you broke my schemes. What I expected did not happen. What you achieved was to take me out from my "pigeonholing" and to make me think further. If we as teachers are rigid and un-imaginative, hardly will we be able to have our pupils free their imagination and become enchanted with mathematics. This is badly needed, that's why they reject maths so much. I have questioned my way of interacting with my pupils and the way of structuring my lessons (Karem, 32, 6 years of practice, 4$^{th}$ generation).

## CONCLUSIONS AND DISCUSSION

Observation of the teacher's performance shows that even those who never had this sort of experience before were able to activate less usual cognitive modes, to transit from one to another and to take advantage of new metaphors to understand better and to efficiently solve problematic situations. In particular, after some prompting, a high percentage of them were able to switch from their dominant verbal-sequential cognitive mode to a non-verbal or non-sequential one. These findings support our optimistic hypothesis that *cognitive flexibility*, i.e. the ability to approach the same object through various cognitive modes and transiting from one cognitive mode to other, is trainable, even for in-service teachers and that it is facilitated by group work.

However, their first person reports suggest that we had sub-estimated the magnitude of the cognitive shock they experience during the first weeks of our programme. It is interesting to note that testimonies of older and younger teachers are surprisingly alike in this respect. The same holds for their reactions thereafter and changes in their professional practice, as reported above. As a typical example, we recall a 50 year old teacher, Yihecika, from our 3d generation, saying at his final Seminar presentation: "I am very moved, because I am an old teacher doing new things!". At least in the case of these primary teachers, this disproves the hypothesis that changes in cognition and professional practice are out of reach for older teachers.

A rather unexpected outcome of the work carried out with our in-service teachers is the dramatic improvement of their student performance, in several cases, in traditional standardized multiple-choice tests like SIMCE [1]. We may notice that the *relative* improvement was approximately the same for Evelyn and Gina (25 and 23 points resp.) albeit *absolute* scores differed noticeably (328 and 304 resp.), as it is on the average the case between fully private schools and state supported private schools in Chile. Although our programme is intended for teachers in service at state-owned or state supported private schools, Evelyn has been teaching at a fully private high income school for 2 years because she was fired from her previous teaching job at a state supported private school right after completing her professional development programme (as it is the case of roughly 10% of our teachers!). On the other hand, Gina teaches in a low income state supported private school whose explicit aim in mathematics was to reach sometime the threshold of 300 points.

In conclusion, we have gathered some new positive experimental evidence related to this "theory-oriented bricolage", that appears to entail significant cognitive transformations in the being of the teachers (Mason, 1998) and as a consequence, changes in their classroom practice and performance of their students, even measured in traditional ways.

1. SIMCE is a national assessment test, applied to K-4 every year and to K-8 every two years. It is much closer in spirit to TIMMS than to PISA. SIMCE national average score in mathematics for K-4 stagnates at 246 in 2006 and 248 in 2007. Standard deviation is about 50 points. In mathematics only 26% of the students attained the advanced level, whose threshold is 286 points.

## REFERENCES

Brousseau, G. (1998). *Théorie des situations didactiques*. Grenoble: La pensée sauvage.

Dehaene, S. (1997). *La bosse des maths*. Paris: Odile Jacob.

Flessas, J., & Lussier F. (2005). *La neuropsychologie de l'enfant*. Paris: Dunod.

Freudenthal, H. (1991). *Revisiting Mathematical Education.* Dordrecht: Kluwer.

Gravemeijer, K., (1998). Development research as a research method. In A. Sierpinska, & J. Kilpatrick (Eds.) *Mathematics Education as a Research Domain: A Search for Identity* (pp. 277-296). Dordrecht: Kluwer.

Halberda, J., Mazzoco, M., Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement, *Nature*. Published online 7 September 2008. doi:10.1038/nature07246

Ifrah, G. (1981). *Histoire universelle des chiffres*. Paris: Seghers.

Lakoff, G., & Nuñez, R. (2000). *Where Mathematics comes from?* New York: Basic Books.

Margolinas, C. (1998). Relations between the theoretical field and the practical field in mathematics education. In A. Sierpinska, & J. Kilpatrick (Eds.) *Mathematics Education as a Research Domain: A Search for Identity* (pp. 351-356). Dordrecht: Kluwer.

Marshall, S. J. (2006). Calling crane in the shade. Retrieved September 29, 2008, from http://www.biroco.com/yijing/index.htm

Masciotra, D., Roth, W. M., & Morel, D. (2007). *Enaction*. Rotterdam: Sense Publishers.

Mason, J. (1998). Researching from the inside in mathematics education. In A. Sierpinska, & J. Kilpatrick (Eds.) *Mathematics education as a research domain: A search for identity* (pp. 357-378). Dordrecht: Kluwer.

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group, *Science*, 306 (5695), 499-503.

Presmeg, N. C. (1997). Reasoning with metaphors and metonymies in mathematics learning. In L. D. English (Ed.*), Mathematical reasoning: Analogies, metaphors, and images* (pp. 267-279). London: Lawrence Erlbaum Associates.

Sfard, A. (1994). Reification as the birth of metaphor. *For the Learning of Mathematics*, 141, 44-54.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher*, *27*(2), 4-13.

Soto-Andrade, J. (2006). Un monde dans un grain de sable: Métaphores et analogies dans l'apprentissage des maths. *Ann. Didactique Sciences Cogn*., 11, 123– 147.

Soto-Andrade, J. (2007). Metaphors and cognitive styles in the teaching-learning of mathematics. In D. Pitta-Pantazi, & J. Philippou (Eds.). *Proceedings CERME* 5 (pp. 191-200). Retrieved May 21, 2008, from http://ermeweb.free.fr/CERME5b/

Tall, D. (2005) A Theory of Mathematical Growth through Embodiment, Symbolism and Proof, *Ann. Didactique Sciences Cogn*. 11, 195-215.

Testing Your Approximate Number Sense. (2008). N. Y. Times. Retrieved October 12, 2008, from http://www.nytimes.com/interactive/2008/09/15/science/20080915_ NUMBER_SENSE_GRAPHIC.html

Varela, F. J., Thomson, E., & Rosch, E. (1991). *The Embodied Mind: Cognitive Science and Human Experience.* Cambridge, MA: MIT Press.

# WHAT DO STUDENT TEACHERS ATTEND TO?

Naďa Stehlíková

Charles University in Prague, Faculty of Education

*The ability to notice key features of teaching is seen as part of student teachers' pedagogical content knowledge. The study shows what student teachers focus on when they have no experience of guided observation of lessons either in reality or on video and when they are not directed by the educator. Some preliminary findings from a wider study are presented which are in line with other existing research: namely, that the student teachers neglect the subtleties of the introduction of the mathematical content.*

*Keywords: pedagogical content knowledge, ability to notice, student teachers, videos*

## THEORETICAL FRAMEWORK

The notion of pedagogical content knowledge (or PCK) was first introduced by Shulman. The teacher needs understanding of the material he/she is teaching, but he/she also needs the "knowledge of the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986). He/she needs to be aware of topics with which pupils might have difficulties and of their common misconceptions and misunderstandings. Bromme (2008) claims that PCK can also be seen in the ways the teacher "takes into account pupils' utterances and their previous knowledge". An (2004) stresses four aspects of the effective teacher's activity in the classroom which are part of PCK: building on students' mathematical ideas, addressing and correcting students' misconceptions, engaging students in mathematics learning and promoting and supporting students' thinking mathematically.

Thus, in my opinion, part of PCK is the *ability to notice*. In order for the teacher to take into account the pupil's utterance and build on his/her understanding, he/she has to notice the importance of this utterance in the first place, put it into the appropriate context, interpret it and only afterwards use it. According to Sherin and van Es (2005), *noticing* involves a) identifying what is important in a teaching situation, b) making connections between specific classroom interactions and the broader concepts and principles of teaching and learning that they represent, c) using what teachers know about their specific teaching context to reason about a given situation. This study is mainly concerned with the first aspect of noticing.

The (student) teachers' ability to notice is important for the development of what Mason and Spence (1999) call *knowing-to*: "Knowing-to is active knowledge which is present in the moment when it is required." They distinguish this kind of knowledge from knowing-that, knowing-how, and knowing-why. Knowing-to triggers the other types of knowing and thus its absence blocks "teachers from responding creatively in the moment" (ibid). While Mason and Spence mostly concentrate on the way knowing-to develops in pupils (e.g., while solving problems), they also touch on educating teachers to be able to know-to: “We propose that knowing-to act in the moment depends on the structure of attention in the moment, depends on what one is aware of. Educating this awareness is most effectively done by labelling experiences in which powers have been exhibited, and developing a rich network of connections and triggers so that actions ‘come to mind’”. (ibid)

In the same spirit, Ainley and Luntley (2006) propose the term *attention-dependent knowledge* for the knowledge that enables teachers to respond effectively to what happens during the lesson. It can only be revealed in the classroom. The analysis of videos can help us to label such events when this kind of knowledge is at play.

To sum up, the ability to notice seems to be an important component of the (student) teacher’s PCK. This ability can be developed, among others, by analysing videorecordings of the teaching of others and our own (e.g., Sherin & van ES, 2005; Star & Strickland, 2008; Muñoz-Catalán, Carrillo & Climent, 2007; Hošpesová, Tichá & Macháčková, 2007). Most of the studies confirm that (student) teachers must learn what to notice. Santagata, Zannoni and Stigler (2007) found out that “more hours of observations per se [...] do not affect the quality of preservice teachers’ analyses” and on the other hand, Star and Strickland (2008) claim that the ability to learn from observations of teaching “(either live or on video) is critically dependent on what is actually noticed (attended to)”.

The study presented here is a part of a wider study aimed at exploring how student teachers' ability to reflect on their own teaching and the teaching of others can be developed and what the characteristics of this development are. Here, I will restrict the questions to:

What do the student teachers focus on in a pedagogical situation, on their own, that is, without any expert drawing their attention to important moments?

How deep are their observations?

How do their evaluations of the same moment differ?

## METHODOLOGY

The participants of the study are student teachers, future mathematics teachers of pupils aged 11 till 19. They are in their 4th or 5th year of study. In particular, the students whose work is dealt with below were in year 4 and had one term of the Mathematics Education (or ME) course previously (partially not taught by me). From now on, “students” will be used for student teachers and “pupils” for pupils taught in the observed lesson.

In order to answer the research questions, we need to put students in a situation in which they will be confronted with a mathematics lesson but in which an educator’s influence is minimal. The first, obvious, type of data are received from *individual students* who are asked to write unstructured reflections about a video recording of the whole mathematics lesson. They watch it at home. However, a discussion between students can perhaps lead to a richer analysis. Thus, the second type of data is gathered from *pairs of students* who are asked to analyse a lesson on video. They do it at school, in an empty office, without the educator's presence, and they are being video recorded. In order to find out their immediate reactions, they are asked to stop the video whenever they feel that something deserves commenting on and to say the comment aloud to each other.

The collected data are organised in two ways: a) According to the lesson observed: the same videos of teaching have been used repeatedly so that reactions from different students are received. b) According to the type of origin, i.e., individuals' reflections, pairs' discussions, my teaching (videorecordings of the ME course in which video analyses are sometimes used), teaching practice (students' descriptions of didactical moments which they consider to be important when they observe lessons; their very choice and evaluation of these moments can be of importance).

The data collection still proceeds. In this article, I will restrict myself to the data connected to one particular lesson (see below) which was analysed by 3 pairs of students and 4 individual students. Their list follows (pseudonyms are used). In parentheses, the students' study results are given, received as a weighted average of their marks from mathematical courses during their first 3 years of study at the Faculty (1 is the best mark): A – 1, B – (1, 2), C – higher than 2.

Pairs (video recordings, transcripts, written reflections): John (B) and James (C), Molly (A) and Mark (B), Lota (A) and Meg (A)

Individuals (written reflections): Zina (B), Jack (B), Lance (C), Paul (B).

The students were told that they would be given a recording of an Australian mathematics lesson from Grade 8 from TIMSS Video Study 1999 and that the topic was the division of a quantity in a given ratio. The lesson in question was used on purpose – I believed that there was a lot to be noticed and, on the other hand, to be missed. Moreover, I supposed that the students would feel more interested in a foreign lesson.

The students were also given the teacher's preparation and self-reflection (written by her after viewing the video recording of her own lesson) and pupils' worksheets. They watched the video in English with the Czech subtitles. Pairs of students could write a reflection if they wanted (to complement their discussion while viewing the video), while the individuals were obliged to write a reflection. It was an unstructured reflection. They were told that they could write whatever they wanted or felt important.

In the data analysis, I had in mind six key moments which, in my opinion, were important from the point of view of the mathematical content and its presentation in the lesson. Their short description together with my perception from the lesson in question follows.

**1. Manipulation.** The division of a quantity in a given ratio is introduced using the model of cubes and boxes. This should help pupils to build an image of the whole process.

Comment: The pupils first work with cubes and create ratios such as 1 : 2, 5 : 8, etc. Then they work with empty boxes. When solving problems, they are asked to first model the situation and only then to calculate.

**2. Block versus box.** While blocks are counted as separate individuals, the empty boxes stand for a certain unknown number (or amount). Each must contain the same number (or amount). The letters *a, b* in the ratio $a : b$ stand not only for a certain number of things but also for groups of (or boxes full of) things.

Comment: The pupils are asked to imagine that there is a certain number of things (or a certain amount of money) in each box and to solve problems such as divide 210 dollars in the ratio of 2 : 5. The teacher often refers to the boxes and asks, e.g., how many things are in one box (when looking for a unit quantity). The pupils are asked to actually move boxes on their desk to the left or right according to the ratio.

**3. Relationship between the ratio and quantity**. In order for the division of a quantity in a given ratio to have integer answers, the whole quantity must be divisible by a unit quantity.

Comment: The teacher wants the pupils to think of their own story problems with ratios but she realises that there might be a problem if they do not see the relationship in question. She probably thinks that a non-integer answer would add to the cognitive burden and unnecessarily lead the pupils away from the idea of ratios. She, therefore, asks them whether they see this relationship. The pupils seem not to know what to do so the teacher points to the already solved ratios and to the numbers which she deliberately chose. When one girl says that the quantity must be "easily divisible", the teacher picks her idea up and explains the relationship. The question remains whether this important idea could have been found by the pupils themselves when trying to think up (and solve) their own story problems.

**4. Simplifying ratios**. We know from the teacher's reflection that the pupils should know about simplifying ratios from the previous lesson.

Comment: In the classwork, the need to simplify ratios does not arise. When the pupils work on posing problems, the teacher moves around and check them. A pupil has a ratio of 4 : 6 and the teacher says that "it would be better as 2 : 3, because we like simple ratios". After a minute, she can see another pupil with a ratio of 6 : 3 and this time, she does not mention this possibility. There is no comment on simplifying ratios later during the classwork.

**5. Two methods**. The unitary method is based on finding the unit and then multiplying it by the numbers in the ratio. The fraction method enables us to calculate each share by multiplying the quantity by a fraction, i.e., given $a : b$, quantity $q$, then the first share is $a / (a + b)$ times $q$, etc.

Comment: The teacher demonstrates the fraction method on 3 examples written on the board and previously solved by the unitary method. In my opinion, it is rather quick and the pupils do not have any opportunity to actually try it. No wonder that, when asked to vote which method they prefer, they vote for the unitary method (which they used throughout the lesson).

**6. Pupils' problem posing (or PP).** When asked to pose their own problems, pupils are encouraged to think about the matter more deeply and the teacher can assess to what extent they understand it and where the problems lie. It is usually motivating for them. In my opinion, it is advisable to ask pupils to solve the problems, too, as it makes them focus on the mathematical part as well as the context.

Comment: The teacher asks the pupils to think of their own question with a ratio and then talks about making a "story". This might have contributed to most pupils producing a story without a question.

The problem posing activity enabled the pupils to grasp the difference between the two types of task: to look for a ratio, and to divide a quantity in a given ratio. The pupils apparently mixed the two types together and the teacher became aware of this fact only on the basis of this activity (based on her reflection).

The above six key moments were the springboard from which I started the data analysis. All the data were uploaded to the software *Atlas.ti* as separate documents. The documents were coded first using the six items (their names were used as the code names) and then open coded in the sense of Strauss and Corbin (1998), analysing a whole sentence or a paragraph rather than line-by-line because, especially in the pair experiment, one idea was spread in students' several utterances.

During the coding process, five more codes emerged as important for some students. Thus, I tracked them in all the reflections.

**7. Involvement of pupils**. It shows to what extent the pupils are actively involved in the construction of new knowledge (as far as we can say that from the video recording only!) and other mathematical work in the lesson. It involves two free codes: Pupils' activity and Pupils' understanding.

Comment: It is difficult to generalize, but at many stages of the lesson I have the impression that the pupils are not given enough time to think the questions over and find the solutions themselves, but rather that they are given the solutions by the teacher immediately. They are almost never encouraged to explain their thinking or strategies, but rather the teacher offers the explanation and corrects their mistakes.

**8. Elaboration – consequences.** It involves the elaboration of the observed teaching practice in terms of its possible consequence for the pupils' understanding or for the flow of the lesson. (See below for examples.)

**9. Elaboration – their teaching**. It concerns the elaboration of the observed teaching practice in terms of its possible connections with the students' future teaching practice.

**10. Alternatives**. It means suggesting an alternative action to what actually happened.

**11. General perception**. It means a general perception of the lesson based on the codes Chaotic versus calm, Teacher's personality, Teaching method, Appraisal / Criticism of the teaching practice, Classroom environment, Empathy for the teacher.

## PRELIMINARY RESULTS

The results will be first presented in the form of two tables and then discussed.

Explanation: "+" – the student mentioned the item (it will sometimes be briefly given in what way), "x" – it did not appear. T stands for the teacher, Ps for pupils. In item 7, | means a reference to pupils' potential understanding. In item 10, | means a reference to the mathematics of the lesson, * to the organisation of the lesson.

<table>
<tr><th>Pairs</th><th>John + James</th><th>Molly + Mark</th><th>Lota + Meg</th></tr>
<tr><td>1. Manipul.</td><td>+ no elaboration</td><td>+ "good idea"</td><td>+ good for Ps, they "see it"</td></tr>
<tr><td>2. Block/box</td><td>+ consider them the same</td><td>x</td><td>+ see the difference</td></tr>
<tr><td>3. Ratio vs. quantity</td><td>x</td><td>x</td><td>+ T should simply say it as a rule</td></tr>
<tr><td>4. Simplify</td><td>x</td><td>x</td><td>x</td></tr>
<tr><td>5. Two methods</td><td>x</td><td>+ very quick, voting nonsense</td><td>+ not 2 methods but a different notation, T should've stressed the common properties; voting nonsense</td></tr>
<tr><td>6. Pupils'PP</td><td>+ consider it nonsense</td><td>+ good, story vs. task</td><td>+ good</td></tr>
<tr><td>7. Involv. of pupils<br>Ps' unders.</td><td>+ T shows the methods, explains where there is a mistake<br>|</td><td>+ pupils are only passively involved<br>|</td><td>x<br>|||||</td></tr>
<tr><td>8. Conseq.</td><td>x</td><td>+ PP – T can see how Ps understand</td><td>x</td></tr>
<tr><td>9. Teaching</td><td>x</td><td>+ "I tried to imagine myself in T's shoes."</td><td>+ "What to do with quick pupils?"</td></tr>
<tr><td>10. Altern.</td><td>****</td><td>**|*|*</td><td>|||||****|*|||</td></tr>
<tr><td>11. General perception</td><td>chaotic, no system, T lacks organis. skills, no bird's view eye, doesn't care what Ps do, doesn't understand what Ps say</td><td>T is calm, does not get angry, no emotions, Ps comfortable with the work</td><td>T changes activities</td></tr>
</table>

| **Individuals** | **Zina** | **Jack** | **Lance** | **Paul** |
|---|---|---|---|---|
| **1. Manipul.** | x | + good for Ps, but not enough time | + good for | x |

| | | | | |
|---|---|---|---|---|
| | | for solution | understanding | |
| **2. Block versus box** | x | x | + can see the difference | x |
| **3. Ratio vs. quantity** | + it is the key question | + thinks that Ps found it | + Ps should have found it in PP | x |
| **4. Simplify** | x | x | x | x |
| **5. Two m.** | x | x | x | + good |
| **6. Pupils' PP** | + good | + good | + good | + good, but above Ps' abilities |
| **7. Involv. of pupils**<br>**Ps' under.** | + Ps discover the knowledge for themselves | + not enough time for own discovery of knowledge | + Ps not involved enough | |
| **8. Conseq.** | + PP – good for cooperation, application of math. in reality, motivating | + PP – good for Ps' understanding | x | + PP – breaks stereotype of problem solving |
| **9. Teaching** | x | x | x | + "I have tried PP with pupils." |
| **10. Altern.** | * | x | *** | x |
| **11. General perception** | T leads Ps from concrete to abstract knowledge, towards relationships, waits till Ps find knowledge themselves | a calm lesson, probably too calm | too noisy, more discipline is needed | x |

If the item is missing from the students' reflections, we can presume that they did not notice it <u>or</u> did not attribute any importance to it.

## DISCUSSION OF RESULTS

### Discussion of individual items

Manipulation was seen as important for the mathematical content of the lesson 4 times out of 7, however, only Lance and one pair could see the difference represented by blocks and boxes. Despite the teacher's frequent reference to it, John and James consider them the same and from their discussion we can infer that they are lost in the mathematical part of the activity. This aspect, which I see as important for the development of pupils' knowledge of ratio, was not mentioned at all 4 times out of 7.

The question about the relationship between the quantity and ratio was noticed 4 times out of 7 but another mathematical item about simplifying ratios was not addressed at all. The "two method" item was only mentioned 3 times and in 2 of them, the vote was rejected as nonsense on the grounds that the pupils did not have time to actually try it.

Pupils' problem posing was commented on by all students and mostly judged positively. John and James have another view but they do not give any reason for it.

The students made interpretative comments, too. In most cases they commented upon the problem posing activity and its advantages. The reason why they actually thought about this type of activity deeper might be that it was novel for them. In Czech schools, problem posing by pupils is quite rare. Only 3 times, the students elaborated a little on what they saw from the point of view of their (future) role as teachers.

In many cases, the students suggested alternative actions for both the organisational and mathematical aspects of the lesson, often after a critical remark about what actually happened in the lesson.

**Comparison of reflections**

Two pairs stand out in the quality of reflection. At the one end of the spectrum, John and James made a lot of critical remarks but only suggested alternatives to the organisational aspects of the lesson. They probably did not give much thought to the mathematical part (except for frequent comments at the beginning of the lesson that "it makes no sense what the teacher does") and did not think about the types of tasks the teacher used. Their dialogue is mainly descriptive without any elaboration of what the event might mean. They are extremely critical about the lesson and, of the 10 students are the only ones to make critical comments on the personality of the teacher and her skills.

At the other end, Meg and Lota also did not understand at first where the teacher was heading with modelling but after much effort and discussion, they grasped it. They comment on nearly all mathematical items. They make the most references to pupils' possible understanding and suggest the most alternatives, most of which are for the mathematics of the lesson. Their level of reflection is deeper than the boys' one. I believe that, among others, their content knowledge might have influenced this difference. While Meg and Lota have A's, John has B and James has C. Their insufficient knowledge of mathematics and thus inability to see where the teacher was leading the pupils might have influenced their appraisal of the lesson.

Finally, quite surprisingly for me, there are opposing views concerning the same items. While Jack believes that the pupils discovered the relationship between the ratio and quantity themselves, Lance suggests otherwise as he points out that the pupils should be allowed to discover it when posing problems.

The involvement of pupils in the development of knowledge is also differently judged. While Molly and Mark, and Lance (and indirectly also John and James) think that the pupils were rather passive and the teacher did the explanation, Zina believes that the pupils were actively involved and Lance suggests that the teacher wants them to be more involved but that allows them little time.

The general impression from the lesson differs widely. John and James, quite understandably considering the above, see the lesson as chaotic, with no system, and have little empathy for the teacher. Molly and Mark as well as Jack consider the lesson calm and the pupils comfortable with the work. For Lance, there is little discipline and too much noise in the lesson.

It might have been illuminating to let the students discuss their opposing views to see on what grounds they put their claims. As it is, we have little information as to the reasons for the discrepancies.

Star and Strickland (2008) also studied preservice teachers' uninfluenced responses to a lesson on video, thus it seems appropriate to compare their results with mine. They let the students watch the video and take notes and then asked them questions concerning 5 aspects of the lesson which they should answer based on their memory and notes. (They did not look, however, into how the students interpreted the events.) The five aspects were: Classroom environment, Classroom management, Communication, Tasks (refer to the activities pupils do in the class; it includes my code Pupils' problem posing), Mathematical content (it includes my codes Manipulation, Block versus box, Relationship between the ratio and quantity, Simplifying ratios, Two methods). The first three dimensions are not among my codes as the students did not mention them. My remaining codes concern interpretation and, as such, cannot be put into the five categories.

Star and Strickland (ibid) found that without any training, the investigated student teachers were good observers of Classroom management, quite attentive to the category of Tasks and did least well on Classroom environment (in my study, the students hardly mentioned it, too) and Mathematical content. The authors say that "preservice teachers largely did not notice subtleties in the ways that the teacher helped students think about content" and "the mathematics of the lesson and the students' understandings of that mathematics were not noticed [...], either in the initial or in the second viewing of the video" (p. 118). This is echoed in the preliminary findings of my study where the mathematics of the lesson was rarely attended to.

## FUTURE WORK

In order to answer my research questions, more analysis is needed. While doing the open coding, the elements of the following stage of analysis, that is axial coding, gradually emerged and some categories began to be assembled. Clearly, some codes are connected with the mathematics in the presented lesson only (e.g., Two methods) while others are more general (e.g., Alternatives). Some codes are closely tied (e.g., Alternatives and Elaboration – their teaching). In my further work, the various types of data for different lessons will be coded. It is assumed that during this process some categories will emerge which would help me to concentrate on some of them not in one type of data or in the data tied to one particular lesson, but more generally. It may also be valuable to compare reflections received from individuals and those from pairs. Does a discussion between students influence the depth of their considerations? This will also be the focus of my future work.

## REFERENCES

Ainley, J. & Luntley, M. (2006). What teachers know: the knowledge bases of classroom practice In M. Bosch (Ed.), *Proceedings of CERME4* (pp. 1410–1419). Spain: Fundemi Business Institute.

An, S. (2004). *The Middle Path in Math Instruction. Solution for Improving Math Education.* Lanham: Scarecrow Education.

Bromme, R. (2008). Pedagogical content knowledge jako konceptuální východisko pro výzkum moudrosti praktiků. In T. Janík a kol. (Eds.), *Metodologické problémy výzkumu didaktických znalostí obsahu* (s. 9–16). Brno : Paido.

Hošpesová, A., Tichá, M. & Macháčková, J. (2005). Differences and similarities in (qualified) pedagogical reflection. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of CERME5* (pp. 1906–1915). Cyprus: University of Cyprus.

Mason, J. & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics, 38(1–3)*, 135–161.

Muñoz-Catalán, M. C., Carrillo, J. & Climent, N. (2007). The professional development of a novice teacher in a collaborative context: an analysis of classroom practice. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of CERME5* (pp. 1935–1944). Cyprus: University of Cyprus.

Santagata, R., Zannoni, C. & Stigler, J. W. (2007). The role of lesson analysis in pre-service teacher education: An empirical investigation of teacher learning from a virtual video-based field experience. *Journal of Mathematics Teacher Education, 10(2)*, 123–140.

Sherin, M. G. & van Es, E. (2005). Using video to support teachers‘ ability to notice classroom interactions. *Journal of Technology and Teacher Education*, *13(3)*, 475–491.

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15(2)*, 4–14.

Star, J. R. & Strickland, S. K. (2008). Learning to observe: using video to improve preservice mathematics teachers' ability to notice. *Journal of Mathematics Teacher Education*, 11, 107–125.

Strauss, A. & Corbin, J. (1998). *Basics of qualitative research. Techniques and procedures for developing grounded theory*. Thousand Oaks: SAGE publications.

The article was supported by the research grant “Teaching profession in the environment of changing education requirements” No. MSM 0021620862.

# THE MATHEMATICAL PREPARATION OF TEACHERS: A FOCUS ON TASKS

Gabriel J. Stylianides — University of Pittsburgh, U.S.A.
Andreas J. Stylianides — University of Cambridge, U.K.

*In this article we elaborate a conceptualization of mathematics for teaching as a form of applied mathematics (building on Bass's idea of characterizing mathematics education as a form of applied mathematics) and we examine implications of this conceptualization for the mathematical preparation of teachers. Specifically, we discuss issues of design and implementation of a special kind of mathematics tasks whose use in teacher education is intended to promote mathematics for teaching.*

The notion of *Mathematics for Teaching* (*MfT*) (Ball & Bass, 2000) describes the mathematical content that is important for teachers to know and be able to use in order to manage successfully the mathematical issues that arise in their practice. According to Ball and Bass (2000), this specialized kind of mathematical knowledge, referred to as *Mathematical Knowledge for Teaching* (*MKfT*), is important for solving the barrage of "mathematical problems of teaching" that teachers face as they teach mathematics: offering mathematically accurate explanations that are understandable to students of particular ages, validating student assertions, etc.

In this article, we focus on the following research question: What kind of learning opportunities might mathematics teacher education programs design to effectively support the development of prospective teachers' MKfT? To address this question, we elaborate a conceptualization of MfT as a form of applied mathematics and probe the implications of this conceptualization for the mathematical preparation of teachers, with particular attention to the nature of mathematics tasks that might be important for use in mathematics (content) courses for prospective teachers. To exemplify the constructs we discuss in the article, we use data from a research-based mathematics course for prospective elementary teachers in the United States.

## CONCEPTUALIZING MATHEMATICS FOR TEACHING AS A FORM OF APPLIED MATHEMATICS

In thinking about the problem of teachers' mathematical preparation, we found useful Bass's (2005) suggestion of viewing mathematics education as a form of applied mathematics: "[Mathematics education] is a domain of professional work that makes fundamental use of highly specialized kinds of mathematical knowledge, and in that sense it can […] be usefully viewed as a kind of applied mathematics" (p. 418). Given that mathematics education makes use of specialized knowledge from several other fields in addition to mathematics (psychology, sociology, linguistics, etc.), we propose that the characterization "form of applied mathematics" be used to refer specifically to the mathematical component of mathematics education, notably MfT.

The conceptualization of MfT as a form of applied mathematics calls attention to the domain of application of MfT (i.e., the work of mathematics teaching) and the specialized nature of “mathematical problems of teaching” (Ball & Bass, 2000). In particular, the conceptualization has two important and interrelated implications for the mathematical preparation of teachers, which are aligned with existing research and theoretical accounts in the area of MKfT.

First, the conceptualization implies that the mathematical preparation of teachers should take seriously the idea that “there is a specificity to the mathematics that teachers need to know and know how to use” (Adler & Davis, 2006, p. 271). This idea relates to broader epistemological issues about the situativity of knowledge (e.g., Perressini et al., 2004) and to research findings that different workplaces require specialized mathematical knowledge by their practitioners (e.g., Hoyles et al., 2001).

Second, the conceptualization implies that the mathematical preparation of teachers should aim to “create opportunities for learning subject matter that would enable teachers not only to know, but to learn to use what they know in the varied contexts of practice” (Ball & Bass, 2000, p. 99). In other words, it underscores the importance of the development of a “pedagogically functional mathematical knowledge” (ibid, p. 95), which can support teachers to solve successfully mathematical problems that arise in their work. The characterization of MKfT as “pedagogically functional” helps clarify further the meaning we assign to the term “applied mathematics” in the proposed conceptualization of MfT. Specifically, our use of this term refers to mathematics that is (or can be) *useful for* and *usable in* mathematics teaching (the domain of application), and thus, important for teachers to know and be able to use when they teach mathematics (i.e., when they function in the domain of application).

Acceptance of the conceptualization of MfT as a form of applied mathematics necessitates that mathematics courses in teacher education design opportunities for prospective teachers to learn and use mathematics from the perspective of a *teacher of mathematics*. How might these opportunities be designed in teacher education?

Given the central role that mathematics tasks can play in individuals’ learning experience in classrooms, we considered fruitful to begin to address the question above (which is a reformulation of our research question) by conceptualizing a special kind of mathematics tasks that we call *Pedagogy-Related mathematics tasks* (*P-R mathematics tasks*). These tasks are intended to embody essential elements of MfT as a form of applied mathematics and support mathematical activity that can enhance the development of prospective teachers’ MKfT.

## “PEDAGOGY-RELATED MATHEMATICS TASKS”: A VEHICLE TO PROMOTING MATHEMATICAL KNOWLEDGE FOR TEACHING

### Feature 1: A primary mathematical object

Like all other kinds of mathematics tasks, P-R mathematics tasks have a *primary mathematical object.* This is intended to be the main focus of prospective teachers’ attention and to engage them in activity that is primarily mathematical (as opposed to pedagogical). The mathematical object of a P-R mathematics task can take different forms such as validation of a conjecture or description of the mathematical relationship between two methods for obtaining the same mathematical result.

**Feature 2: A focus on important aspects of MKfT**

Like most other kinds of mathematics tasks used in mathematics courses for prospective teachers, the mathematical object of a P-R mathematics task relates to one or more mathematical ideas that have been suggested by theory or research on MKfT as being important for teachers to know (see, e.g., Stylianides & Ball, 2008). In our work with prospective teachers we pay special attention to such ideas that are also *fundamental* (Ma, 1999) and *hard-to-learn* for both students and teachers.

**Feature 3: A secondary but substantial pedagogical object and a corresponding pedagogical space**

The defining feature of P-R mathematics tasks is that they have a *secondary pedagogical object.* This object is substantial (i.e., it is an integral part of the task and important for its solution) and situates the mathematical object of the task in a particular *pedagogical space* that relates to school mathematics and, ideally, derives from actual classroom records. The pedagogical object and the corresponding pedagogical space of a P-R mathematics task help engage prospective teachers in mathematical activity from the perspective of a *teacher of mathematics*.

Consider for example a P-R mathematics task whose mathematical object is the development of a proof for a conjecture. The pedagogical object of this task could be a teacher's need that the proof be appropriate for the students in his/her class. The corresponding pedagogical space could be a description (scenario) of what the solvers of the P-R mathematics task might assume the students in the class to know in relation to mathematical content that is relevant to the task. Thus the solution of the task cannot be sought in a purely mathematical space, but rather in a space that intertwines content and pedagogy. As a result, the task can generate mathematical activity that is attuned to particular mathematical demands of mathematics teaching.

Next we discuss four points related to feature 3 of P-R mathematics tasks. First, the pedagogical object/space of a P-R mathematics task, and especially its connection to (actual) classroom records, can embody the ideas of "situativity of knowledge" and "pedagogical functionality" that we discussed earlier in relation to MfT as a form of applied mathematics. Specifically, the pedagogical object can support development of mathematical knowledge that is applicable in a particular context (pedagogical space) within the broader work of mathematics teaching.

Second, the pedagogical space of a P-R mathematics task determines to great extent what counts as an acceptable/appropriate solution to the task, because it provides a set of conditions with which a possible solution to the task needs to comply. This is important, because, almost always in teaching, a purely mathematical approach to a "mathematical problem of teaching" does not address adequately the different aspects of the pedagogical space in which the problem is embedded.

Third, given the complexities of any pedagogical situation, it is often impractical (if not impossible) to specify all the parameters of the situation that can be relevant to the mathematical object of a P-R mathematics task. This lack of specificity can be useful for teacher educators who implement P-R mathematics tasks with their prospective teachers: teacher educators can use the endemic ambiguity surrounding the pedagogical space in order to vary some of its conditions and create opportunities for prospective teachers to engage in related mathematical activities within the particular pedagogical space. The variation of conditions of the pedagogical space (and the mathematical activities that can result from this variation) can offer prospective teachers practice with grappling with the barrage of mathematical issues that arise (often unexpectedly) in almost every instance of a teacher's practice.

Fourth, the pedagogical object/space of a P-R mathematics task have the potential to motivate prospective teachers' engagement in the task by helping them see and appreciate why the mathematical ideas in the task are or might be important for their future work as teachers of mathematics. According to Harel (1998), "[s]tudents are most likely to learn when they see a need for what we intend to teach them, where by 'need' is meant intellectual need, as opposed to social or economic need" (p. 501; the original was in italics). In the case of prospective teachers, a "need" for learning mathematics may be defined in terms of developing mathematical knowledge that is useful for and usable in the work of teaching. By helping prospective teachers see a need for, and thus develop an interest in, the material that teacher educators engage them with, teacher educators increase the likelihood that prospective teachers will learn this material. This is particularly useful in relation to material that prospective teachers tend to have difficulty to see as relevant to their future teaching practices.

## EXEMPLIFYING THE USE OF P-R MATHEMATICS TASKS IN A MATHEMATICS COURSE FOR PROSPECTIVE TEACHERS

### General description of the course

The course was the context of a *design experiment* (see, e.g., Cobb et al., 2003) that we conducted over a period of four years and that aimed to develop practical and theoretical knowledge about ways to promote prospective teachers' MKfT. It was a three-credit undergraduate-level mathematics course for prospective elementary teachers, prerequisite for admission to the masters-level elementary teaching certification program at a large state university in the United States. It was the only mathematics content course in the admission requirements for the program,[1] and so it was designed to cover a wide range of mathematical topics. The students in the course pursued undergraduate majors in different fields and tended to have weak mathematical backgrounds. Also, given that the students were not yet in the teaching certification program, they had limited or no background in pedagogy.

[1] The students who are admitted to the teaching certification program take also a mathematics pedagogy course, but the focus of this course is on teaching methods.

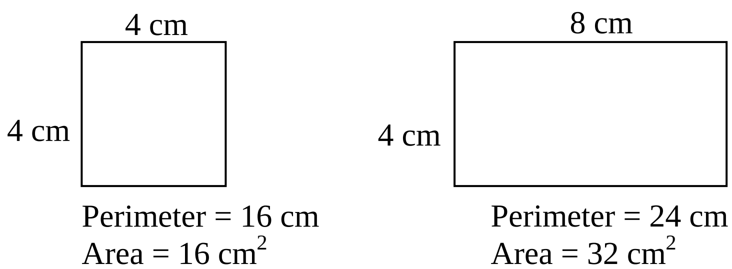
The most relevant aspect to this article of the approach we took in the course to promote MKfT is the design and implementation of task sequences that included both P-R mathematics tasks and *typical mathematics tasks*, which embody only features 1 and 2 of P-R mathematics tasks. A common task sequence in the course began with a typical mathematics task that engaged prospective teachers in mathematical activity from an adult's point of view. The P-R mathematics task that followed described some pedagogical factors that prospective teachers needed to consider in their mathematical activity. To satisfy feature 3 of P-R mathematics tasks about situating prospective teachers' mathematical activity in a pedagogical space, we used a range of actual classroom records such as video records or written descriptions (as in scholarly publications) of classroom episodes, excerpts from student interviews or textbooks, etc. Less frequently and when actual classroom records were unavailable, we used (similar to Biza et al., 2007) fictional but plausible classroom records.

## An example of a task sequence and its implementation in the course

We illustrate the use of P-R mathematics tasks in the course with a task sequence that included a typical and a P-R mathematics task. To develop this and other task sequences in the course we followed a series of five research cycles of implementation, analysis, and refinement over the years of our design experiment. In this article we use data from the last research cycle that involved enactment of the course in two sections; these sections were attended by a total of 39 prospective teachers and were taught by the first author. Specifically, the data come from one of the two sections and include video and audio records of relevant classroom episodes, and fieldnotes that focused on prospective teachers' small group work.

The focal task sequence aimed to promote prospective teachers' knowledge about a possible relation between the area and perimeter of rectangles, with special attention to the ideas of generalization and proof by counterexample, which are considered important for elementary mathematics teaching (see feature 2 of P-R mathematics tasks in relation to Stylianides and Ball, 2008). The task sequence is an adaptation of an interview task used by Ma (1999) and developed originally by Ball (1988).

> Imagine that one of your students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that <u>as the perimeter of a rectangle increases, the area also increases</u>. She shows you this picture to prove what she is doing:




1. Evaluate mathematically the student statement? (underlined)

2. How would you respond to this student?

Although question 1 refers to a student statement, it is essentially a typical mathematics task because the prompt asks prospective teachers to evaluate mathematically the statement, without asking (or expecting) them to take account of the fact that the statement was produced by a student. Question 2, on the other hand, is a P-R mathematics task because it introduces a student consideration that prospective teachers need to consider in their mathematical activity. The *mathematical object* of this P-R mathematics task is to evaluate mathematically the underlined statement, which is essentially what the prospective teachers were asked to do in question 1 (a teacher would need to know about the correctness of the statement before deciding how to respond to the student who produced it). The *pedagogical object* of the task is the teacher's need to respond to the student who produced the statement. The *pedagogical space* is the (fictional) scenario in the task with a student announcing enthusiastically to the teacher a mathematical "discovery," which was supported by a single example in the domain of the corresponding statement. Although an appropriate response to question 1 could say that the statement is false and provide a counterexample to it, an appropriate response to question 2 would need to include more than that. Specifically, from a pedagogical standpoint, it would be useful and important for the student's learning if the teacher did not just prove her statement false, but also helped her understand why the statement is false and the mathematical conditions under which the statement is true.

The prospective teachers in the course worked on the two questions first individually, then in small groups, and later in the whole class. The whole class discussion started with the teacher educator asking different small groups to report their work on the task, beginning with question 1 (all prospective teacher names are pseudonyms).

*Andria:* We said that it [the student statement] was mathematically sound because as you increase the size of the figure, the area is going to increase as well.

*Tiffany:* We thought the same, because as the sides are getting bigger… [inaudible]

*Stylianides:* Does anybody disagree? [no group expressed a disagreement]

*Evans:* I agree. [Evans was in a different small group than both Andria and Tiffany]

*Stylianides:* And how would you respond to the student?

*Melissa:* I think it's true but they haven't proved it for all numbers so it's not really a proof.

*Andria:* I think that you don't have to try every number [she means every possible case in the domain of the statement] to be able to prove it because if the student can explain why it works like we just did, like if you increase the length then the area increases. [pause]

*Stylianides:* Yeah, so it's impossible to check all possible cases [of different rectangles].

*Meredith:* I'd say that it's an interesting idea, and I'd see if they can explain why it works.

As the excerpt shows, all small groups believed that the student statement was true, but at the same time they realized that the evidence the student provided for her claim was not a proof (see, e.g., Melissa's comment). As a result, the prospective teachers started to think how they could prove the statement and what they could respond to the student. For example, Andria observed that it would be impossible to check every possible case. Also, both Andria and Meredith pointed out that the student needed to explain why (i.e., prove that) the area of a rectangle increases as its perimeter increases. Yet, the teacher educator knew that the statement was false, and so he probed the prospective teachers to check more cases and see whether they could find an example where the student statement failed. All small groups found quickly at least one counterexample to the statement and concluded that it was actually false.[2]

The prospective teachers did not expect this intuitively "obvious" statement to be false, so they became motivated to work further on question 2. The teacher educator gave them more time to think about this question in their small groups. The excerpt below is from the whole class discussion that followed the small group work.

*Natasha:* We said that the way that they [the students] are doing it, where they're just increasing the length of one side, it's always going to work for them but if they try examples where they change the length on both sides that's the only way it's going to prove that it doesn't work all the time. So you should try examples by changing both sides.

*Stylianides:* What do you think about Natasha's response? Does it make sense? [the class nodded in agreement] So what else? What else do you think about this?

*Evans:* You can kind of ask them to restructure the proof so that it would work.

*Stylianides:* What do you mean by "restructure the proof"?

*Evans:* Like once they figure out that it doesn't work for all cases they could say it's still like… if they saw it and if they revise it like the wording or just add a statement in there that if they can come up with a mathematically correct statement…

*Stylianides:* Anything else? [no response from the class]
I think [that] both ideas [mentioned earlier] are really important. So when you have something [a statement] that doesn't work, then it's clear that this student would be interested to know more. For example, why it doesn't work or under what conditions does it work because, obviously, some of the examples that the student checked worked. […]

Natasha and Evans proposed two related issues that the elementary teacher in the task scenario could address when responding to the student: why the statement is false and the conditions under which the statement would be true. Based on our planning for the implementation of the task, the teacher educator would raise these issues anyway, because, as we explained earlier, a teacher response to the student that would consist only of a counterexample to the statement would be mathematically sufficient but pedagogically inconsiderate. The fact that the two issues were raised by prospective

[2] The prospective teachers had opportunities earlier in the course to discuss the idea that one counterexample suffices to show that a general statement is false.

teachers instead of the teacher educator is noteworthy, because Natasha and Evans had no teaching experience and also the issues they raised were requiring further mathematical work for themselves and the teacher education class. Take for example Evans's contribution, which raised essentially the following new mathematical question: Under what conditions would the statement be true? It is hard to explain what provoked Natasha and Evans's contributions, but we hypothesize that the pedagogical object/space of the P-R mathematics task played an important role in this. Specifically, we hypothesize that the need to respond to a false but plausible student statement made the prospective teachers think hard about related mathematical issues and how to "unpack" them in pedagogically meaningful ways (Ball & Bass, 2000; see also Adler & Davis, 2006).

Following the summary of the two issues as in the previous excerpt, the teacher educator engaged the prospective teachers in an examination of the conditions under which the student statement would be true. A more detailed discussion of the prospective teachers' work on the task sequence is beyond the scope of this article.

To conclude, our discussion in this section exemplified the idea that the application of mathematical knowledge in contextualized teaching situations can be different than its application in similar but purely mathematical contexts. Although the mathematical objects of the typical and P-R mathematics tasks in the sequence were the same (namely, the mathematical evaluation of a statement about a possible relation between the area and perimeter of rectangles), the pedagogical space in which the P-R mathematics task was embedded changed what could count as an appropriate solution to it, thereby generating mathematical activity in a combined mathematical and pedagogical space.

## CONCLUDING REMARKS

Although the primary object of P-R mathematics tasks is mathematical, their design, implementation, and solution require some knowledge of pedagogy. This requirement derives primarily from the pedagogical objects of P-R mathematics tasks, which, although secondary to the tasks, determine to great extent what counts as acceptable/appropriate solutions to the tasks and influence the mathematical activity (to be) generated by the primary objects of the tasks. For example, the design of the P-R mathematics task that we discussed earlier used knowledge about a common student misconception regarding the relation between the area and perimeter of rectangles. Furthermore, successful implementation and solution of this task required appreciation of the pedagogical idea that a mere counterexample might be a limited teacher response to a flawed but plausible student statement.

The pedagogical demands implicated by the design, implementation, and solution of P-R mathematics tasks make it reasonable to say that instructors of mathematics courses for prospective teachers need to have, in addition to good knowledge of mathematics, knowledge of some important pedagogical ideas. This requirement might be hard to fulfill in contexts such as the North American where mathematics courses for prospective teachers are typically offered by mathematics departments and are taught by (research) mathematicians. However, if such knowledge is agreed to be essential for teaching MfT to prospective teachers, then the field of mathematics teacher education needs to find ways to support the work of instructors of mathematics courses for prospective teachers. One way might be to offer instructors access to what we may call *educative teacher education curriculum materials*. This is the teacher education equivalent of the notion of educative curriculum materials, i.e., curriculum materials that aim to promote teacher learning in addition to student learning at the school level (see, e.g., Davis & Krajcik, 2005).

The pedagogical aspects of P-R mathematics tasks raise also the following question: Would it make sense to promote MKfT in mathematics courses designed specifically for prospective teachers, or would it make more sense to promote it in combined mathematics/pedagogy courses, which, by definition, pay attention to both pedagogical and mathematical issues? The idea of promoting MKfT in combined mathematics/pedagogy courses may be attractive to some given the potential of P-R mathematics tasks to intertwine mathematics and pedagogy. Yet a possible decision to eliminate mathematics courses designed specifically for teachers in favor of combined mathematics/pedagogy courses might create different kinds of problems. In their examination of different types of tasks in formal assessments used across a range of mathematics teacher education courses in South Africa, Adler and Davis (2006) reported the concern that in combined mathematics/pedagogy courses the mathematical and pedagogical objects lose their clarity and that evaluation in these courses tends to condense meaning toward pedagogy.

The conceptualization of MfT as a form of applied mathematics that we elaborated in this article highlights the idea that, irrespectively of whether MfT is promoted in specialized mathematics courses or combined mathematics/pedagogy courses, prospective teachers' learning of MfT should not happen in isolation from pedagogy. P-R mathematics tasks can facilitate the integration of mathematics and pedagogy in prospective teachers' learning: although these tasks make mathematics the focus of prospective teachers' activity, they situate this activity in a substantial pedagogical space that shapes and influences the activity. Future research may explore ways in which to facilitate the integration of mathematics and pedagogy from the opposite direction, i.e., by making pedagogy the focus of prospective teachers' activity and having mathematics play a secondary but substantial role in this activity. Towards this end, one can reverse the relative importance of mathematical and pedagogical objects in P-R mathematics tasks to coin the twin notion of *Mathematics-Related pedagogy tasks.* Specifically, these tasks can be defined to have a primary pedagogical object (with a corresponding pedagogical space) and a secondary but substantial mathematical object, and can be used to generate activity that is predominantly pedagogical (as opposed to mathematical in P-R mathematics tasks).

## AUTHOR NOTE

The two authors contributed equally to the preparation of this article. The research was supported by funds from the Spencer Foundation (Grant Numbers: 200700100, 200800104). The opinions expressed in the article are those of the authors and do not necessarily reflect the position, policy, or endorsement of the Spencer Foundation.

## REFERENCES

Adler, J., & Davis, Z. (2006). Opening another black box: Research Mathematics for Teaching in Mathematics Teacher Education. *Journal for Research in Mathematics Education, 37,* 270-296.

Ball, D. L. (1988). Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education. Unpublished doctoral dissertation, Michigan State University, East Lancing.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 83-104). Westport, CT: Ablex Publishing.

Bass, H. (2005). Mathematics, mathematicians, and mathematics education. *Bulletin of the American Mathematical Society, 42*, 417-430.

Biza, I., Nardi, E., & Zachariades, T. (2007). Using tasks to explore teacher knowledge in situation-specific contexts. *Journal of Mathematics Teacher Education*, *10*, 301-309.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

Davis, E. A., & Krajcik, J. S. (2005). Designing educative curriculum materials to promote teacher learning. *Educational Researcher, 34*(3), 3-14.

Harel, G. (1998). Two dual assertions: The first on learning and the second on teaching (or vice versa). *American Mathematical Monthly*, *105*, 497-507.

Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education, 32,* 4-27.

Ma, L. (1999). *Knowing and teaching elementary mathematics.* Mahwah, NJ: Lawrence Erlbaum.

Perressini, D., Borko, H., Romagnano, L., Knuth, E., & Willis, C. (2004). A conceptual framework for learning to teach secondary mathematics: A situative perspective. *Educational Studies in Mathematics*, *56*, 67-96.

Stylianides, A. J., & Ball, D. L. (2008). Understanding and describing mathematical knowledge for teaching: Knowledge about proof for engaging students in the activity of proving. *Journal of Mathematics Teacher Education*, *11*, 307-332.

# PROBLEM POSING AND DEVELOPMENT OF PEDAGOGICAL CONTENT KNOWLEDGE IN PRE-SERVICE TEACHER TRAINING

Marie Tichá, Alena Hošpesová

Institute of Mathematics of the Academy of Sciences of the Czech Republic, University of South Bohemia České Budějovice

*The paper focuses on problem posing as the possible method leading to development of pedagogical content knowledge of mathematics education in pre-service training of primary school teachers. In the background there is our belief that this knowledge is of utter importance for quality of the education process. Using samples of (a) problems posed by teacher students, (b) students' assessment of the problems posed, (c) students' opinions on the significance of "problem posing" in teacher training, we will demonstrate how we employed problem posing in pre-service teacher training. We start from the belief (proved in our previous work) that an analysis of the posed problems is a good diagnostic tool; it gives the opportunity to discover the level of understanding as well as the causes of misconceptions and errors.*

Keywords: mathematics education, teacher training, content knowledge, problem posing

## INTRODUCTORY REMARKS: MATHEMATISATION OF THE SOCIETY AND MATHEMATICAL LITERACY

On many different occasions we come across the signs of an increasing importance of mathematics in contemporary life, the opinion that the society is being "mathematised". We must understand mathematics if we are to be able to understand the world that surrounds us. That is why the need of mathematical literacy is more and more emphasized. These trends also impact the focus of the research in the field of didactics of mathematics (e.g. the central topic of PME 30 conference in 2006 was "Mathematics in the centre").

We understand mathematical literacy as functional. It begins with the ability to understand a mathematical text, the ability to recall mathematical terms, procedures and theory, to master the necessary mathematical apparatus and with the ability to apply it, to solve problems. However, in our view to be mathematically literate also means to "understand mathematics", to perceive it as an abstract discipline. Development of mathematical literacy triggers perfection of the ability to reason, of critical thinking, it teaches how to apply mathematics efficiently. To be functionally mathematically literate means to see the mathematics that surrounds us; to see the questions and problems arising both from real and mathematical situations. In order to educate mathematically literate pupils we need professionally competent teachers.

In our previous work we have been focusing on the potential of a qualified pedagogical reflection and we have showed that it is one of the possible ways of development of professional competence of primary school teachers (Tichá, Hošpesová, 2006). In this paper we show that problem posing represents another possible way. We also show the potential of problem posing in diagnosis of the teacher-students' subject didactic knowledge.

## THEORETICAL FRAMEWORK

### Professional competence and content knowledge

The calls for development of mathematical literacy make demands on professional competences of the teacher. In our previous research, especially the need for a good level of *subject didactic competence* appeared very strongly, i.e. the knowledge of mathematical content and its didactic elaboration as well as its realization in school practice (Tichá, Hošpesová, 2006). It corresponds with the following generally accepted Shulman's idea: if teaching should become a profession, it is necessary to aim at creating a *knowledge base for teaching* which encapsulates, in particular, *subject-matter content knowledge, pedagogical content knowledge, and curriculum knowledge* (Shulman, 1986). It is the knowledge of mathematical content that most authors place in prominent positions on their lists of items of knowledge required from teachers (e.g. Bromme, 1994; Harel, Kien, 2004). The need of solid niveau of subject didactic competence is extremely demanding for primary school teachers. Especially if we realize that the content of mathematical education at primary school level is a system of propaedeutic to many fields (arithmetic, algebra, geometry, …, functions, statistics, …). Yet these teachers are not specialists in the subject – on the contrary, they must master many more subjects than mathematics.

What is often emphasized is the need to create an "amalgam" of the components of the teacher's education. "The two basic elements of teacher knowledge are mathematics and pedagogical knowledge. When these two elements are separated and remain at a general level, mathematics teaching does not share the characteristics of ... a good teaching. The blending of mathematics and pedagogy is necessary for developing mathematics knowledge for teaching." (Potari et al., 2007, p. 1962). In other words "... mathematical experiences and pedagogical experiences cannot be two distinct forms of knowledge in teacher education." (Potari et al., 2007, p. 1963).

### Problem posing as a way to refinement of competences

#### *Opinions on the employment of problem posing*

Our existing experience indicates that one of the beneficial ways of improving subject didactic competences of pre-service teachers of mathematics is development of the ability to pose problems (and the related activities). Already Freudenthal and Polya emphasize the significance of activities aiming at problem posing as a part of mathematics training. The same need is referred to by many others (Silver, Cai, 1996; English, 1997; Pittalis et al., 2004 etc.). Apart from "problem solving" (in the sense of "learning mathematics on the basis and through problem solving") they emphasize the need and significance of development of the ability to pose problems. There is an agreement among many authors that "problem formulating should be viewed not only as a goal of instruction but also as a means of instruction. The experience of discovering and creating one's own mathematics problems ought to be a part of every student's mathematics education" (Kilpatrick, 1987, p. 123).

Teacher educators show and stress links between problem posing and problem solving, and problem posing and mathematical literacy (competence). That is why stress is on the inclusion of activities in which students generate their own problems in mathematics education. At the same time most literature points out that the treatment of issues regarding problem posing has by no means been satisfactory so far. For example, Christou *et al*. (2005) bring forward the fact that "little is known about the nature of the underlying thinking processes that constitute problem posing and schemes through which students' mathematical problem posing can be analysed and assessed" (p. 150). And Crespo (2003, p. 267) adds "... while a lot of attention has been focused on teacher candidate's own ability to solve mathematical problems, little attention has been paid to their ability to construct and pose mathematical problems to their pupils."

### *Problem posing in the frame of grasping of situations*

We started to pursue the issue of problem posing while studying the process of grasping situations (Koman & Tichá, 1998). What we understand by grasping situations is the search for questions and problems growing from a mathematical or "non-mathematical" situation, i.e. also problem posing. We define problem posing similarly to a number of other teacher educators as (a) creation of new problems or (b) re-formulation of a given problem, e.g. by "loosening the parameters of the problem" (by modifying the input conditions), by generalization, on the basis of the question "What if (not)?", etc. The problem may be worded or re-worded either before its solution or during the solving process or after it. We perceive the process of problem solving as a dialogue of the solver with the problem, we ask: How to begin? How to continue at the point reached? The solver reacts to the "behaviour, response of the problem", chooses a particular strategy, creates an easier problem, changes the conditions of the assignment to be able to continue.

Our experience from work with teacher students (and also with 10-15 year old students) confirms that their effort to pose problems guides them to deeper understanding of mathematical concepts and development of their mathematical and general literacy. Problem posing enriches both the teaching and the learning.

## TEACHER STUDENTS AND PROBLEM POSING (INVESTIGATION)

### The focus of the investigation: goals and questions

In our ongoing research we look for the ways leading to development and refinement of professional competences of both teacher students and in-service teachers. We try to show if and to what extent “problem posing” and “the level of subject didactical competence” and also “mathematical knowledge” influence each other, i.e. in presently research we look for answers to the following questions: How rich knowledge base (general as well as specific, mathematical) is needed for proficiency in problem posing? How does systematic application of problem posing contribute to development of subject didactical competence / mathematical knowledge?

## The topic of the investigation: translation between representations of fractions

We believe that problem posing can be regarded as a translation between representations e.g. as posing problems that correspond to a given calculation (Silver, 1994). The incentive to this focus was investigations that confirm the great significance of utilization of various modes of representation for the development and deepening of the level of understanding. Many authors (see e.g. Janvier, 1987; Tichá, 2003) stress that the level of understanding is related to the continuous enrichment of a set of representations and emphasize the development of the student’s capability of translation between various modes of representation.

One of the key topics of mathematical education in primary school is the foundation of the base for understanding the relations between a part and the whole. In the process of division of the whole into equal parts, the preconception of the concept of fraction is formed. The concept of fraction is one of the most difficult concepts in mathematics education at primary school level. The subject mater is difficult not only for pupils and teacher students but often also for in-service teachers who face problems regarding both the mathematical content and its didactic treatment. That is why we paid so much attention to this topic in teacher training. The core of our work lay in the construction of the concept of fractions and in posing problems with fractions. We focused on formation of preconceptions and intuitive perception of fractions, on problem solving and the potential of problem posing.

## The procedure and findings of the investigation

The stress in the course of didactics of mathematics for primary school teacher students was continuously on problem posing, thus on the development of the students’ proficiency in problem posing (the seminar was attended by 24 teacher students). One of the components of the work in the course was realization of an investigation whose aim was to show the students that problem posing can also be employed as a diagnostic tool, thus which on the basis of the problems posed it is possible to investigate the level of understanding as well as the obstacles in understanding and misconceptions.

The investigation was carried out in several steps: posing problems corresponding to a given calculation; individual reflection on the posed problems; joint reflection on a chosen set of the posed problems; evaluation of the activity “problem posing”.

### *Posing problems corresponding to a given calculation*

The students were assigned the task**:** *Pose and record such three word-problems to whose solution it is sufficient to calculate* 1/4 • 2/3.

The problems were posed during work within one of the last seminars. What is satisfactory is the immediate finding that problem posing competence can be developed in appropriate conditions; teacher students who attended the course in which stress was put on the development of proficiency in problem posing were able to pose several problems. On the contrary students who came in contact with problem posing more or less haphazardly were not able to pose any problems if asked to do so. Some of the latter even did not understand what the point of the activity was and refused to pose any problems – in their opinion they should only solve such problems that were assigned to them and had been formulated by somebody else. The same can be observed in mathematics education at schools.

### *Reflection on the posed problems*

A database of the posed problems was formed (without giving the author's name); each of the participants had access to the database. The participants of the course assessed the suitability and correctness of the posed problems that they had chosen themselves.

Then the lecturer selected a triplet of problems posed by one student. This triplet was then assessed and analyzed by all participants (the lecturer found this triplet of problems very interesting and asked their author for permission to use them in the subsequent work). The following step was joint reflection; joint assessment of individual problems, comparison and justification of opinions.

The following triplet of problems was chosen

1. *There was 2/3 of the cake on the table. David ate 1/4 of the 2/3 of the cake. How much cake was left?*
2. *There was 2/3 kg of oranges on the table. Veronika ate 1/4 kg. How many oranges remained (kg)?*
3. *The glass was full to 2/3. Gabriel drank 1/4. What part of the glass remained full?*

In advance, the lecturer went through the problems with their author. It was only in this dialogue that the student began to consider correctness of the posed problems. (It is interesting that all students began to ponder over correctness of the posed problems only after being asked to do so. However, to our gratification the students generally found and corrected their mistakes themselves.) Let us quote an extract from the dialogue between the student (S) and the lecturer (L).

S: Here (*she points at problems 2 and 3*) I don't count a part of something, I reduce, take away. ... Actually I don't know what I meant by it.

L: What could you have meant?

S: Something like this (*she sketches an illustration – a circle*) – I divide in into quarters and take away one. But, somebody could understand it that he drank a quarter of the glass. Well, I posed only one correctly. ... I should have checked.

L: How would you have checked?

S: Well, it seems I should have calculated it somehow. Or have somebody else to calculate it. Somebody who is better at it.

***Samples of student assessment of the triplet of the posed problems***

The **third problem** can be, according to some students, accepted on the condition that its wording is modified / supplemented; the given wording is regarded by many as confusing. However, the students only stated that it was confusing, they did not specify why or where.

The **first problem** was evaluated by a majority of the students positively. But the arguments of some of the evaluators reveal misconceptions: *If we have 2/3 of a cake, we can eat ¼, but the denominators do not equate. If he ate 1/3 out of the 2/3, then they would. It would be possible in real life but it is not correct mathematically.*

This statement was illustrated by a picture (Fig. 1) and by the word problem: *There are 1/4 of all pupils present in class A today and 2/3 of all pupils present in class B. If we multiply the number of pupils from both classes present today, what will the result be?*

Another student wrote and claimed: *The problem is correct. David ate 1/4 out of 2/3 of a cake ... = 1/4 • 2/3 = 1/6 of the cake.*
However, the student supplemented his statement with a picture (Fig. 2) that testifies his wrong interpretation of the whole (1/4 and 2/3 out of the same whole).

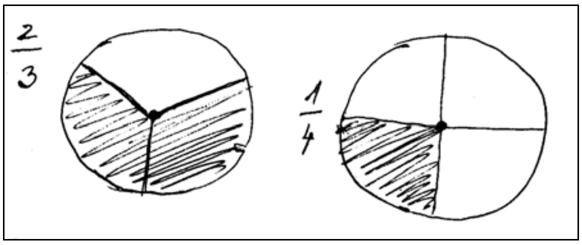



Fig. 1

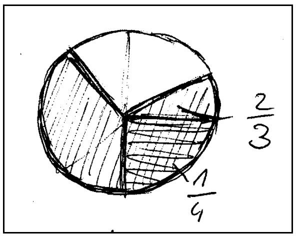



Fig. 2

When assessing the **second problem**, the students stated that this problem did not meet the condition from the assignment. However, their justification reveals that the conceptions of the evaluators themselves are also incorrect. Several illustrating examples of such evaluation follow.

- *Problem 2 is incorrect. There was 2/3 kg of oranges = 2/3 out of one (out of 3/3). Veronika ate 1/4 kg – but out of what? Out of 2/3? of 1/3?*

- *Number two is incorrect. From the total 2/3 kg of oranges, she ate 1/4 kg. She ate 1/4 but it does not say out of what.*

- *The second word problem isn't correct; it's not a suitable problem. I am not* *interested in the number of oranges but their weight. This wording would require that the oranges should be cut to pieces.*

***What does the students' production show?***

The subsequent joint reflection on the posed problems was of utmost benefit both to the participants and the lecturers. It enabled the students to become aware of their own weaknesses and it pointed to the teacher educators what they should focus on. Some of the findings follow.

The individual assessment and especially the following joint reflection show that many students do not have any idea of "what is in the background" of a particular simple calculation that they perform mechanically. They are not able to place it into a specific real life context. They did not pose problems in accord with the given calculation (what become transparent here are obstacles as far as multiplicative structure is concerned). A considerable proportion of the students posed additive problems corresponding with the calculation 1/4 + 2/3.

What comes to surface is the students' difficulty as far as interpretation of fractions is concerned. The offered formulations show that when assessing the second problem they for example do not realize that they understand and interpret the fraction alternately as an operator and as quantity *"she ate 1/4 kg"* vs. *"She ate 1/4 but it does not say out of what."*).

If the students were asked to pose more than one problem, we could observe stereotypical nature of these problems. Students often set their problems either only into discreet space (sets consisting of isolated elements) or only into continuous space. We could also observe a monotony of the motives: marbles and cakes (those are the models most often used in our textbooks).

***What do the students think of problem posing?***

The students were also asked to express their opinion on these, for them often unusual, activities. Let us present here several statements from individual reflections which illustrate how the students perceive "problem posing".

- *I have problems with word problems. To pose a word problem on my own ,…, was extremely difficult. The difficulty is not in posing a problem, but in being able to solve it myself. It was toil and moil for me.*

- *What I personally found most difficult was to ask the question correctly, when I posed the third one, I could think of no further questions and that's why I only managed to pose the most banal ones.*

- *As soon as I came to understand the assignment of this task, I was immediately full of various ideas ... I was delighted because I love discovery … that there were no limits.*

- *My first reaction was that of fear. However, I started from what first came to my mind – a simple problem and then I began to toy with it. It was very pleasant to look* *for and discover various combinations...*

In the discussion the students indicated that it was easy to formulate a great number of problems of the same type but it was difficult to formulate a sequence of problems (cascade) of a growing difficulty or a problem for whose solution it was necessary to connect various pieces of knowledge or problems in which the role of the fraction alternates (i.e. various sub-constructs of fractions, …, Behr et al., 1983).

## CONCLUDING REMARKS ON THE BENEFIT OF PROBLEM POSING AND ON THE PERFORMED INVESTIGATION

Our experience from work with teacher students (and also from our long-term cooperation with in-service teachers) proves that poor level of pre-service mathematical training is pervasive and the flaws are difficult to overcome (Hošpesová & Tichá, 2005; Hošpesová *et al.*, 2007). Problem posing is in our opinion one of the beneficial possibilities.

### The detection of a change in the "nature, climate" of work in the seminar

It seems to us that problem posing contributed to a change in approach to work in the seminar – the students gradually overcame their fears or anxiety and many of them gained self-confidence.

The character of the problems posed by the participants also changed. Before their participation in the seminar they posed simple, "textbook-like" problems, predominantly drill. The wording of the problems was often erroneous and the problems were uninteresting and demotivating from mathematical point of view. Many of the problems had no solution, despite the author's intention.

After the course finished, a great variety of assignments of the problems could be observed (including charts, graphs etc.). There were also problems enabling different ways of solution and problems demanding explanation, reasoning, argumentation, allowing different answers with respect to the solver's interests.

It turned out that it is not enough to demand from the students to pose a problem if one is to detect the quality of their understanding. It is crucial that it should be possible to assess the posed problems individually and/or collectively. This certifies the need to carry out joint reflection. If the authors are given the chance to assess the problems of each other, their insight into the situation deepens and the ability to handle reality, i.e. to "see mathematics in the world surrounding us" develops.

### The benefit for students

The analysis of the posed problems makes the participants map the level of their own notions and concepts, understanding, various interpretations and makes them realize possible misconceptions and erroneous reasoning. It is an impulse for work on themselves (reeducation).

It was confirmed that the result of inclusion of problem posing into the curricula is better approach to problem solving. It stimulates the use of various representations, construction of knowledge nets, development of creative thinking, improvement of attitude to mathematics and increase in self-confidence.

**The benefit for teacher educators and researchers**

From the point of view of teacher trainers and researchers problem posing provides an opportunity to get an insight into natural differentiation of students' understanding of mathematical concepts and processes and to find obstacles in understanding and misunderstandings that already exist.

Our belief that problem posing supplemented with reflection is the path to development and enhancement of subject didactical competence, i.e. of pedagogical content knowledge was confirmed.

**Open questions**

There still exist many questions which ask for deeper investigations, e. g. How can be the benefit that problem posing brings to its authors and the shift in their (pedagogical) content knowledge detected and measured? Which teacher's and/or student's competences are developed? What conditions are essential for introduction of problem posing? What help and guidance can be offered when incorporating problem posing?

**NOTE**

This research was partially supported by the grant projects: GACR 406/08/0710; AS CR, Institutional Research Plan AV0Z 10190503; 142453-LLP-1-2008-1-PL-COMENIUS–CMP.

**REFERENCES**

Behr, M., Lesh, R., Post, T. & Silver, E.A. (1983). Rational number concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of Mathematics Concepts and* processes (pp. 91-125). New York: Academic Press.

Bromme, R. (1994). Beyond subject matter: A psychological topology of teachers' professional knowledge. In R. Biehler *et al*. (Eds.). *Didactics of mathematics as a scientific discipline* (pp. 73-88). Dordrecht: Kluwer Academic Publishers.

Crespo, S. (2003). Learning to pose mathematical problems: exploring changes in preservice teachers' practices. *Educational Studies in Mathematics, 52*, 243-270.

Christou, C., Mousolides, N., Pittalis, M., Pitta-Pantazi, D. & Shiraman, B. (2005). An empirical taxonomy of problem posing processes. *ZDM, 37(3*), 149-158.

English, L. D. (1997). The development of fifth-grade children's problem-posing abilities. *Educational Studies in Mathematics, 34*, 183-217.

Harel, G. & Kien, H. L. (2004). Mathematics teachers' knowledge base: preliminary results. In M. J. Hoines & A. J. Bishop (Eds.), *Proceedings of 28th PME* (pp. 25-32). Bergen, Norway.

Hošpesová, A. & Tichá, M. (2005). Developing mathematics teacher's competence. In M. Bosch (Ed), *Proceedings CERME 4* (CD ROM) (pp. 1483-1493). Barcelona: FundEmi IQS.

Hošpesová, A., Tichá, M. & Macháčková, J. (2007). Differences and similarities in (qualified) pedagogical reflection. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings CERME* 5 (CD ROM) (pp. 1906-1915). Larnaca: University of Cyprus.

Janvier, C. (1987). Representation and understanding: the notion of function as an example. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 67-71). Hillsdalle: Lawrence Erlbaum Associates.

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123-147). Hillsdale: Lawrence Erlbaum Associates.

Koman, M. & Tichá, M. (1998). On travelling together and sharing expenses. In D. Burghes (Ed.), *Teaching mathematics and its applications* (pp. 117-122). Oxford University Press.

Pittalis, M., Christou, C., Mousolides, N. & Pitta-Pantazi, D. (2004). A structural model for problem posing. In M. J. Hoines & A. J. Bishop (Eds.), *Proceedings of 28th PME* (pp. 49-56). Bergen, Norway.

Potari, D., Zacharides, T., Christou, C., Kyriazis, G. & Pitta-Pantazi, D. (2007). Teachers' mathematical knowledge and pedagogical practices in the teaching of derivative. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings CERME* 5 (pp. 1955-1964). Larnaca: University of Cyprus.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*, 2, 4-14.

Silver, E.A. & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27(5*), 521-539.

Silver, E.A. (1994). On mathematical problem posing. *For the Learning of Mathematics*, *14(1*), 19-28.

Tichá, M. (2003). Following the path of discovering fractions. In J. Novotná (Ed.), *Proceedings of SEMT '03* (pp. 17-27). Praha : UK PedF.

Tichá, M. & Hošpesová, A. (2006). Qualified Pedagogical Reflection as a Way to Improve Mathematics Education. *Journal for Mathematics Teachers Education. Special Issue: Inter-Relating Theory and Practice in Mathematics Teacher Education.* Springer Netherlands, 9, 2, 129-156.

# SUSTAINABILITY OF PROFESSIONAL DEVELOPMENT

Stefan Zehetmeier

University of Klagenfurt, Austria

*This contribution addresses the issue of sustainable impact of professional development projects. It claims for widening the scope from evaluations of short-term effects to analyses of long-term impact. For that, the contribution discusses various types of effects and possible levels of impact. In particular, an overview concerning factors promoting the impact of professional development projects is provided. A case study that analysed the impact of an Austrian professional development project three years after its termination is introduced. The paper closes with further research questions that emerged from this study.*

Key-words: professional development, sustainable impact, promoting factors, case study

## INTRODUCTION

The quality of teaching and learning represents a recurring key issue of research. In particular, teachers are considered to be playing a central role when addressing this topic: „Teachers are necessarily at the center of reform, for they must carry out the demands of high standards in the classroom” (Garet, Porter, Desimone, Birman, & Yoon, 2001, p. 916). Various types of professional development projects are offered to support and qualify these teachers. The expected effects of such projects by both the facilitators and the participants are not only related to the professional development of individual teachers to improve teacher quality, but also to the enhancement of the quality of whole schools, regions and nations. The desideratum of all such projects providing teachers support and qualification is to enhance the learning of students. As Kerka (2003) states, “Funders, providers, and practitioners tend to agree that the ultimate goal of professional development is improved outcomes for learners” (p. 1). This strategy, to achieve change at the level of students (improved outcomes) by fostering change at the teachers’ level (professional development), is based on the assumption of a causal relationship between students’ and teachers’ classroom performance: “High quality professional development will produce superior teaching in classrooms, which will, in turn, translate into higher levels of student achievement” (Supovits, 2001, p. 81). Similarly, Hattie (2003) states, “It is what teachers know, do, and care about which is very powerful in this learning equation” (p. 2). Ingvarson, Meiers, and Beavis (2005) sum up: “Professional development for teachers is now recognised as a vital component of policies to enhance the quality of teaching and learning in our schools. Consequently, there is increased interest in research that identifies features of effective professional learning” (p. 2).

## TYPES OF EFFECTS

The expected outcomes of professional development projects are not only focused on short-term effects that occur during or at the end of the project, but also on long-term effects that emerge (even some years) after the project's termination (Peter, 1996). Effects that are both short-term and long-term can be considered to be sustainable. So sustainability can be defined as the lasting continuation of achieved benefits and effects of a project or initiative beyond its termination (DEZA, 2005). As Fullan (2006) points out, short-term effects are "necessary to build trust with the public or shareholders for longer-term investments" (p. 120). Besides these short-term effects also long-term effects need to be considered; otherwise the result could be to "win the battle, [but] lose the war" (ibid.). Hargreaves and Fink (2003) state, "Sustainable improvement requires investment in building long term capacity for improvement, such as the development of teachers' skills, which will stay with them forever, long after the project money has gone" (p. 3). Moreover, analysis of sustainable impact should not be limited to effects that were planned at the beginning of the project; it is also important to examine the unintended effects and unanticipated consequences that were not known at the beginning of the project (Rogers, 2003; Stockmann, 1992).

## SUSTAINABLE IMPACT

Evaluations and impact analyses of professional development projects are formative or summative in nature; in most cases they are conducted during or at the end of a project and exclusively provide results regarding short-term effects. These findings are highly relevant for critical reflection of the terminated project and necessary for the conception of similar projects in the future. But apart from and beyond that, an analysis of sustainable effects is crucial: "Too many resources are invested in professional development to ignore its impact over time" (Loucks-Horsley, Stiles, & Hewson, 1996, p. 5). This kind of sustainability analysis is often missing because of a lack of material, financial and personal resources. "Reformers and reform advocates, policymakers and funders often pay little attention to the problem and requirements of sustaining a reform, when they move their attention to new implementation sites or end active involvement with the project" (McLaughlin & Mitra, 2001, p. 303). Despite its central importance, research on this issue is generally lacking (Rogers, 2003) and "Few studies have actually examined the sustainability of reforms over long periods of time" (Datnow, 2006, p. 133). Hargreaves (2002) summarises the situation as follows: "As a result, many writers and reformers have begun to worry and write about not just how to effect snapshots of change at any particular point, but how to sustain them, keep them going, make them last. The sustainability of educational change has, in this sense, become one of the key priorities in the field" (p. 120).

Zehetmeier (2008) summarises the literature concerning the sustainability of change and provides a case study of four teachers from one school, analyzing the impact of a professional development project three years after its termination. For that, he develops a theoretical model which allows analysing both the various characteristics of the project, the different levels of impact, and the factors promoting or hindering the sustainability of impact (see also Zehetmeier, in prep.).

## LEVELS OF IMPACT

When analyzing possible effects of professional development, the question of possible levels of impact arises. Which levels of impact are possible and/or most important? How can impact be classified? Recent literature provides some answers to these questions; the following levels of impact are identified (Lipowsky, 2004):

*Teachers' knowledge:* This level can be defined in different ways, for example, referring to content knowledge, pedagogical knowledge, and pedagogical content knowledge (Shulman, 1987), or attention-based knowledge (Ainley & Luntley, 2005), or knowledge about learning and teaching processes, assessment, evaluation methods, and classroom management (Ingvarson et al., 2005).

*Teachers' beliefs:* This level includes a variety of different aspects of beliefs about mathematics as a subject and its teaching and learning (Leder, Pehkonen, & Törner, 2002), as well as the perceived professional growth, the satisfaction of the participating teachers (Lipowsky, 2004), perceived teacher efficacy (Ingvarson et al., 2005) and the teachers' opinions and values (Bromme, 1997).

*Teachers' practice:* At this level, the focus is on classroom activities and structures, teaching and learning strategies, methods or contents (Ingvarson et al., 2005).

*Students' outcomes:* Many papers highlight that students' outcomes are related to the central task of professional development programmes: namely to the improved learning and knowledge of the students (Kerka, 2003; Mundry, 2005; Weiss & Klein, 2006).

Zehetmeier (2008) points out that the complexity of possible impact is not fully covered by this taxonomy. For example, results of an impact analysis in the context of the Austrian IMST project (Krainer, 2005, 2007) show that the project made impact also on students' beliefs or other – non participating – teachers' practice. In particular, the findings of this analysis demonstrate that the taxonomy of levels of impact (see above) needs to be extended (Zehetmeier, 2008): The categories knowledge, beliefs, and practice are suitable to cover the impact in the teachers' level. But also on the levels of pupils, colleagues, principals, and parents all three categories (knowledge, beliefs, and practice) are respectively necessary to gather possible levels of impact. Moreover, in addition to these in-school levels, also beyond-school levels need to be considered when analyzing the impact of professional development projects: e.g., other schools, media, policy, or scholarship. These results lead to a grid of possible levels of impact (Zehetmeier, 2008, p. 197):

| | In-School Levels | | | | | Beyond-School Levels | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Teachers | Pupils | Colleagues | Principals | Parents | Schools | Media | Policy | Scholarship |
| **Knowledge** | | | | | | | | | |
| **Beliefs** | | | | | | | | | |
| **Practice** | | | | | | | | | |

## FOSTERING FACTORS

What are the factors that promote and foster the impact of professional development projects? Literature and research findings concerning this question point to a variety of different factors. To give an overview, in the following section Borko's (2004) four elements of professional development projects are used to organize and classify these factors: *participating teachers, participating facilitators, the programme itself*, and the *context* that embeds the former three elements.

Within the element of *participating teachers* the following factors are fostering the impact of professional development programmes: If the teachers are involved in the conception and implementation of the programme, they can develop an affective relationship towards the programme by developing ownership of the proposed change (Clarke, 1991; Peter, 1996). They can be empowered to influence their own development process (Harvey & Green, 2000). Teachers should be prepared and supported to serve in leadership roles (Loucks-Horsley et al., 1996). An "inquiry stance", taken by the participating teachers, also fosters the sustainability of impact (Farmer, Gerretson, & Lassak, 2003, p. 343): If teachers understand their role as learners in their own teaching process, they can reflect and improve their practice. Cochran-Smith and Lytle (1999) also use this notion for describing teachers' attitude towards the relationship of theory and practice: "Teachers and student teachers who take an inquiry stance work within inquiry communities to generate local knowledge, envision and theorise their practice, and interpret and interrogate the theory and research of others" (p. 289). Altrichter and Krainer (1996) recommend a reorientation of professional development programmes from "teachers to be taught" towards "teachers as researchers" (p. 41) and refer to Posch and Altrichter (1992) who state: „The most important part of teacher professional development takes place on site: by reflection and development of the own instructional practice and by school development" (p. 166).

Similar to the teachers, also the *participating facilitators* of the professional development programme should take a "stance of inquiry" (Ball, 1995, p. 29) towards their activities. They should reflect on their practice and evaluate its impact (Farmer et al., 2003). The facilitators' knowledge, understanding, and their image of effective learning and teaching also foster the initiative's impact (Loucks-Horsley et al., 1996). The development of mutual trust between the facilitators and the participating teachers represents a further fostering factor (Zehetmeier, 2008).

The *programme itself* should fit into the context in which the teachers operate, and provide direct links to teachers' curriculum (Mundry, 2005). It should focus on content knowledge and use content-specific material (Garet et al., 2001; Ingvarson et al., 2005; Maldonado, 2002), and should provide teachers with opportunities to develop both content and pedagogical content knowledge and skills (Loucks-Horsley et al., 1996; Mundry, 2005). Moreover, an effective professional development programme includes opportunities for active and inquiry-based learning (Garet et al., 2001; Ingvarson et al., 2005; Maldonado, 2002), authentic and readily adaptable student-centered mathematics learning activities, and an open, learner-centered implementation component (Farmer et al., 2003). Further factors fostering the effectiveness and sustainability of the programme are: prolonged duration of the activity (Garet et al., 2001; Maldonado, 2002), ongoing and follow-up support opportunities (Ingvarson et al., 2005; Maldonado, 2002; Mundry, 2005), and continuous evaluation, assessment, and feedback (Ingvarson et al., 2005; Loucks-Horsley et al., 1996; Maldonado, 2002).

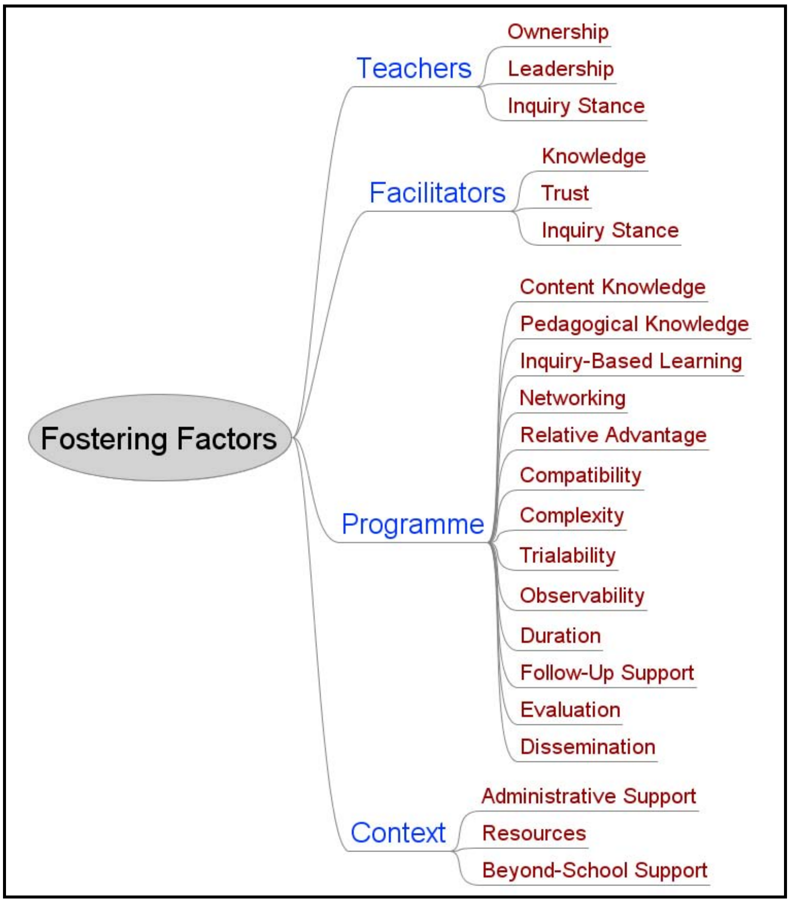
Lerman and Zehetmeier (2008) highlight that community building and networking represent further factors fostering sustainability. This claim is supported by several authors and studies, even if the categories used to describe these activities are sometimes different: Clarke (1991), Peter (1996), and Mundry (2005) point to cooperation and joint practice of teachers, Loucks-Horsley et al. (1996) and Maldonado (2002) highlight the importance of learning communities, Wenger (1998) and McLaughlin and Mitra (2001) identify supportive communities of practice, Arbaugh (2003) refers to study groups, and Ingvarson et al. (2005) stress professional communities as factors contributing to the sustainability of effects. In particular, providing rich opportunities for collaborative reflection and discussion (e.g., of teachers' practice, students' work, or other artefacts) presents a core feature of effective change processes (Clarke, 1991; Farmer et al., 2003; Hospesova & Ticha, 2006; Ingvarson et al., 2005; Park-Rogers et al., 2007; Zehetmeier, 2008).

The dissemination of innovations or innovative teaching projects is another factor that fosters the sustainability of professional development programmes (Zehetmeier, 2008). E.g., teachers participating in the Austrian IMST project (Krainer, 2005, 2007) write down and publish reflective papers or project reports. As Schuster (2008) shows, teachers' writings have a positive impact on their reflection skills and knowledge base. The dissemination of good practice projects and ideas requires a structural framework that allows teachers to publish or actively present their projects and results. E.g., the Austrian IMST project created a web-based wiki where some hundreds of project reports written by Austrian teachers can be easily accessed. Moreover, an annual nation-wide conference is set up, where teachers can share their projects, ideas, and results. A professional development programme aiming at sustainable impact should provide these possibilities for dissemination even after the programme is terminated. Otherwise the possibility of dissemination along with the involved advantages for teachers' professional growth is likely to fade away (Zehetmeier, 2008).

Rogers (2003) highlights that the diffusion of an innovation depends on different characteristics: Relative advantage, compatibility, complexity, trialability, and observability. Fullan (2001) describes similar characteristics (need, clarity, complexity, quality and practicality) that influence the acceptance and impact of innovations. Relative Advantage includes the perceived advantage of the innovation (which is not necessarily the same as the objective one). An innovation with greater relative advantage will be adopted more rapidly. Compatibility and need denote the degree to which the innovation is perceived by the adopters as consistent with their needs, values and experiences. Complexity and clarity include the teachers' perception of how difficult the innovation is to be understood or used. Thus, more complex innovations are adopted rather slowly, compared to less complicated ones. Trialability denotes the possibility of participating teachers to experiment and test the innovation (at least on a limited basis). Innovations that can be tested in small steps represent less uncertainty and will be adopted as a whole more rapidly. Quality and practicality make an impact on the change process. High quality innovations that are easily applicable in practice are more rapidly accepted. Observability points to the claim that innovations which are visible to other persons (e.g., parents or principals) and organisations are more likely to be rapidly accepted and adopted.

The *context* which embeds teachers, facilitators, and the programme itself, is of particular importance regarding the sustainability of innovations and change processes (e.g., McNamara, Jaworski, Rowland, Hodgen, & Prestage, 2002; Noddings, 1992; Owston, 2007). Teachers need administrative support and resources (McLaughlin & Mitra, 2001). School-based support can be provided by students and colleagues (Ingvarson et al., 2005; Owston, 2007), and in particular by the principal (Clarke, 1991; Fullan, 2006). To foster sustainability not only at the individual (teacher's) level but also at the organisational (school's) level, Fullan (2006) proposes a new type of leadership that "needs to go beyond the successes of increasing student achievement and move toward leading organizations to sustainability" (p. 113). In particular, these "system thinkers in action" should "widen their sphere of engagement by interacting with other schools" (p. 113) and should engage in "capacity-building through networks" (p. 115). Support from outside the school (e.g., by national or district policies) is also an important factor fostering the programme's impact (McLaughlin & Mitra, 2001; Owston, 2007).

The following figure sums up and illustrates these factors that promote and foster the impact of professional development projects:




## FUTURE RESEARCH

Impact analysis that combines and compares various cases and bigger samples could help answering the following questions (see also Zehetmeier, 2008):

- Do different professional development projects make different sustainable impact? Are there any patterns of impact?
- Does a professional development project show different sustainable impact on different participating teachers? Are there any patterns?
- Are there any hierarchical structures within the different levels of impact? Does one level require another one to occur?
- Are there any factors that promote certain levels of impact in a particular way?
- Are there any "universal" factors fostering sustainable impact?

Upcoming impact analyses dealing with these and similar questions appear to be necessary and promising; from the perspective of both scholarship and practice.

## REFERENCES

Ainley, J., & Luntley, M. (2005). What teachers know: The knowledge base of classroom practice. In M. Bosch (Ed.), *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (pp. 1410-1419). Sant Feliu de Guíxols, Spain: European Research in Mathematics Education.

Altrichter, H., & Krainer, K. (1996). Wandel von Lehrerarbeit und Lehrer/innenfort-bildung [Change of teachers' work and professional development]. In K. Krainer & P. Posch (Org.), *Lehrer/innenfortbildung zwischen Prozessen und Produkten* (pp. 33-52). Bad Heilbrunn: Klinkhardt.

Arbaugh, F. (2003). Study groups as a form of professional development for secondary mathematics teachers. *Journal of Mathematics Teacher Education, 6*, 139-163.

Ball, D. L. (1995). Developing mathematics reform: What don't we know about teacher learning – but would make good working hypotheses? *NCRTL Craft Paper, 95*(4). (ERIC Document Reproduction Service No. ED399262).

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33(8*), 3-15.

Bromme, R. (1997). Kompetenzen, Funktionen und unterrichtliches Handeln des Lehrers [Expertise, tasks and instructional practice of teachers]. In F. Weinert (Eds.), *Enzyklopädie der Psychologie. Band 3. Psychologie des Unterrichts und der Schule* (pp. 177-212). Göttingen, Germany: Hogrefe.

Clarke, D. M. (1991). *The role of staff development programs in facilitating professional growth*. Madison, WI: University of Wisconsin.

Cochran-Smith, M., & Lytle, S. (1999). Relationships of knowledge and practice: Teacher learning in communities. *Review of Research in Education, 24*, 249-305.

Datnow, A. (2006). Comments on Michael Fullan's, "The future of educational change: System thinkers in action". *Journal of Educational Change, 7*, 133-135.

Direktion für Entwicklungshilfe und Zusammenarbeit / DEZA. (2005). *Glossar deutsch*. Bern: DEZA.

Farmer, J., Gerretson, H., & Lassak, M. (2003). What teachers take from professional development: Cases and implications. *Journal of Mathematics Teacher Education, 6*, 331-360.

Fullan, M. (1990). Staff development, innovation and institutional development. In B. Joyce (Ed.), *Changing school culture through staff development* (pp. 3-25). Alexandria, VA: Association for Supervision and Curriculum Development.

Fullan, M. (2001). *The new meaning of educational change* (3rd edition). New York: Teachers College Press.

Fullan, M. (2006).The future of educational change: System thinkers in action. *Journal of Educational Change, 7*, 113-122.

Garet, M., Porter, A., Desimone, L., Birman, B., & Yoon, K. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal, 38*, 915-945.

Hargreaves, A. (2002). Sustainability of educational change: The role of social geographies. *Journal of Educational Change, 3*, 189-214.

Hargreaves, A., & Fink, D. (2003). Sustaining leadership. *Phi Delta Kappan, 84(9*), 693-700.

Harvey, L., & Green, D. (2000). Qualität definieren [Defining quality]. *Zeitschrift für Pädagogik, Beiheft, 41*, 17-37.

Hattie, J. (2003, October). *Teachers make a difference. What is the research evidence?* Paper presented at the Australian Council of Educational Research conference: Building Teacher Quality. Melbourne, Australia.

Hospesova, A., & Ticha, M. (2006). Qualified pedagogical reflection as a way to improve mathematics education. *Journal of Mathematics Teacher Education*, *9*, 129-156.

Ingvarson, L., Meiers, M., & Beavis, A. (2005). Factors affecting the impact of professional development programs on teachers' knowledge, practice, student outcomes and efficacy. *Education Policy Analysis Archives, 13(10*), 1-28.

Kerka, S. (2003). Does adult educator professional development make a difference? *ERIC Myths and Realities*, *28*, 1-2.

Krainer, K. (2005). IMST3 – A sustainable support system. In Austrian Federal Ministry of Education (Ed.), *Austrian Education News 44* (pp. 8-14). Vienna: Austrian Federal Ministry of Education.

Krainer, K. (2007). *Beiträge von IMST zur Steigerung der Attraktivität des MNI-Unterrichts in Österreich* [Contributions of IMST² to enhance the attractiveness of mathematics and science education in Austria]. Unpublished manuscript.

Leder, G., Pehkonen, E., & Törner, G. (2002). *Beliefs: A hidden variable in mathematics education?* Dordrecht, The Netherlands: Kluwer Academic Publishers.

Lerman, S., & Zehetmeier, S. (2008). Face-to-face communites and networks of practising mathematics teachers. In K. Krainer (Ed.), *International handbook of mathematics teacher* education: Vol. 3. Participants in mathematics teacher education: individuals, teams, communities, and networks (pp. ???). Rotterdam: Sense Publishers.

Lipowsky, F. (2004). Was macht Fortbildungen für Lehrkräfte erfolgreich? [What makes professional development for teachers successful?]. *Die deutsche Schule, 96*, 462-479.

Loucks-Horsley, S., Stiles, K., & Hewson, P. (1996). Principles of effective professional development for mathematics and science education: A synthesis of standards. *NISE Brief, 1(1)*, 1-6.

Maldonado, L. (2002). *Effective professional development. Findings from research*. Retrieved 12.1.2007 from www.collegeboard.com.

McLaughlin, M., & Mitra, D. (2001). Theory-based change and change-based theory: Going deeper, going broader. *Journal of Educational Change, 2,* 301-323.

McNamara, O., Jaworski, B., Rowland, T., Hodgen, J., & Prestage, S. (2002). *Developing mathematics teaching and teachers*. Unpublished manuscript.

Mundry, S. (2005). What experience has taught us about professional development. National Network of Eisenhower Regional Consortia and Clearinghouse. ???

Noddings, N. (1992). Professionalization and mathematics teaching. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 197-208). New York: Macmillan.

Owston, R. (2007). Contextual factors that sustain innovative pedagogical practice using technology: An international study. *Journal of Educational Change, 8(1*), 61-77.

Park-Rogers, M., Abell, S., Lannin, J., Wang, C., Musikul, K., Barker, D., & Dingman, S. (2007). Effective professional development in science and mathematics education: Teachers' and facilitators' views. *Journal of Science and Mathematics Education*, *7*, 507-532.

Peter, A. (1996). *Aktion und Reflexion – Lehrerfortbildung aus international vergleichender Perspektive* [Action and reflection – teacher education from an international comparative perspective]. Weinheim, Germany: Deutscher Studien Verlag.

Rogers, E. (2003). *Diffusion of innovations*. New York: Free Press.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57(1),* 1-22.

Stockmann, R. (1992). *Die Nachhaltigkeit von Entwicklungsprojekten* [The sustainability of development projects]. Opladen, Germany: Westdeutscher Verlag.

Supovitz, J. (2001). Translating teaching practice into improved student achievement. In S. Fuhrman (Ed.), *From the capitol to the classroom. Standards-based reforms in the states* (pp. 81-98). Chicago: University of Chicago Press.

Weiss, H., & Klein, L. (2006). Pathways from workforce development to child outcomes. The Evaluation Exchange, 11(4), 2-4.

Zehetmeier, S. (2008). *Zur Nachhaltigkeit von Lerher/innenfortbildung* [The sustainability of teacher professional development]. Doctoral thesis. Klagenfurt, Austria: University of Klagenfurt.

Zehetmeier, S. (in preparation). What remains? The sustainability of professional development programmes. *Journal of Mathematics Teacher Education*.

# A COLLABORATIVE PROJECT AS A LEARNING OPPORTUNITY FOR MATHEMATICS TEACHERS

Maria Helena Martinho

CIEd (Centro de Investigação em Educação), Universidade do Minho

João Pedro da Ponte

Instituto de Educação Universidade de Lisboa,

*This paper analyses the evolution of Maria, a mathematics teacher involved in a long term collaborative project together with a researcher and two other teachers. The study aimed to understand teaching practices and to develop richer classroom communication processes. It follows a qualitative-interpretative approach, with data gathered through recording of meetings and interviews. We discuss to what extent this project became relevant for the professional practice of Maria. The results indicate the potential of collaboration to understand communication phenomena in the classroom, putting practices under scrutiny and developing richer communication interaction patterns between teacher and students.*

Key-words: Mathematics communication; collaboration; professional development.

## INTRODUCTION

The possibilities of collaboration between teachers and researchers as a research strategy are receiving increasing attention. Collaboration is an opportunity to combine joint work with individual input, taking advantage of the potential of different individuals building a common experience (Hargreaves, 1994). In this paper, we take collaboration as an experience shared by a set of people who identify a common interest and establish and implement a working agreement, providing mutual support and challenging each other. This perspective defined a collaborative project involving a researcher (the first author of this paper) and three mathematics middle school teachers, whose purpose was to understand classroom communication, putting practices under scrutiny and developing richer communication processes.

Our research question enquires what are the influences, if any, of a collaborative project on the conceptions and practices of a teacher regarding classroom communication. It links concerns emerging in recent research on collaborative work, (Boavida & Ponte, 2002; Jaworski; 1986) and classroom communication (Alro & Skovsmose, 2004; Lampert & Cobb, 2003; Sherin, 2002). Here we restrict the scope of analysis to Maria, one of the teachers. In particular, we discuss to what extent the project became relevant to her professional practice. First, we discuss the meaning of collaboration in educational research and how communication was understood within the project group. Then, we present the methodology and analyse the “case” of Maria.

Finally, we end with a discussion concerning issues that arise in collaboration as a research strategy in mathematics education.

## BACKGROUND

*Collaboration.* Collaboration plays an increasing role in educational research. In a collaborative project, participants may take advantage of working together (Kapuscinski, 1997), but often tensions emerge along such a process. They arise, for example, from the different attitudes teachers and researchers maintain towards practice, planning, motivations or use of knowledge (Kapuscinski, 1997; Olson, 1997). There are, of course, a variety of collaborative structures, and corresponding different degrees of individual commitment – as indicated, for example, by Clift and Say (1988), Day (1999), Goulet and Aubichon (1997), and Wagner (1997). A number of aspects, however, are recognised as consensual as characteristic of any true collaboration. One of them is that the relationships between the participants should not be hierarchical. Mutual support requires some sort of egalitarian base (Boavida & Ponte, 2002). There are, of course, different roles, a difference which, moreover, should be made clear in the group, but all roles must have similar relevance. Another element concerns diversity, understood as an added value to collaboration, which should be assumed as such by the group (John-Steiner, Weber & Minnis, 1998).

In a collaborative context, participants do not waste time to promote what they believe to be their own image (Fullan & Hargreaves, 1991). Disagreements are frequent and welcome, given that discussions are centred in values, purposes and practices. A considerable effort is required to build a collaborative culture, which always supposes an effective personal development. In particular, it requires making explicit some common objectives inside the group. Each participant must be aware of her/his own role in the way he/she relates to the others and cares about such a relationship (Drake & Basaraba, 1997). Teachers' involvement in a project depends on how they perceive its relevance, namely to practice, as well as on the way decisions are made inside the group (Bonals, 1996). Essential to the success of a collaborative project is also the ability to carry on reflection exercises together (Day, 1999). To develop such an ability to think critically with others requires some degree of maturation in dealing with doubt and incertitude (Fernandes & Vieira, 2006).

The benefits of collaboration are well documented in the literature. Fullan and Hargreaves (1991) and Maeers and Robison (1997), for example, mention how it helps teachers to feel less isolated and impotent. It is also a factor for change in educational practices, namely when the collaborative experience is made public (Olson, 1997). Active involvement in a collaboration and sharing of concerns and experiences promotes personal and professional development (Lafleur & MacFadden, 2001) as it leads to increased self-knowledge. Collaboration increases the self confidence of every participant (Maeers & Robison, 1997).

Collaborating along a reasonable period of time is not an easy task. Collaborations are fragile, by definition, requiring balances that often are difficult to set up and maintain (Fullan & Hargreaves, 1991; Olson, 1997). Therefore, planning and flexibility, dialog and negotiation, are essential to any collaborative project. Finally, managing expectations, emotions, personal differences, becomes fundamental whenever a collaboration is to be maintained.

*Communication in the mathematics classroom.* Several authors underline the importance of communication processes in the mathematics classroom (Bishop & Goffree, 1986; Ponte & Santos, 1998; Yackel & Cobb, 1998). Communication is a social process along which participants interact, sharing information and mutually constraining their activity and evolution. It concerns not only an heterogeneous set of interactive processes evolving in a classroom but also their contexts, underlying denotations and expressive resources. Such a perspective includes two issues clearly identified in the literature (Ponte, Boavida, Graça & Abrantes, 1997) in the study of communication in the mathematics classroom: (i) *continuous interaction* between the actors in a classroom, and (ii) *negotiation of meanings* understood as the processes such actors set to share their own ways of making sense of mathematical concepts and procedures, and their evolution and relation to the formal curriculum contents.

Mathematical learning requires a stepwise construction of a reference framework through which students construct their own personal account of mathematics in a dynamic tension between old and newly acquired knowledge. This is achieved along the countless interaction processes taking place in the classroom. Of especial import are the interactions between students and teacher, which simultaneously constrain and are constrained by the kind of lesson. For example, in a learning context in which the teacher stresses exposition and solving exercises, he/she tends to control the whole process. In other contexts he/she may assume instead the role of a coordinator. The nature of the questions posed by the teacher is particularly relevant, leading to the development of communication and reasoning skills (Barrody, 1993).

It is widely recognised the fundamental role that the teacher plays either in enabling or limiting the communicative processes within the classroom (Barrody, 1993; Lappan & Schram, 1989; Pimm, 1987). Such a role makes itself explicit from the outset, for example, when selecting challenging tasks or encouraging students to express and argue their own views (Lampert & Cobb, 2003; Ponte & Santos, 1998), or else when resorting to tasks and educational materials that put the focus of the lessons on mathematical ideas, conjectures or intuitions, instead of calculations and procedures. The teacher is also responsible for creating an atmosphere of self-esteem and mutual respect, so that students feel comfortable to participate, as well as for structuring the classroom discourse.

## METHODOLOGY

This paper reports a study, qualitative and interpretative, based on a case-study design (Yin, 1989). This is part of a broader research project involving three case studies developed within the context of a long term collaborative project on communication in the mathematics classroom (Martinho, 2007). The project involved a researcher, the first author of this paper, and three mathematics teachers, Maria being one of them. This group was initiated by the first author who invited a teacher with whom she had already collaborated, who later invited two other teachers to join. Along a year and a half, the project involved regular meetings devoted to a variety of tasks, namely, analysis of documents, lesson planning and review, free debates on communication issues, and project planning and evaluation. Each teacher selected a number of lessons to be observed and recorded by the researcher, and finally these lessons were discussed in group meetings. Data gathering for this research study was based on two semi-structured interviews and on the recordings of group meetings and the researcher's field notes. The aim of the interviews was to get a deep understanding of the way the teacher reasoned about her own communication practices. The focus was on creating a friendly environment to allow a natural flow of conversation about the topics of interest. The recordings of group meetings and the researcher's field notes provided complementary data about the teacher activity, concerns and reflections at each moment. Data collection and analysis were carried simultaneously during collaborative work, mutually influencing each other. The research adopted the interactive model of analysis (Huberman & Miles, 1994).

The project started in 2004, with regular working meetings taking place every fortnight (in a total of 25 meetings), along the whole academic year of 2004/05. From September 2005 onwards, meeting periodicity changed to a weekly basis. Even today, after the formal closing of the original project, the group still meets every week, including now two more teachers. All of them, except the researcher, work in the same middle school.

## RESULTS AND DISCUSSION

*Maria*. Maria is 52 years old and counts 31 years as a teacher. She is married and has two children, already grown up. She assumes her work with professionalism and commitment. For 6 years she served as a school principal and is quite active in a trade union. She has an accurate sense of public service and citizenship. In general, Maria is resolute, determined, and always exigent with herself. She concluded a bachelor degree in chemical engineering in 1974. Becoming a teacher was not her first professional option; only later, she completed another degree on teaching biology and geology. At present, she teaches mathematics and natural sciences. This background may explain her main concern as a mathematics teacher: to provide evidence of the usefulness of this subject.

Maria feels some difficulties in several mathematical topics (she often says that she is not a mathematician) and this clearly influences her teaching practice. She has a deep respect for mathematics as a wonderful world that, however, she is not able to master easily. Mathematics, in her view, is a network of abstractions, concepts and methods, tightly connected. Therefore, she fears that her way of teaching, emphasizing a detached view of each concept or sub-area, may not contribute to make mathematics an interesting and motivating subject for her students. Therefore, she seeks possible links among the topics she teaches, but recognizes her difficulties in improving her practice just by herself. In the group meetings she eagerly took notes of any observation seeming profitable regarding mathematical connections. To some extent, this feeling of inability in giving a unified view of mathematics was challenged (and altered) during the collaborative work.

*Maria within the collaborative project*. From the outset, Maria played an active role in the project, assuming the group as a personal learning experience. Among the topics addressed she mentions the joint discussion of lessons and their previous planning. In such a context, she said, “it becomes easier to try new experiences” (M15, January 05)[1]. Moreover, in several occasions she values the importance of group discussions: “The interest of this sort of work, even if not to learn a lot of new things, is to put us thinking and to raise new questions” (M23, June 05).

We describe several influences of the project on Maria’s communication conceptions and practices. First, she acknowledges how fundamental it is to recognise one’s own communication failures so that effective change becomes possible. She values the group discussion of past lessons as a step in building such awareness: “I guess what matters to identify communication problems in the classroom is to be able to identify failures. Often, the daily routine is so pressing that we are unable even to recognise them” (M25, July 05). She also points out that it is too easy to blame students when a lesson fails, instead of recognising communication problems. For Maria, the role of discussions in small groups became increasingly clear: “Only when we meet in a small group, like this, and begin to ask what’s going wrong, one becomes aware of difficulties in communicating with our students” (M25, July 05).

Second, Maria also emphasizes that our joint work helped in breaking the daily routine of isolated teachers which tends to obfuscate the real problems. Among these problems she underlines how difficult it is to respect students in their heterogeneity:

> We talk to the average student, forgetting those with extra difficulties or kids with different ways of making progress. We still plan lessons in a sort of canonical format that is the format we have rationalised from our previous experience as students ourselves. In the absence of sharing experiences and mutual questioning, we still go on the same way. (M25, July 05)

[1] (M15, January 05) stands for the transcript of the 15th group meeting, hold on January 2005.

Third, Maria focused in some particular elements of her practice. For example, she was challenged to address the issue of students working in groups in mathematics classes. She had already some experience of group work in natural sciences classes, but wondered how this could be done in mathematics. Along the project she tried a number of experiences with group work, allowing students to work by themselves and discussing results afterwards. The project was most helpful in modifying her initial conviction that this requires much more time than conventional lectures to cover about the same contents. She used to say:

> Sure, these steps [group work] help students to build deeper mathematical insight. My doubt is: and time? (…) How much time can one devote to discovery, building insight, mastering mathematical reasoning? My dilemma is: build mathematics or follow [successfully] the national curriculum. (M18, March 05)

Later she comments on an experience carried out on a statistics unit: “It took five lessons; normally I need less than that for this topic” (M22, May 05). But she acknowledges the fact that this activity was a training experience for herself. Training for developing more careful lesson plans and a few routines enabling her to “waste less time”, or, as she notes, “to use the available time with increased quality” (M24, July 05).

Finally, we observed her effort to take into account in her own practice the main concerns shared in the project group. For example, she indicates that she does a serious effort to reduce the number of interventions she has in the classroom: “A number of things inside my own mind are already working. For example, reminding me: let’s see what they think, what they say” (M22, May 05). She became more attentive to what her students say. Similarly, she sought her students to listen more carefully to each other. She points out episodes illustrating her greater willingness to give more time for students answering and reasoning in class: “before [the project]”, she commented, “I used to guide their answers, suggesting a possible way of handling the question straight away” (M21, May 05). Note that she recognized that such an attitude “was made possible because of the discussions within the group” (M21, May 05). Moreover, she said “my concerning with negotiation of meaning increased as a consequence of our work. Now I require students to give proper and detailed explanations and raise themselves new questions” (M22, May 05).

The project was lived by Maria as an opportunity to think about the impact of communication issues in the classroom and their relevance as a source of common difficulties in teaching. This is further illustrated by her comment in the last meeting of the academic year 2004/05: “A fundamental issue is to be aware that several daily difficulties in our professional life are related to communication” (M25, July 05). And, later, she wrote concerning the work developed:

> (…) Discussing together what classroom communication effectively means, studying a few theoretical papers as well as experience reports, our own availability to share our classes with others, to reflect in a critical way about our own practices, all this made the

> project sessions a true opportunity of professional development. Several connections were built at different levels (pedagogical, scientific, didactical), giving to this group a real sense of what needs to be changed and how. (June 06)

An indicator of the relevance this collaborative project had for the three teachers involved was the decision they took to extend it behind the initial closing data: The group still goes on at the moment of writing. Quite recently Maria wrote in an email concerning group planning for 2008-09: “I am completely available for this project. Actually, it is an irreplaceable space for sharing, knowledge building, and friendship” (September 08).

Maria always supported the project with enthusiasm and a pro-active attitude: sharing plans, discussing suggestions, inviting others to assist to her lectures. She never neglected the possibility to discuss a lesson, sharing her own thoughts and taking care to make explicit the strategies used and her motivations underlying them. The project influenced her practice with respect to the sort of discourse and interactions with students, but mainly, as she stresses, in what concerns her ability to bring variety to her lessons and relationships with students. Maria understood this project as a personal challenge, not always easy to follow. But she was always willing to share: “I have to wait so much, until Friday, to tell you…” And this led another teacher in the group to comment: “This group is our therapy” (May 06).

## CONCLUSION

The purpose of this study is to illustrate how a collaborative project can influence its participants and have an impact on their practices. Maria was chosen as the focus of this paper since she was the teacher who was most influenced by the project. Probably that happened because she took such a decision from the outset: To be open to the group influence and look into it in a positive, pro-active way. We shall now extend the discussion to the group level.

The focus of this research was communication in the mathematics class, a broad theme that may include a variety of issues and experiences. As it developed, however, it became clear to the researcher that a collaborative research entails the need for never avoiding or ignoring the questions raised by the participants or the issues that they think are most relevant, even if this implies taking less obvious “routes”.

Allowing others to come into their own classroom as well as sharing and discussing their experiences had a deep significance to all the teachers involved in this project. Maria was no exception. But this did not evolve without concern, and the feeling that something that used to be “private” was now made available to others. A number of fragments of a discourse seeking auto-justification provide evidence that collaboration is a process that extends itself in time. As underlined by several authors (e.g., Fullan & Hargreaves, 1991), mutual support in the group is essential to get through, or at least to control, our own difficulties and vulnerabilities. Just as it happened with Maria, the project helped all the others to increase self-confidence, reducing the feeling of impotence and solitude. This role, which is central in a collaborative project (Maeers & Robison, 1997), was recognised by all the participants, with different degrees.

A collaborative project is a social construction. As such, it entails the need for all participants to share their different ways to approach a situation or experience (John-Steiner, Weber and Minnis, 1998). The relative heterogeneity of participants made mutual influence possible and played an important role in the perception that the group has of its own development.

For the researcher, this was a rich experience, namely as an opportunity to approach very closely school reality and the way it is experienced by teachers. Nothing is given once and for all, and so sometimes she felt tired, unable, almost lost. But progress was made because in the group we have always felt that, in spite of difficulties, we needed to go on because it was exactly from our disagreements that we evolved as a group.

This research study shows that, even with a highly motivated group, changing is always slow. The steps to undertake cannot be too large. Often, the researcher felt that her attempt to propose a number of experiences and activities was fruitless: What is really necessary is that every teacher in a collaborative group takes the group objectives as his/her own.

Along the project, Maria assumed herself the role of researching her own practice and provided evidence of how that entailed changes in her professional practice. This seems to be consistent with related research (e.g., Fernandes & Vieira, 2006) which shows that collaborative work fosters an attitude of serious enquiry about the teacher's own practice. As a consequence, Maria considers herself now more able to challenge her students, to develop their autonomy and to explore their mutual interactions in the classroom. She feels them more active and responsible towards their own learning. She is confident about her stance, but keeps saying that to make changes effective one needs a reflexive attitude and time to mature.

## REFERENCES


Alrø, H., & Skovsmose, O. (2004). Dialogic learning in collaborative investigation. *Nordisk Matematikkdidaktikk, 9*(2): 39-62.

Barrody, A. (1993). *Problem solving, reasoning, and communicating, K-8: Helping children think mathematically*. New York, NY: Macmillan.

Bishop, A. J., & Goffree, F. (1986). Classroom organization and dinamics. In B. Christiansen, A. G. Howson & M. Otte (Eds.) *Perspectives on mathematics education* (pp. 309-365). Dordrecht: Reidel.

Boavida, A. M., & Ponte, J. P. (2002). Investigação colaborativa: Potencialidades e problemas. In GTI (Ed.), *Reflectir e investigar sobre a prática profissional* (pp. 43-55). Lisboa: APM.

Bonals, J. (1996). *El trabajo en equipo del profesorado*. Barcelona: Graó.

Chazan, D., & Ball, D. (1995). *Beyond exhortations not to tell: The teacher's role in discussion-intensive mathematics classes*. East Lansing: Michigan St. University.

Clift, R. T., & Say, M. (1988). Teacher education: Collaboration or conflict? *Journal of Teacher Education, 39*(3), 2-7.

Day, C. (1999). *Developing teachers: The challenges of lifelong learning*. London: Falmer.

Drake, S., & Basaraba, J. (1997). School-university research partnership: In search of the essence. In H. Christiansen, L. Goulet, C. Krentz & M. Maeers (Eds.), *Recreating relationships: Collaboration and educational reform* (pp. 13-25). New York, NY: SUNY Press.

Fernandes, I. S., & Vieira, F. (2006). A supervisão como um jogo de subversão de regras. In F. Vieira, M. A. Moreira, I. Barbosa, M. Paiva & I. S. Fernandes (Eds.), *No caleidoscópio da supervisão: Imagens da formação e da pedagogia* (pp. 233-237). Mangualde: Pedago.

Fullan, M., & Hargreaves, A. (1991). *What's worth fighting for? Working together for your school*. Buckingham: Open University Press.

Goulet, L., & Aubichon, B. (1997). Learning collaboration: Research in a first nations teacher education program. In H. Christiansen et al. (Eds.), *Recreating relationships: Collaboration and educational reform* (pp. 115-127). New York, NY: SUNY Press.

Hargreaves, A. (1994). *Changing teachers, changing times: Teachers' work and culture in the postmodern age*. Toronto: University of Toronto Press.

Huberman, M., & Miles, M. (1994). Data management and analysis methods. In N. Denzin & Y. Lincoln (Eds.), *Handbook of qualitative research* (pp. 428-444). London: Sage.

Jaworski, B. (2004). Grappling with complexity: Co-learning in inquiry communities in mathematics teaching development. In M. J. Høines & A. B. Fuglestad (Eds.), *Proceedings of 28th PME Conference* (Vol. 1, pp. 17-36). Bergen.

John-Steiner, V., Weber, R., & Minnis, M. (1998). The challenge of studying collaboration. *American Educational Research Journal*, *35*(4), 773-783.

Kapuscinski, P. (1997). The collaborative lens: A new look at an old research study. In H. Christiansen et al. (Eds.), *Recreating relationships: Collaboration and educational reform* (pp. 3-12). New York, NY: SUNY Press.

Lafleur, C., & MacFadden, J. (2001). The angst, ecstasy and opportunities of collaborative inquiry. In R. Ravid & M. G. Handler (Eds.), *The many faces of school-university collaboration: Characteristics of successful partnerships* (pp. 207-221). Englewood, CO: Teacher Ideas Press.

Lampert, M., & Cobb. P. (2003). Communication and language. In J. Kilpatrick, W. G. Martin & D. Shifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 237-249). Reston, VA: NCTM.

Lappan, G., & Schram, P. (1989). Communication and reasoning: Critical dimensions of sense making in mathematics. In P. R. Trafton, & A. P. Shulte (Eds.), *New directions for elementary school mathematics* (pp. 14-30). Reston, VA: NCTM.

Maeers, M. "Vi", & Robison, L. (1997). Making sense of mathematics within collaborative communities. In In H. Christiansen, L. Goulet, C. Krentz & M. Maeers (Eds.), *Recreating relationships: Collaboration and educational reform* (pp. 153-165). New York, NY: SUNY Press.

Martinho, M. H. (2007). *A comunicação na sala de aula de Matemática* (Tese de doutoramento. Universidade de Lisboa).

Olson, M. (1997). Collaboration: An epistemological shift. In H. Christiansen et al. (Eds.), *Recreating relationships: Collaboration and educational reform* (pp. 13-25). New York, NY: SUNY Press.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms.* London: Routledge.

Ponte, J. P., Boavida, A. M., Graça, M., & Abrantes, P. (1997). *Didáctica da Matemática: Ensino secundário*. Lisboa: Ministério da Educação.

Ponte, J. P., & Santos, L. (1998). Práticas lectivas num contexto de reforma curricular. *Quadrante*, *7*(1), 3-32.

Sherin, M. G. (2002). A balancing act: Developing a discourse community in a mathematics classroom. *Journal of Mathematics Teacher Education, 5,* 205-233.

Wagner, J. (1997). The unavoidable intervention of educational research: A framework for reconsidering researcher-practitioner cooperation. *Educational Researcher*, *26*(7), 13-22.

Yackel, E., & Cobb, P. (1998). Sociomathematical norms, argumentation, and autonomy in mathematics. In *Journal for Research in Mathematics Education, 27*(4), 458-477.

Yin, R. K. (1989). *Case study research: Design and methods*. London: Sage.

# REFLECTION ON PRACTICE: CONTENT AND DEPTH

**Cristina Martins**, *Escola Superior de Educação de Bragança*

**Leonor Santos**, *Faculdade de Ciências da Universidade de Lisboa*

## ABSTRACT

*This text is based upon an ongoing investigation with the main goal of studying the professional development of primary school teachers, specifically the ability to reflect, within a continuous training program.*

*This study follows a methodological approach of a qualitative type, comprising case study, with recourse to interviews, participant observation and documental analysis.*

*A first analysis of the written reflection of one of the participants, included in the reflection portfolio, points, in terms of content, towards less spreading of the themes approached, the ones considered the most significant being subsequently extracted and correlated. A greater depth in the reflection is also noted, with the teacher having concern to justify her statements, present a critical analysis of her role and rethink her practice.*

Key-words: Professional development, mathematics' teacher, teacher training, reflection, practice

## INTRODUCTION

Reflection is one of the activities most frequently considered to contribute to the professional development of teachers, since it may be presented as a means to improve classroom practices.

The Program for Continuous Training in Mathematics for Primary School Teachers, launched by the Board of Education and the Board for Science Technology and Higher Education, has been under development in Portugal since the academic year of 2005/2006. This program aims at an improvement in the teaching and learning of Mathematics as well as developing a more positive attitude towards this branch of knowledge. It involves conducting group training sessions, classroom supervision sessions and one final plenary meeting for a final appraisal of the program. Participant evaluation is undertaken through the elaboration of a portfolio, over the duration of the program. Contents of this program include the nature of the tasks, namely problem solving, and the use of physical resources, in which manipulative materials are included.

This paper is based in an ongoing investigation, whose goal is to study the professional development of primary school teachers through participation in the program. Specifically, we aim here to answer the following question: (i) In what way does the teacher's ability to reflect evolve throughout the training program?

## THEORETICAL FRAMEWORK

"The professional development of teachers, both inside and outside the classroom, is the result of their reflection and participation in training opportunities which improve and increase their development and progress." (National Council of Teachers of Mathematics, 1994, p. 175). Reflection is an activity which may contribute towards the teacher's professional development. The term reflection is, however, polysemic. To Dewey (1933), in the field of education, the "active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusions to which it tends, constitutes reflexive thought" (p.7), appearing as an activity thoughtfully and directly connected to practice. Zeichner (1993), although stressing that terms such as reflexive practitioner and reflexive teaching have become slogans for teaching reform and teacher training, attributes a strong personal angle to reflection, considering that there are no recipes to teach the teacher how to reflect. Schön (1983) also contributes in clarifying this concept, considering three kinds of reflection: in action; on action and upon reflection in action.

Addressing teacher training programs, Lee (2005) finds differences in the content and depth of the reflection undertaken by future teachers. Specifically he identifies the following as factors related to the depth of the reflection: personal context, professional experiences encountered and ways of communicating.

To Day (2001), just conceiving the existence of reflection as a means of learning does not demonstrate the depth, reach and goals of the process, as "good teachers are technically competent and reflect upon matters pertaining to the goals, the process, the content and results" (p. 72).

One of the contexts which may be supportive in producing reflection is the one involving portfolios. Written reflection is one of its basic components, particularly if one is examining documented teaching, and is focused on what the teacher and the student have learned (Santos, 2005; Wolf, 1996). Reflection is, thus, "the critical heart of the record" [contained in the portfolio] (Lyons, 2002).

Summing up, this study considers that reflection helps to looking backwards and rethinking one's own practices (Muñoz-Catalán *et al.*, 2007; Oliveira & Serrazina, 2002), although it is possible to find idiosyncratic differences in the process of reflection (Hospesova *et al.*, 2007). Moreover, reflection as analytical thought is above all associated with unsolved problems (Dewey, 1933), or rethinking meanings previously associated with educational situations.

## INVESTIGATION METHODOLOGY

This work takes place in a natural environment, in which the researcher is also the leader of a working group made up of nine teachers. We have chosen to adopt a qualitative methodological approach (Bogdan & Biklen, 1994), undertaking three case studies (Gall, Borg & Gall, 1996), with the help, in data gathering, of semi-structured interviews, participant observation and documental analysis.

Initial, intermediate and final interviews have as a main goal the gathering of data pertaining to the participant teachers, on the basis of the issues under consideration. Interviews, after each class has taken place, are related to points emerging from the experimental classroom activity. Group training sessions and classroom supervision sessions were observed. Interviews and observations undertaken were fully audio taped and transcribed. Documental analysis focused on the records included in the portfolios (planning, material used, student production and reflections), in the field notes about supervision sessions and in the reflections about group training sessions

In her portfolio, Sara, one of the participants, has included three reflections on tasks tried out in the classroom during the course of the program, although she was only compelled to include two. In this paper we present the analysis of the first reflection, which took place in December 2006, and of the third, in April 2007.

To address the presentation of written reflections to be included in the portfolio, guidelines, followed in the training program, were provided, consisting of the following points: 1. Activity goals; 2. Activity description; 3. Reflection on the activity, including four aspects: (i) activity planning; (ii) evaluation of what the students might have learned with the activity; (iii) importance of the activity for the teacher; and (iv) the teacher´s future perspectives regarding Mathematics.

Analysis of information gathered started after completion of the training program and consisted of organizing and interpreting data, considering the problem under investigation, theoretical framework and the empirical work which had taken place. Specifically, fields of analysis considered were content and depth (Lee, 2005). Regarding content, we have defined as categories for analysis the ones included in the guidelines. Regarding depth, we have considered: (i) Confrontation with one's own practice (identification and description of what one considers important or problematic); (ii) Interpretation (why does one perform the way one does?); (iii) Putting into perspective (confrontation of action with what one thinks and feels about it) and (iv) Reconstruction (what ought to be kept? What can be different? what can be changed, why?)

## TEACHER SARA'S WRITTEN REFLECTION

Sara is around forty, and has twenty to twenty five years of professional experience. She has a Primary School Teacher's degree and the Scientific and Pedagogical Training Complement for Primary School Teachers, which bestows a license level degree.

Sara tells us she has always liked mathematics. Although she considers herself as having enough knowledge to teach she has invested time in keeping herself up to date through attendance at training sessions and programs.

Regarding the sort of tasks she planned and put into practice in the field of Mathematics, before attending the program, Sara said she *sometimes* uses problem solving. She states that she is aware of not using a lot of materials in the tasks she puts forward, relating this idea to the need to keep up with the program:

> I am, I am aware I don't use much. I think we are rather limited concerning time because we are always concerned with keeping up with the program and then we may get one day behind, which we may need later. [initial interview]

Specifically, regarding reflection upon practice, before attending the program Sara explains she did not reflect much and that she had never made a written reflection:

> Also, it is not that one completely overlooks it. But, when returning home, one puts school somewhat aside because we must also support our family a bit (:…) Perhaps, after several activities, I sit down and reflect a bit to myself. Not on paper, but to myself [initial interview]

The first reflection she presents in her portfolio is based on the students solving the following problem: Francisco raises chickens and rabbits. He has in all 16 heads and 48 legs. How many chickens and how many rabbits does Francisco own? The third one relates to constructing and identifying geometrical figures using the *Tangram*.

Sara has respected the guidelines in both reflections. Specifically, in point 3 – Reflection on the activity – of the written reflection that she produced, and related to the item – *activity planning* – she begins by making reference to what she considers essential to someone who solves a problem and stresses the difficulties to the one proposing it (speech 1). She presents, succinctly, the goals of the task she has put forward (speech 2):

> 1. Interest in the problem and its ownership by the one who solves it are essential. The hardest step for the one presenting it, might be to choose the problem or even to make it up.
>
> 2. When presenting the problem to the students I wished them to explore the context, gather data and find differences [Sara's portfolio 1st reflection]

The third reflection begins with her expectations in relation to the fulfillment of the task, regarding her previous knowledge of the class.:

> As I was aware that the tangram had already been used in the classroom, I was led to think that free activities and the relationships between the pieces had already been explored. So, I started the class aware it would be a noisy class, but that it would be easy to reach the projected goals within the time allotted. [Sara's portfolio 3rd reflection]

She mentions some flaws regarding planning, especially regarding the sequence of the proposed activities:

> In the course of the class I noticed that planning had some flaws, namely regarding the order of activities. I came to the conclusion that I should have started the class with a deeper exploration of the tangram.

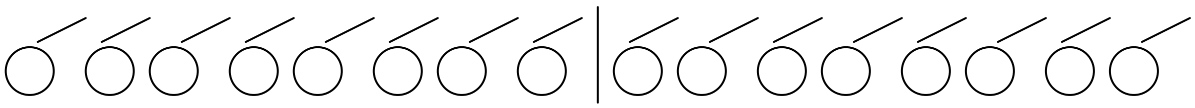
> Activity 2 should have taken place more towards the end of the class, because they were very worried about drawing, which caused it to last for a long time and some of them only managed it with help. [Sara's portfolio 3rd reflection]

Concerning the item – *evaluation of what the students might have learned with the activity* – in the first reflection she identifies what she considers to be the main concern of students during the activity and explains her reaction regarding that concern (speech 1) She also mentions the students' reactions regarding difficulties felt in the beginning of the task; she tries to account for them and explains her way of reacting in face of the situation (speech 2):

> 1. During the course of the class I noticed a huge concern of the students to place the data and perform an operation. I read the problem once more and showed them that the results were not dependent on adding or subtracting these figures.
>
> 2. I noticed they were having trouble with starting the task on paper. They asked a lot of questions such as "I did not understand this here", I guess to call for the teacher's attention, to see if they could get a little help. At first, the idea was not to interfere or help the students but due to the number of requests I finally decided to lend a little hand [Sara's portfolio 1st reflection]

As a matter of fact, at the beginning of the task, just after Sara had handed over the problem's instructions, some comments were heard: "I know the operation!", "It's too much!", and "I already know the problem!" While she read the problem aloud some students interrupted with questions: "What are heads?", "What are chickens?" Sara explained: "16 heads means 16 animals". And she asked: "How many legs does a chicken have? And a rabbit?" After the reading she informed them: "Each one of you does it as you want" The students tried to solve the problem individually, always requesting the assistance of the teacher and even of the researcher.

She noticed that that although the students remained restless and constantly requested the teacher's assistance they started designing their strategies. Sara moved about the room in order to see the work the students were performing. After some time Sara asked some students to explain their ways of solving the problem on the blackboard. One of the students made the following sketch:



He began by making 16 circles and made a dividing slash in the middle and counted the "number of chicken" and the "number of rabbits" making a jot over each circle and simultaneously explained his reasoning.

Another student made drawings. She started by drawing a child and two sets of eight animals some with two feet others with four. In the end she explained her reasoning to the colleagues. Another student drew an animal with four legs, another with two, and so forth, up to a total of 16 animals.

Only the students who had come up with the correct answer were asked to come up to the blackboard.

In the course of her reflection, besides identifying the solving procedure used by most students, she comments on it and stresses a strategy used by just one student:

> I realized that many students started by dividing the number 16 in two groups and then added the legs. I think that choosing this method is related to the 8 multiplication table, which we had studied recently and was on the board.
>
> I was sorry Cláudio couldn't go up on the blackboard to show the method he had used to solve the problem. He did not come up with 8 rabbits and 8 chickens because he got lost in counting but his representation was different and interesting. [Sara's portfolio 1st reflection]

Sara also mentions time management, specifically lack of time to communicate the different solving procedures used. "I think I gave too much time to individual solving, which did not allow the children to go up on the blackboard to explain their reasoning and to check for the existence of diverging results".

She mentions that "not many of the students managed to come up with valid reasoning to get to the result one wished for" and she points out, justifying this, that the students felt some difficulties in problem solving, although there was some development in competencies (speech 1). She also indicates the main learning outcomes the students achieved (speech 2):

> 1. I noticed the students felt some difficulties in solving these sorts of problems, perhaps because they were not used to them, even so, there was a development of competencies which led to the building up of personal strategies. Problem solving placed the students in an active learning attitude, both by giving them the possibility of constructing notions as an answer to the questions raised, and by urging them to use the acquisitions made and to test their efficacy.
>
> 2. They have learned to show curiosity and the taste for exploring and solving simple problems;
>
> To solve situations and daily problems using representations and schemes;
>
> They have learned to make simulations of real life events [Sara's portfolio 1st reflection]

In the third task, Sara began by giving some information about the origin and use of tangrams. The students listened attentively. Many of them said they had already worked with that material. After distributing the tangrams among the students, these at once started building free figures. Sara passed around a work sheet with the instructions for the task. Some students remained interested in figure building. Sara asked a student to read the introductory text about tangram and she read the first questions in the work sheet. "1.Which is the tangram's original shape? 2. In how many parts is it divided? and 3. Which geometrical shape does each component represent?" The students recorded their answers in their work sheets. Next, Sara asked the students to perform the second task indicated in the sheet: “using all the elements, build and record the figures built: a) a square; b) a rectangle and c) a right triangle”.

Several students mention not understanding what they are supposed to do. Others say: “I can’t make it” and ask the teacher’s help. Others advance on their own and solve the problem. Some students also show difficulties in recording the results and concentrate on this point, failing to advance in building the various figures requested. Many appear seriously worried about not being able to perform the task and some give up. Several students find it difficult to know what a right triangle is.

In her reflection, and regarding this point, she correctly evaluates the mathematical output of students, indicating learning outcomes achieved:

> They rememorized geometrical figures and defined them regarding the number of sides;
>
> They learned that one of the seven elements of the tangram is called a parallelogram;
>
> They were able to find out that you can build squares out of the several elements of the tangram;
>
> They have learned that you can build a lot of figures with the tangram.
>
> There were also learning acquisitions in other areas such as Portuguese Language, because besides having to communicate they also had to read and write. And they also learned some trivia, for instance, that the Chinese tangram is not the only one [Sara’s portfolio 3rd reflection]

She identifies, justifying this, two particular cases of students which surprised her when performing the task:

> Two students surprised me, one for the better, one for the worse. Hélia surprised me for the worse because she has shown she is a participating student who likes to commit herself to solving the activities and in this particular class she needed a lot of help to solve the activities I put forward;
>
> Pedro surprised me for the better because he showed himself to be more committed in solving the activities, did not interrupt the class as often, and managed to solve what was asked of him [Sara’s portfolio 3rd reflection]

Regarding the item – *importance of the activity had for the teacher* – in her first reflection Sara only presents a brief remark:

> For me, as a teacher, it was an important class, as it allowed me to see that children felt a lot of difficulties in translating real and everyday language into Mathematical, symbolic language [Sara’s portfolio 1st reflection]

In her third reflection, she explains in a detailed way how important the activity had been for her, connecting it to the learning outcomes achieved by the students:

> One of the factors which either contributed to or made some students’ learning difficult was the fact that it was an individual task, as it became complex for me to provide

> answers to all requests as quickly as possible, which was what they wanted. Even so, this activity was very important for me, as I think I left the students motivated to work with the tangram, a material with which many mathematical themes or contents can be associated [Sara's portfolio 3rd reflection]

As a matter of fact, Sara was widely called on by students, either to help them build shapes or to draw them. She tried to answer all requests, by giving them some clues but, mostly, by reminding them that they had to try to build the shapes themselves. It was apparent that Sara experienced some difficulty in providing assistance to all the students, as, on one hand, the class was made up of over twenty students and, on the other, as she repeatedly mentioned during the activity, she wanted the students themselves to find out the answer.

Regarding the item – *the teacher's future perspectives regarding Mathematics* –, in her first reflection Sara presents future valuation of problem solving:

> I think that in this class one must pay more attention to problem solving because it will help them to develop reasoning and prepare them for a future where they can more easily develop personal problem solving strategies and to, step by step, assume a critical attitude in face of the results [Sara's portfolio 1st reflection]

In the third reflection, she presents future classroom work perspectives, showing a definite interest on resorting to the use of manipulative materials:

> Although it is a large and noisy class I would have no qualms about proposing a similar activity. I think it would be very useful for these children to work more with manipulative materials as they allow mathematical abilities to develop and to broaden knowledge in every area. They also allow imagination, reasoning and communicative skills to develop. [Sara's portfolio 3rd reflection]

Throughout the academic year, Sara has tried out problem solving more often, for instance using problems originating from the National Examinations.

Regarding this matter, in her final interview Sara stated there had been some changes in her teaching practice compared to the program's beginning and pointed out some aspects she had started placing more value on:

> There have been several changes from the beginning of the program because I started giving more value to verbal interactions and the nature of the tasks put forward, to value learning more and to value reflection much more [final interview]

Also, regarding the use of manipulative materials, after attending the training program she greatly stressed their use, namely with regard to awareness of their capabilities:

> I learned I can use known material such as the tangram and the geoboard, to teach concepts with I never formerly associated with them (…) We came into contact with new materials and with how to work with already known ones such as the tangram and the

geoboard but which were underused, which we had in the classroom but which we did not use as they could be used [final interview]

## FINAL CONSIDERATIONS

Regarding the written reflections presented, although she always based herself upon the guidelines, Sara does not reason in both of them in the same way, either with regard to content or to depth.

With regard to content, there are some distinguishing aspects which naturally arise from each task's specificity, for instance: expectations regarding the noise to be naturally experienced while performing a task involving manipulative materials. However, in the first reflection, the diversity of themes approached within each category is very large. For instances, in item – *evaluation of what the students might have learned* – Sara highlights the students' main concern within the development of the task, identifying her own reaction and ways of handling the situation as well as the students' reactions. She also identifies solving procedures used by the students, difficulties felt and main learning outcomes of the students.

In her third reflection, there is a more restricted range of subjects approached. However, in general, she covers the main items of the guidelines and, essentially, focuses on her role in what she identifies as having developed below or against expectations. She specifies the aspects approached, directing them in a sustained way towards her students and towards more specific mathematical acquisitions. She tries to explain her statements in length.

Concerning the depth in her first reflection, there are contents which are only briefly touched upon (for instance, communication of the problem solving procedures), there are others in which she presents some justification for certain events (for instance, students' difficulties concerning problem solving). Thus, the first reflection is marked by confrontation with her own practice, some interpretation and very little putting into perspective, thus focusing on a retrospective dimension. In her third reflection, it seems possible to state that Sara has by now absorbed that which was fundamental to obtain from the activity undertaken, showing some distance from the specific items mentioned in the guidelines. She establishes connections among different items and always tries to account for her statements. She reflects upon the described points, showing her role in the development of the task and rethinking her future practice. She thus shows herself as having reached the level of appropriation and some approximation to the level of reconstruction, situating herself, in consequence, in a prospective dimension.

## REFERENCIES

Bogdan, R. e Biklen, S. (1994). *Investigação qualitativa em educação*. Porto: Porto Editora.

Day, C (2001). *Desenvolvimento profissional de professores: Os desafios da aprendizagem permanente*. Porto. Porto Editora.

Dewey, J. (1933). *How we think*. Mineola, New York: Dover Publications, Inc.

Gall, M. D., Borg, W. R. e Gall, J. P. (1996). *Educational research: An introduction*. New York, NY: Longman.

Hospesova, A.; Tichá, M. & Macháčková, J. (2007). Differences and similarities in (qualified) pedagogical reflection. *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 1906-1915). Larnaca: University of Cyprus.

Lee, H. (2005). Understanding and assessing pre-service teachers' reflective thinking. *Teaching and Teacher Education, 21*, 699-715.

Lyons, N. (2002). The Project: Interrogating. Documenting, and representing the scholarship of teaching through a reflective portfolio process. In N. Lyons, A. Hyland & N. Ryan (Eds.), *Advancing the scholarship of teaching and learning through a reflective portfolio process: The University College Cork experience* (pp. 19-28). Available online at http://www.ucc.ie/Teaching_and_Learning/Scholar.doc (accessed 8 March 2007).

Muñoz-Catalán; M.; Carrillo, J. & Climent, N. (2007). The professional development of a novice teacher in a collaborative context: an analysis of a classroom practice. *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 1935-1944). Larnaca: University of Cyprus.

National Council of Teachers of Mathematics (1994). *Normas profissionais para o ensino da matemática*. Lisboa: APM e IIE.

Schön, D. (1983). The *reflective practitioner: How professionals think in action*. Aldershot Hants: Avebury.

Santos, L. (2005). The portfolio in teacher education. *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (pp. 1579-1587). Sant Feliu de Guílos: Universitat Ramon Llull.

Wolf, K. (1996) Developing an effective teaching portfolio. *Educational Leadership,* 5*3*(6), 34–37.

Zeichner, K. (1993). *A formação reflexiva de professores: Ideias e práticas*. Lisboa: Educa.

# DEVELOPING MATHEMATICS TEACHERS' EDUCATION THROUGH PERSONAL REFLECTION AND COLLABORATIVE INQUIRY: WHICH KINDS OF TASKS?

Angela Pesci

Department of Mathematics, University of Pavia

*Abstract. After the reprise of a model of intervention for the training of mathematics teachers (both initial and in-service) developed after experiences carried out in a cooperative modality (Pesci, 2007a), several tasks are presented for encouraging the development of disciplinary, didactic, and relational competences of the teachers. The theoretical framework related to these tasks puts in evidence the reasons of their choice: the importance, for teachers, of collaboration in sharing personal experiences, difficulties, and resource, the importance of autobiographical reflection, of reflection on one's own classroom practices, and of epistemological reflection on the disciplinary contents. The connection to the debate about tasks which is developing considerably in relation to the education of teachers (Javorski, 2007) is underlined.*

Key-words: mathematics, teachers, cooperation, collaboration, tasks.

## INTRODUCTION

This paper has two goals, that of developing and specifying the model of intervention on teachers delineated in the contribution at Cerme5 (Pesci, 2007a) and that of explicitly connecting the model to some crucial ideas for the education of mathematics teachers which the literature is highlighting with growing intensity. How do I intend to reach the two goals? By supplying examples of tasks for teachers which, on the basis of the mathematical contents proposed, on the didactic modalities adopted and on the requested personal reflections, make evident their theoretical motivations and their connection to the debate delineated by Jaworski (2007) and synthesized by Watson and Mason, in the same special issue of JMTE. More specifically, this paper foresees a brief look back at the model of intervention on mathematics teachers already outlined (Pesci, 2007a) and the description of some tasks for teachers which have the goal of promoting personal reflection on their own relationship with mathematics and the encouraging of epistemological reflection on specific mathematical contents. Then there are the synthesis of the theoretical background for the choice of such tasks, with reference to related literature, and some suggestions for future research.

## A MODEL OF INTERVENTION ON MATHEMATICS TEACHERS

The main issues of the model described in Pesci (2007a) are summarized shortly in this section, with the aim to make evident the frame in which the following tasks should be placed. The model was developed in the framework of situated cognition and distributed cognition:

> The frame of reference is that of social constructivism, which emphasises discussion, negotiation of meanings, collaboration, and development of positive personal relationships (Ernest, 1995, Bauersfeld, 1995) and the concept of cognition is that formulated both as "situated cognition" (Nunez, 1999) with relevance to the context, and as "distributed cognition" (Crawford, 1997) with relevance to interrelationship and to sharing. (Pesci, 2007a, p. 1946).

The model was based also on cooperative modality, which gives special importance to relational and social aspects: in their different interpretations, all cooperative models share their explicit attention to both disciplinary dimension and social one. The goals to be reached along the educational process are not placed only at the disciplinary level but also at personal and social ones, with a special attention to the quality of the relationships established amongst people (Johnson, Johnson & Holubec, 1994, Cohen, 1984). At the base of the model (interpreted both for students and for teachers), there was, therefore, the idea of a co-construction of knowledge, a social construction, with the principles that for several decades, even with different accents, pervade the most diffuse teaching-learning models. At the centre of the learning process, managed by an expert, there are the learners and the inter-relationships (between learners and with the expert) with the consequent emphasis on the role of language and on the phases of discussion, argumentation, confutation, comparison, and sharing. What is suggested by teaching-learning cooperative models is also coherent to what is underlined by neuroscience (Damasio, 1999) and by epistemology (Polanyi, 1958): in each process of building or revisiting knowledge it is necessary, as a matter of fact, to keep track of the close connection between emotion, sentiment and cognition. This is valid not only for the students, in class, but also for the teachers, in their training meetings. In each training intervention, therefore, there was a special attention to the affective-relational aspects.

> With reference to relational and social aspects, I consider essential that a meaningful intervention on mathematics teachers (a) could give time and space to their reality as teachers in that precise moment of their professional history through the autobiographical discourse; (b) could constitute a direct experience of what is proposed, with wide possibility of dialogue with the other participants; (c) could be, in each case, attentive to the modalities of communication. (Pesci, 2007a, p. 1952)

The main goal, in planning meetings for teachers, was to promote their personal reflection, taking account of disciplinary, didactic and relational aspects:

> The basic idea is that of creating, in each encounter, occasions for personal reflection and for dialogic inquiry, with the same spirit stressed in the project *Learning Communities in Mathematics* (Jaworski, 2004), where the main objective is that both researchers and practitioners are engaged in action and reflection for mutual growing. (Pesci, 2007a, p. 1952).

The following tasks for mathematics teachers are examples of how it could be possible to foster their reflection and inquiry on the three different and essential aspects of their competence: *disciplinary*, *didactic* and *relational*.

## EXAMPLES OF TASKS FOR MATHEMATICS TEACHERS

**Autobiographical reflection.** Every time that it is possible, in particular when the training meeting foresees more than one session, I organize the initial phase with the teachers starting with their personal relationship with mathematics, both with reference to their own history as *student* and to their own history as *teacher*.

In the first case, I propose answering several written questions, which have to do with their recollection of a pleasant episode (and respectively an unpleasant one) during a mathematics lesson, referring to all of their pre-university scholastic life. Sometimes I turn to the request for an opportune metaphor, such as "to do mathematics was like entering a jungle, or a challenging game, or a long marathon, etc.", described in Pesci (2006).

In the second case, the activity of reflection on one's own "history" as a mathematics teacher can come about through a choice of metaphor or with the request to complete a questionnaire of this kind:

***From my "history" as a teacher***

*An episode to remember*
*An episode to forget*
*A moment of change*
*A wish that came true*
*A wish that didn't come true*

In both cases, the task, by its nature, is individual, but I usually invite the participants to share within their own group (of 4-5 people), if they want, the interpretation of the task or some experiences, both before writing and at the conclusion of the writing. The only recommendation is that, in each case, there is a period of silence, during which each person can collect his own thoughts and write calmly. To this aim, it is essential that, right from the beginning, each commits to observing the others attentively, being aware of when it is opportune to intervene with their own contribution or give space to the intervention of another person or remain silent.

The personal reflections which are asked for are of various natures and, obviously, depend a lot on the characteristics of the group itself. For example, in a group of teachers who have been in-service for several years, but were not yet confirmed, it came out that more than a third of the participants (there were about 60) highlighted, as a 'wish not yet come true' that of didactic continuity. It is clear that the same kind of wish does not appear anymore with teachers in regular service for years. It is not important, in this context, to list the different kinds of responses collected. Instead, it seems interesting to observe, at least, these two facts:

- the tasks of an autobiographical nature, followed by the sharing of personal experiences, have as a consequence to immediately orientate teachers' attention toward the other members of the group, reducing the attention which, at the beginning of the activity, everyone has toward the presenter of the training, and encouraging the perception of the others' resources, at the level of disciplinary competence and interpersonal qualities. When the activities are carried out together, it is, without a doubt, the most productive starting point;

- to put into play the one's own memories and one's own history is unusual but manages to capture the participants in an absorbing way: the result is a sort of requalification of the way of being present at the training event. Often I perceive in the teachers, also during the following activities, a less superficial, more meaningful, and more profound, involvement, as if the autobiographical connotation were able to give greater strength and authenticity to the actions that they share.

**Reflection on one's own classroom practice.** Amongst the tasks proposed to the teachers to encourage their reflection on their own classroom practice, I'll quickly cite two examples connected to two different kinds of experiences. During a cycle of seminars on how to confront the difficulties in mathematics, in the secondary school, it came out that all the participants (about 20) had already adopted specific strategies to help students overcome difficulties in mathematics. Therefore, I held it to be opportune to dedicate an entire meeting to the specific reflection on such strategies, inviting each one to respond to some questions, amongst which were the following:

> *You have already adopted specific strategies to help your pupils overcome difficulties in mathematics: choose, in the case of several strategies, the one which you hold to have been the most effective and describe how you realized it in class, according to the following chart:*
> *a) Strategy used*
> *b) With what frequency?*
> *c) With pupils of which classes?*
> *d) Briefly describe how you develop such strategy in class*
> *e) For which mathematical contents did you turn to such strategy?*
> *f) Which are, in your opinion, the strong points of such a strategy?*
> *g) Which are, in your opinion, the weak points of such a strategy?*

Naturally, it was only the beginning of a longer path, certainly not exhausted in one meeting. Still, I noted that the participants were not used to reflecting on the methodology of their own practices, but they were almost exclusively worried about the mathematical content to develop in class. For example, it came out that whoever had tried to make the young people work in groups, had not structured the activity in any way, not foreseeing specific roles for the pupils and not planning sufficient time and adequate space for the activity. Even the mathematical questions were chosen without specific motivations. Analogously, whoever had proposed a learning experience of peer tutoring, had not programmed any form of collection of the work carried out, neither for the pupil in the role of the teacher, nor for the one in the role of the pupil. Not having a clear idea that a key element for success, in these cases, is precisely the awareness of the importance of *setting*, they did not share with the pupils the methodology of the activity to be carried out and they did not put the right emphasis on it. The results, in fact, were not satisfactory.

In another case, following experiences conducted in classes with the cooperative learning modality, after a rather long period (more than a year), I had foreseen with the teachers specific instances of reflection on the perceived effects (positive or negative at the disciplinary or relational level) on the pupils and on themselves. Several questions and several results, which are not necessary to take up here, are described in detail in Pesci (2007a). Here, I would like to put in evidence some general observations, also in relation to what I noted during the seminars on the difficulty of learning cited before.

The modality that I put into effect with the teachers is usually that of sharing and discussion in small groups (4-5 people) before the general discussion and debate. I noted that this encourages, in a decisive way, the participation of everyone. Each one, in the small group, feels more welcome, safer, and freer therefore to express their own difficulties, their own fears, their own experiences and desires. Realizing that a fear (for example, that of not being up to maintaining control of the class) or a difficulty (for example, that of managing the time in class well) is common to others, gives greater strength to each one in the search for and sharing of the best strategies for confronting them. The requested reflections on the practices of the teachers go on to involve their acting in class and out of class and the sharing with colleagues shapes itself as an important occasion of comparison and growth. The relational competences of the teachers, specifically the ability to communicate with their colleagues, to share resources, and to confront together the obstacles has, without a doubt, a central role in the building of a team of prepared, reflective and able to change teachers (Dozza, 2006). In other words, it seems necessary to give time and space to such activity of personal reflection.

**Reflection on specific mathematical contents.** I will describe briefly two different situations as examples of the tasks proposed to the mathematics teachers for reflection on their teaching discipline. The first is more appropriate for a single intervention, which can be completed in one meeting and the second is more appropriate for starting a longer activity, which can be developed in successive meetings. Both tasks have the characteristic of a simple enough presentation, which makes the teachers curious and therefore easily involves them, but continuing on they become more complex. These tasks are therefore right to be confronted in collaboration, in the direction of the discovery of their didactic values and of the variety of the mathematical themes from which to choose possible developments. Besides, both the tasks could be proposed to the students, the first example starting from the upper classes of the primary school and the second starting from the secondary school.

The first problem is placed in ZxZ (the "pointed" plane) and proposes a search for isosceles triangles with the oblique side assigned AB, limited to those with all three vertices in points of ZxZ. The investigation, apparently very simple, proves to be quite demanding, both for the geometric questions and the arithmetic questions involved. Besides, it can be developed with questions of isoperimetry, of equiextension, and of congruence between the triangles found, going on to weave together, in a single context, the use of arithmetic and geometric competences and of argumentative and demonstrative procedures. It is evident, therefore, that also the discussion about the didactic value of the problem turns out to be quite full and interesting.

The second problematic context looks at Euler's formula and its validity; to be explored in several models proposed concretely or drawn on the blackboard. It is well known that in simpler cases, for example for regular polyhedra or for convex polyhedra, it is easy to count *faces, corners*, and *vertices* and immediately to verify the validity of the well known numerical relationship $V - S + F = 2$ . In more complex situations, instead, one encounters some difficulties. It is necessary to clarify, on the one hand, which are the figures in the space that can be considered "polyhedra", and on the other hand, which are the elements in the space that can be considered "faces" or "vertices" or "corners". The two questions are obviously connected and it is well known how much they are not banal, as is highlighted by the historic reconstruction of the attempts to demonstrate Euler's formula described in Lakatos' book (1976). When this activity is proposed to the teachers, it usually turns out to be evident how it is right for encouraging collaboration and the sharing of resources. It has to do, in fact, with an investigation that is not taken for granted, with an obligatory end, but rather open to further reflections, of a theoretical type or also an epistemological one (Pesci, 2007b).

## THEORETICAL FRAMEWORK FOR THE CHOICE OF TASKS

In this section there are the basic ideas which constitute the theoretical framework for the choice of the kinds of tasks described.

a) On the cooperative methodology to put into effect with the teachers, I have already described the theoretical references in the second section of this presentation. Here, I would add some reflections which could clarify better the features of the model proposed. It is important to remember that, in general, when one speaks of the shared principles of the models of social construction of knowledge, one has not yet arrived at outlining a standard didactical procedure, because for this it is necessary to choose the fundamental values which one intends to promote. As Ernest (1995) observes, standard didactical procedure is defined in each case on the basis of the *values* which one intends to promote. To define better the model of intervention experimented with mathematics teachers, I would like to stress that the fundamental *value* that I chose to promote is the *collaboration* amongst all the participants (teachers and didacticians) at the educational moment. The goal is that of more easily arriving together at a higher result than that which each one could reach alone, whatever the proposed task could be. The term *collaboration*, here, could be interpreted as a synonymous of *cooperation* in reference to the fact of sharing the urgency to develop, in a symmetric way, both the cognitive-disciplinary and the affective-relational competences of the subjects. But here the term *collaboration* has a more general meaning: a positive inter-relationship amongst the people involved, not necessarily connected to a specific modality of acting in groups. The *collaboration* amongst the participants (teachers and didacticians) has the following goals: to encourage the sharing of personal experiences, of resources, of difficulties, and to encourage reflection on the mathematical contents, on their epistemological meaning, on their classroom practice, and on their own professional history. In short, the *collaboration* with peers, interpreted at the level of teachers, seems the most efficient road for covering the role of teacher, which lies within the competence in projecting the educational path and the reflection-evaluation of the processes activated.

I would like to add one last characteristic of this model. The interaction between equals, in a climate of positive *collaboration*, implies a particular *setting*, that is the organization of time, space, and modes of interaction which allow the progressive evolution of the disciplinary and relational competences. All that is a privileged environment also for the well-being and for the mental health of the participants (Dozza, 2006). Trust in oneself, generosity in the welcoming and helping of the others and the recognition of oneself in the others, contribute to affirming and enriching one's own identity in the community to which one belongs, supporting the development of personal potentialities.

b) Autobiographical reflection, by means of the use of metaphors or narrations of meaningful episodes from one's life, turns out to be a preferred tool for accessing the deepest parts of self, allowing that decentralization which is necessary to be able to tell about oneself (Barker, 1987; Darrault-Harris & Klein, 1993). The narration of self was rediscovered in the last 10-15 years as an educational modality which is important for both students and teachers (the first direct references to the autobiographical practice in adults' education can be found in French studies, i.e. Pineau, 1983, the Italian studies have been developed mostly starting with Demetrio, 1996). Amongst the objectives that can be pursued, there is fundamentally the reflection on one's own experience, in particular, on its attributive implications and on the causal links to the events of one's history. This allows the recognition that the narration of oneself is not a simple report of events, but rather a reinterpretation of them, in the light of the present. Telling about self means giving meaning, coherence, and continuity to one's various experiences and also encourages the definition or the reformulation of one's identity. Autobiographical reflection, elaborated for oneself, but also communicated to and shared with others, encourages a positive development of interpersonal communication, the recognition and re-evaluation of personal facts and characteristics, the ability to listen to oneself and understand oneself, and a consequent openness to listening to and welcoming of others. So, it seems that autobiographical activity emerges as a fundamental tool in the work with teachers, a work which has at its centre the teachers in their totality, personal and professional at the same time.

c) The tasks of the disciplinary type proposed in the preceding section are, on the basis of the experiences carried out, particularly appropriate for developing epistemological reflection on mathematics in an *inquiry* style (Javorski, 2004), in a climate of investigation of mathematics which could be transferred to the class. With reference to this I would like to link to a question proposed by Watson and Mason (2007, p. 213).

> We question whether tasks need to be structured in ways which require ‘inquiry’ or whether instead ‘inquiry’ is the mindset with which teachers, and ultimately their students, need to approach all tasks.

I would say that both things are necessary. A task must be interesting enough to stimulate involvement and action. It must be open enough, that is, appropriate to being developable in several ways and therefore with personalized in-depth study. In other words, the task has to be *generative* of several different possibilities of development (as Borasi well described in the 21 examples showed in detail in her book, 1996). Besides, the structuring of the environment in which the task is proposed must be adequate, in the sense that it must foresee times, materials, and attitudes which can fully support the investigative activity. In other words, the *milieu* (Brousseau, 1997), in which a task and the following activity take place, has to be suitable for the intended work. It is still evident that also the attitudes of the participants in the investigation must be appropriate, that is, ready to participate in the activity, allowing themselves to be involved in the problem and putting into play their own time and their own resources. The two aspects (the characteristics of the task and the attitude of the one who confronts it) turn out to be, in my opinion, strongly intertwined and they influence each other in turn. A task which does not have the characteristics cited cannot give rise to *inquiry* and on the other hand an appropriate task, proposed in an unprepared *milieu* for the inquiry, will not be developed and unlikely will not become object of research.

d) The last observation that I would like to propose is relative to the general sense of a training experience proposed to the teachers, with the modalities and by means of the tasks described. As shown also by the analysis conducted by Watson and Mason (2007, p. 208):

> Tasks are often designed so that teachers can experience for themselves at their own level something of what their learners might experience and hence become more sensitive to their learners. The fundamental issue in working with teachers is to resonate with their experience so that they can imagine themselves ‘doing something’ in their own situation,

through having particularised a general strategy for themselves ... their professional choices of actions are the manifestation of what they have learned or are learning.

It is precisely in this direction that I develop each intervention on the teachers. I am convinced that a training meeting can be effective in the measure in which it can be set up, for the participants, as *metaphor* of experiences of living in class; a *metaphor* therefore understood not as verbal construction, but as life experience (Pesci, 2003, 2005, 2006, Fabbri & Munari, 2000).

## CONCLUSION

The model of intervention on teachers and the tasks here described put an explicit accent on the necessity to intertwine *disciplinary*, *methodological* and *relational* aspects for teachers' professional preparation, without leaving out a special care for structuring an adequate *setting* for the intervention itself. A theoretical frame for this complexity can not be simple and, of course, it could be different from that here described. It could be the occasion for further investigation and analyses, for instance in the direction: a) to formulate different models which could describe the same complex "scenario" of mathematics teachers' professional education; b) to elaborate specific and adequate instruments of analysis of teachers' interaction, at the different levels of competences involved by the model proposed. A final observation refers to the importance the model puts on the necessity to take account of teachers' personal biographies (their personal stories, their preferences, their expectations). I believe this is a feature not yet explored in depth for teacher education (see for instance the review about the common assumptions related to mathematical tasks in teacher education in Watson & Mason, 2007). Such orientation could be of interest for research, with possible fruitful resonance from perspectives of teachers' educators.

## REFERENCES


Barker, P. (1987). *L'uso della metafora in psicoterapia.* Roma: Astrolabio.

Bauersfeld, H. (1995). The structuring of the structures: Development and function of mathematizing as a social practice. In Steffe P. Leslie & J. Gale (Eds.), *Constructivism in* Education (pp. 137–158). Hillsdale, New Jersey: LEA.

Brousseau, G. (1997). *Theory of didactical situations in mathematics.* Dordrecht: Kluwer Academic Publishers.

Borasi, R. (1996). *Reconceiving mathematics instruction: A focus on errors*, Norwood, NJ: Ablex P. C.

Cohen, E. G. (1994). *Designing group work*, New York: Teachers College Columbia University.

Crawford, K. **(**1997). Distributed cognition, technology and chance. *Proceedings PME 23*, Vol. 2, 137-144.

Damasio, A. R. (1999). *Emozione e Coscienza*. Milano: Adelphi.

Darrault-Harris, I. & Klein, J. P. (1993). *Pour une psychiatrie de l'ellipse. Les aventures du sujet en création.* Paris: PUF.

Demetrio, D. (1996). *Raccontarsi. L'autobiografia come cura di sé.* Milano: Cortina.
Dozza, L. (2006). *Relazioni cooperative a scuola.* Trento: Erickson.
Ernest, P. (1995). The one and the many. In Steffe P. Leslie & J. Gale (Eds.), *Constructivism in* Education (pp. 459-486). Hillsdale, New Jersey: LEA.
Fabbri, D. & Munari, A. (2000). *I laboratori di epistemologia operativa, a cura di D. Demetrio*. Roma: Carocci.
Javorski, B. (2004). Grappling with complexity: co-learning in inquiry communities in mathematics teaching development, *Proceedings PME 28*, vol. 1, 17-32.
Jaworski, B. (2007). Tasks: a fitting end to an era, *Journal of Mathematics Teacher Education, 10,* 201-204.
Johnson, D. W., Johnson, R. T. & Holubec, E. J. (1994). *The nuts and bolts of cooperative learning*. Interaction Book Company
Lakatos, I. (1976). *Proofs and refutations. The logic of mathematical discovery.* CUP
Nunez, R. E. **(**1999). Embodied cognition as grounding for situatedness and context in mathematics education, *Educational Studies in Mathematics*, *39,* 45-65.
Pesci, A. (2003). Could metaphorical discourse be useful for analysing and transforming individuals' relationship with mathematics?, *Proc. 6th International Conference of the Mathematics Education into the 21st Century Project, The Decidable and the Undecidable in Mathematics*, 224-230.
Pesci, A. (2005). Mediation of metaphorical discourse in the reflection on one's own individual relationship with the taught discipline: an experience with mathematics teachers, *Proceedings of CERME 4 (2005)* (pp. 122-131). Sant Feliu de Guixols. Universitat Ramon Llull.
Pesci, A. (2006). Mathematics teaching: Toward a more human view. *Thinking Classroom, 7–1*, 20-25.
Pesci, A. **(**2007a). From studies of cooperative learning practices towards a model of intervention on mathematics teachers, *Proceedings of CERME 5 (2007)* (pp. 1945-1954). Lanarca: University of Cyprus.
Pesci, A. (2007b). Lakatos e la formula di Eulero, *L'insegnamento della matematica e delle scienze integrate*, *30(A-B 8*), 797-806.
Pineau, G. (1983). *Produire sa vie: autoformation et autobiographie,* Collection formation permanente. Paris: Editions Saint-Martin, Edilig.
Polanyi, M. (1958). *Personal knowledge*. Chicago: The University of Chicago Press.
Watson, A. & Mason, J. (2007). Taken-as-shared: a review of common assumption about tasks in teacher education, *Journal of Mathematics Teacher Education*, *10*, 205-215.

# THE LEARNING OF MATHEMATICS TEACHERS WORKING IN A PEER GROUP

**Martha Witterholt** and Martin Goedhart
*University of Groningen, the Netherlands*

*The research described in this paper is part of a study in which we will follow mathematics teachers during a certain period and describe the development of their practical knowledge. Teachers' practical knowledge is their knowledge and beliefs that underlie their actions. In this study we are focused on what teachers know and believe about learning and teaching statistical investigation skills. Concept maps and semi-structured interviews are used to represent and archive teachers' practical knowledge. In addition, a system of four categories is developed which, in our view, is appropriate for exploring mathematics teachers' practical knowledge. The results show that although changes in practical knowledge occur within a year, not all changes are due to working together in a peer group.*

## INTRODUCTION

Because of educational changes teachers should be able to learn permanently, individually as well as together with fellow teachers. This study reports on the learning of mathematics teachers from the same school, collaborating in a peer group for a longer period. The area of interest is the development of teachers' practical knowledge by collaborating in a peer group in order to achieve an educational design in statistics for students in lower secondary school. By creating an environment in which teachers can learn and develop, they have an opportunity to revise their practical knowledge by using each others expertise. The researcher guides the meetings, but the teachers are making the final decisions in order to create ownership. This kind of professional development is new to the teachers involved. During the peer group meetings, teachers are developing a research task for students which also will be implemented and evaluated. The research task is aimed at students doing statistical investigations about a theme of their own choice. Implementing research tasks is one of the goals of mathematics education in The Netherlands.

## THEORETICAL BACKGROUND

### Learning of experienced teachers in a peer group

A considerable amount of current research on teaching and teacher education focuses on teacher collaboration. Teacher collaboration is presumed to be a powerful learning environment for teachers' professional development (Meirink, Meijer & Verloop, 2007). However, empirical research about how teachers actually learn in collaborative settings is lacking. Learning in collaborative settings stimulates teachers to use the expertise of colleagues for improving their own teaching practice, and therefore adjust, enlarge or change their practical knowledge (Borko, Mayfield, Marion, Flexer & Cumbo, 1997). Borko et al. (1997) mention: "We believe that teachers would learn best by actively constructing new assessment ideas and practices based, in part, on their existing knowledge and beliefs, and sharing ownership of the workshop content and processes”. Furthermore, learning in a peer group is more intense when people with different ideas and opinions cooperate (Putnam & Borko, 2000). Verloop, Van Driel & Meijer (2001, p.453) mention that exploring teachers’ practical knowledge can be relevant in consideration of educational changes. In certain educational innovations teachers were only the executors instead of also the developers (see Van den Akker, 2003). To commit ownership in this study, teachers are developers and implementers of an educational design for learning and teaching statistics for students of the 7th grade of secondary school. Teachers afterwards evaluate the implementation of the design. Because they work together we expect an increased teacher learning, leading to more in-depth practical knowledge.

## Development of practical knowledge

The research presented in this paper is focused on the development of teachers’ educational goals and practical knowledge of mathematics teachers when they collaborate in a peer group. The term *knowledge* as well as the term *beliefs* may frequently be found in studies about teachers’ cognitions. The concepts that these terms refer to are often not easily distinguishable. On the other hand, to explore and analyse the learning of teachers, the term practical knowledge is frequently found in studies about teachers’cognitions (Kagan, 1990; Pajares, 1992) In most studies, only one term is used to refer to both knowledge and beliefs. Kagan (1990) states that: “Readers should note that I often use beliefs and knowledge interchangeably (…)”. Pajares (1992) also pretends that knowledge and beliefs are not distinguishable. He states that teachers’ beliefs are personal values, attitudes or ideologies and knowledge is a teacher’s more factual proposition, sometimes formal and sometimes practical. Meijer (1999, p.22) puts forward that: “Taken together, teachers’ knowledge and beliefs are a huge body of personal theories, values, fractional propositions, and so forth, that is to be found in teachers’ minds, and that teachers can, sometimes more easily than other times, call up and make explicit”. In this study, following Pajares (1992) and also Meijer (1999), teachers’ beliefs and teachers’ knowledge are viewed as inseparable. This will be referred to as teachers’ practical knowledge.

In this study we developed and used a system of four categories which, in our view, are the most appropriate for exploring mathematics teachers’ practical knowledge. Statements of teachers will be classified into the named categories. These categories are derived from the categories used by Meijer (1999, p.61) and Van Driel, Verloop & De Vos (1998). The categories will be described and explained below.

*1. Educational philosophy*

The category ‘Educational philosophy’ includes the vision of teachers on education in general, what motivates him or her to teach. Teacher’s educational philosophy can deviate from, for example, his actions in the classroom and does not need to correspond with reality. This category is an extension of the categories used by Meijer (1999). Meijer used the category ‘Student knowledge’, this are thoughts about students in general, which is part of the category ‘Educational philosophy’ in this study. Teachers’ educational philosophy is of great importance on his actions and thoughts.

Teachers’ former experiences in the classroom have a strong hold on their educational philosophy, just like experiences with professional development and consultation between fellow teachers (see Meijer, 1999). Ernest (1989) mentions that the mathematics teacher's mental contents or schemes includes the vision on mathematical knowledge, beliefs concerning mathematics and its teaching and learning. Ernest states that educational changes only can take place when teacher’s deep-rooted beliefs about mathematics and about the learning and teaching of mathematics will change. We expect to find particularly deep-rooted beliefs in this category, and therefore we expect the fewest changes in practical knowledge.

*2. Learning and teaching statistics*

This category includes teachers’ practical knowledge of  school mathematics, in particular of statistics. Within the scope of pedagogical content knowledge (PCK) also specific perception of statistics, learning difficulties and learning strategies of students within the domain of statistics are gathered in this category. Knowledge of teaching statistics is therefore also part of this category. This category is a combination of the categories ‘Subject matter knowledge’, ‘Curriculum knowledge’ and ‘Knowledge of student learning and understanding’ in the research project of Meijer (1999).

Next to practical knowledge, teachers need understanding of the subject matter content to teach a subject (Sowder, 2007). Shulman (1986, p.25) mentioned: “Where the teacher cognition program has clearly fallen short is in the elucidation of teachers’ cognitive understanding of the subject matter content (..)”. He thereby introduced the term pedagogical content knowledge (PCK). Verloop et al. (2001, p.449) indicated that PCK can be considered as a specific form of teachers’ knowledge due to the focus on students and on subject matter. The category ‘Learning and teaching statistics’ is strongly related to teachers’ working together in a peer group on the educational design and its implementation in the classroom. The teachers in this study are not used to working in a peer group. We therefore expect important changes in this category.

*3. Student activities*

This category describes teachers’ practical knowledge about students in the first class of secondary school and students in general, their activities during the lessons of this course and their learning activities. A direct relation with the subject matter (statistics) is not necessary. This category is an extension of the category ‘Knowledge of purposes' used by Meijer (1999).

Together with the category ‘Learning and teaching statistics’, this category is expected to be strongly influenced by teachers’ collaboration in a peer group. We expect a connection between the objectives of the design formulated by the teachers, how important teachers think research tasks are in math classes and the student activities during the course.

*4. Teacher activities*

On the one hand this category describes teachers’ practical knowledge of the use of materials during the math classes and the practical knowledge of statistical research assignments. On the other hand this category contains teachers’ practical knowledge of designing, implementing and evaluating lessons in statistics and teachers’ role during the implementation. This category is a combination of the categories ‘Curriculum knowledge’ and ‘Knowledge of instructional techniques’ by Meijer (1999).

**Research questions**

The main question presented in this paper is: How does the practical knowledge of mathematics teachers develop as a consequence of designing, implementing and evaluating an educational design (altogether this is called the intervention) for learning statistical investigation skills by working in a peer group?

The main question can be determined by answering three basic subquestions:

1. What is the practical knowledge of the participating teachers prior to and after the intervention?
2. What are the changes in practical knowledge of the participating teachers during the intervention?
3. Which are possible causes of changes in practical knowledge?

## METHODOLOGY

In this study four mathematics teachers of the same school are collaborating in a peer group. During the seven peer group meetings they are developing an educational design in statistics for students in lower secondary school. After the implementation of the design, the last peer group meeting serves to evaluate the design in order to improve the content.

In the study presented in this paper, we use two of the three instruments Meijer (1999) used, completed with three other instruments. The instruments below were used in this study and are at the same time provided with an explanation:

1. *A questionnaire about teacher background variables*

   Just like Meijer, Verloop & Beijaard (1999) we use a list with questions about the teacher’s background. There are patterns that indicate that it is of crucial importance how a teacher deals with his or her experience, training, and consultation with colleagues.

2. Two *concept maps* by each teacher referring to the teaching and learning of investigative skills: one concept map was drawn before the intervention (this is called CM[0]). The other concept map was drawn afterwards (CM[1]). *Explanations* by the teachers about their concept maps, directly after the drawing of the concept maps. The explanations of the teachers are all recorded on tape and are used as an additional source of information to the concept map.
3. *Semi-structured interviews*. Like the concept maps we had two interviews: one before (Int[0]) and one after the intervention (Int[1]). The interviews were hold immediately after the explanation of the concept map, in one session.
4. *Registrations* and *evaluations* of all seven peer group meetings. All peer group meetings are recorded on a voice recorder and evaluated through written evaluation forms filled in by each teacher.
5. *Observations* of the lessons taught within the project. All the nine lessons of all the teachers were observed and recorded on videotape.

The first source of information gives an idea of teacher's experiences with teaching investigative skills during the past years. This will be used for an explanation of the teacher's development. The next two sources of information will be used to determine changes in practical knowledge of teachers. The fourth source of information serves to find causes for the observed changes or to indicate professional development. The fifth source of information serves as a validation-check and is meant to see if teachers 'teach as they preach'.

The combining and analyzing of data from the different sources of information was a procedure with six phases (Morine-Dershimer, 1993; Meirink et al., 2007). In this paper not all the phases will be described, only phase four, where we look at the similarities and the changes in practical knowledge by first comparing CM[0] with CM[1] and Int[0] with Int[1] and after that divide teachers' statements and answers over the named categories. To describe possible changes in practical knowledge and to find out what causes these changes, we use two interesting cases. The first case is a less experienced teacher, Ann, and the second case is an experienced teacher, Bart. The names of the teachers mentioned here are fictitious.

## RESULTS

### Case Ann

### Teacher background variables

Female, 48 years old, ten years of experience in adult education and three years of experience in grades 7-10 of secondary school. Little experience with implementation of research tasks.

## Changes in practical knowledge

Below, in table 1, a list of differences in pre- en post-concept maps and in pre- en post-interviews from Ann is presented. The differences are divided over categories and the instrument concerned is also specified in table 1. There is also a list of similarities, but this list will not be given here. We will focus on the differences, because the differences are more interesting.

Table 1: *Differences in pre- en post-concept maps and in pre- en post-interviews from Ann in categories*

| Category | Differences |
|---|---|
| Educational philosophy | 1. These students are too young to state a hypothesis (from CM[1]).<br>2. “How did I learn it myself?” (from CM[0]). |
| Learning and teaching statistics | 1. The introduction assignment was not applicable, there was no relationship between variables (from Int[1])<br>2. Nowadays you need a computer for presenting and processing data. (from CM[0])<br>3. Statistical concepts should come up for discussion during the introduction (from Int[1]).<br>4. Evaluating the process with students is important (from CM[1]).<br>5. Implementation of statistical research requires a systematic routine (from CM[0]). |
| Student activities | 1. Some children could not work together at all (from CM[1]).<br>2. Students can ask each other critical questions about their posters (from CM[1]). |
| Teacher activities | 1. The role of the teacher is to guide the students (from CM[0]) |

Looking at the differences in table 1 it is obvious that the differences in the category ‘Learning and teaching statistics’ are dominantly present. This is partly a consequence of the used methods. The focus question of the concept maps is ‘Learning and teaching statistics’ and the interviews are also focused on the learning and teaching of statistics. Furthermore, the differences are mainly caused by Ann's basic assumption. Before the implementation of the educational design, in CM[0], she noticed “to be blank”. Afterwards, in CM[1], she changed her basic assumption and noticed that the implementation of the design was the most important. Ann’s teaching experiences in the past play an important role, enforced by experiences during the implementation of the educational design. However, Ann’s research experiences do not play an important role anymore, though this was often a success (see CM[0]). During the evaluative peer group meeting it becomes clear that Ann still is enthusiastic about the educational design, although she proposed a few revisions like more interest in students working together and adjust the introduction assignments. Ann composed the student groups herself. She mentioned that she would do that again, because she is convinced that students have learned a lot by this way of working. Observations of lessons show that Ann is a good coach. She encourages her students to reflect on choices made and she is able to revise her goals if necessary. Repeatedly, she succeeds in creating a good atmosphere, in which students are able to work undisturbed.

**Case Bart**

**Teacher background variables**

Male, 47 years old, eighteen years of experience in teaching in secondary schools. In the past, he implemented two small research tasks, of which one was a statistical task.

**Changes in practical knowledge**

Table 2 below shows a list of differences in pre- en post-concept maps and in pre- en post-interviews with Bart.

Table 2: *Differences in pre- en post-concept maps and in pre- en post-interviews from Bart in categories*

| Categories | Differences |
|---|---|
| Educational philosophy | 1. “Students understanding of the subject matter is very important. I didn’t mention that because I haven’t the impression that they really understood what they were doing” (from CM[1]).<br>2. “In any case, in my view students must have learned enough. There has to be a sufficient amount of data, the result has to be satisfactory and the teamwork should be good”(from Int[1]).<br>3. The factor time is important: “How labour-intensive is it?” (from Int[1]). |
| Learning and teaching statistics | 1. Strenghten that which is in the newspaper and on tv. Bart mentions: “That did go wrong. I couldn’t make that clear either” (from CM[1]).<br>2. In CM[1] Bart is focused on students: “You now know what it was. You do not know that in advance. I automatically focus on the students. That is correct. intended or unintended” (from CM[1]).<br>3. Statistics in the observation period is not really hard: “We use the chapter Statistics to catch up in time” (from CM[0]). |
| Student activities | |
| Teacher activities | 1. “I found the teaching part rather awkward. In fact, I had no time left because of the method we used. Perhaps therefore I skipped it unintended” (from CM[1]). |

Looking at the differences in table 2 it is obvious that the amount of differences in the category ‘Educational Philosophy’ and the category ‘Learning and teaching statistics’ are the same. It is remarkable that there are no differences in the category ‘Student activities’, while Bart is focused on the students during the construction of CM[1]. During the construction of CM[0] he also focuses on the teacher by adding the term ‘teaching'.

The differences are mainly caused by the experiences of Bart preliminary to the implementation of the educational design. Bart is skeptic about students working in a team, because he experienced teamwork as unsatisfying. He thinks the lessons are more chaotic and that he looses control. However, lesson observations give another impression. Bart’s lessons are well prepared with clear explanations and a great deal of structure. From the explanation of CM[1] and during the evaluative peer group meeting, it appeared that Bart doubts whether students learnt the statistical concepts sufficiently and if it would be better to use a more didactic teaching method. At least, that will save him a lot of time. It is remarkable that, although Bart does not believe in teamwork, he once again would choose for students working in teams. Next time, he will choose smaller groups (two students) and let students compose the groups themselves.

## CONCLUSIONS

To get an accurate insight into teachers' practical knowledge and its changes, the construction of concept maps combined with the semi-structured interviews give important information. The classification used here gives a structural description of the practical knowledge of Ann and Bart. It turns out that this knowledge of both Ann and Bart is deep-rooted; it is derived from former experiences and confirmed by implementing the educational design (see Ernest, 1989). The category 'Learning and teaching statistics' embodies the most similarities in practical knowledge, but also the most differences. The practical knowledge in the category 'Learning and teaching statistics' depends highly on the experiences perceived during the intervention. Besides, the changes in this category are probably due to the experimental design. Even though he had a less positive experience before the implementation of the design, Bart's ideas about teamwork do not change. He maintains his opinion that direct instruction is more effective than teamwork. On the other hand, Ann could adjust the goals easily during the lessons. She was more flexible and she showed more persistence during the selected trajectory (see Pajares, 1997). Both Ann and Bart, however, were willing to make concessions during the peer group meetings. They experienced the interest of combining each other's ideas and constructing an educational design to which everybody could commit.

In a follow-up study it would be interesting to look at the different roles teachers play in peer group meetings. Ann, for example, appeared to be a leader, highly committed and motivated. Bart appeared to be a follower, trusting the ideas of Ann (Shamir, 1991). We also need to look more closely at the categories involved in this study. It is difficult to categorise teachers' statements. Furthermore we may need to use sub-categories or rename existing categories.

## REFERENCES

Akker, J. van den (2003), Curriculum perspectives: An introduction. In J. van den Akker, W. Kuiper & U. Hameyer (Eds.). *Curriculum landscapes and trends*, pp. 1-10. Dordrecht Kluwer.

Borko, H., Mayfield, V., Marion, S., Flexer, R. & Cumbo, K. (1997), Teachers' developing ideas and practices about mathematics performance assessment: successes, stumbling blocks, and implications for professional development, *Teaching and Teacher Education*, *13*, 259-278.

Driel, J. H. Van, Verloop, N. & Vos. W. De. (1998), Developing science teachers' pedagogical content knowledge. *Journal of Research in Science Teaching*, *35*(6), 673-695.

Ernest, P. (1989), The impact of beliefs on the teaching of mathematics. In P. Ernest (Eds.), *Mathematics teaching: the state of art*, pp. 249-254. London: Falmer Press.

Kagan, D. M. (1990). Ways of evaluating teacher cognition: Interferences concerning the Goldilocks Principle. *Review of Educational Research, 60*(3), 419-469.

Meijer, P. (1999). *Teachers practical knowledge*. Universitair proefschrift, Universiteit van Leiden.

Meijer, P. C., Verloop, N. & Beijaard, D. (1999), Exploring language teachers' practical knowledge about teaching reading comprehension. *Teaching and Teacher Education, 15*(1), 59-84.

Meirink, J. A., Meijer, P. C. & Verloop, N. (2007), A closer look at teachers' individual learning in collaborative settings. *Teachers and Teaching: theory and practice, 13*(2), 145-164.

Morine-Dershimer, G. (1993). Tracing conceptual change in preservice teachers. *Teaching and Teacher Education, 9*(1). 15-26.

Pajares, M. F. (1992). Teachers' beliefs and educational research. Cleaning up a messy construct. *Review of Educational Research*, *62*, 307–332.

Putnam, R. T. & Borko, H. (2000), What do new views of knowledge and thinking have to say about research on teacher learning?, *Educational Researcher*, *29*(1), 4-15.

Shamir, B. (1991). The charismatic relationship: Alternatice explanations and predictions. *Leadership Quarterly, 2*(2), 81-104 .

Shulman, L. S. (1986). Paradigms and research programs for the study of teaching. In M. C. Wittrock (Ed.), *Handbook of research on teaching*, pp. 3-36. New York: Macmillan.

Sowder, J. T. (2007). The Mathematical Education and Development of Teachers. *Second Handbook of Research on Mathematics Teaching and Learning*, p. 157-223. Frank K. Lester (Eds.). NCTM, 2007.

Verloop, N., Driel, J. Van. & Meijer, P. (2001). Teacher knowledge and the knowledge base of teaching. *International Journal of Educational Research*, *35*(5), 441-461.

# USE OF FOCUS GROUP INTERVIEWS IN MATHEMATICS EDUCATIONAL RESEARCH

Bodil Kleve

Oslo University College

*In my doctoral work I studied three mathematics teachers in lower secondary school in Norway and how they interpreted a curriculum reform, L97 (Hagness & Veiteberg, 1999). This study included methods as focus group interviews and individual interviews with teachers, teachers' self estimations and classroom observations (Kleve, 2007). In this paper I discuss how I used focus group interviews both for the purpose of obtaining information from teachers about their mathematics teaching, about their beliefs about teaching and learning mathematics and also for the purpose of validating the whole research and its findings.*

Keywords: Mathematics, Ethnography, Beliefs, Focus groups, Curriculum reform

## RESEARCH METHODS FITTING INTO AN ETHNOGRAPHIC APPROACH

If one wants to find out something, one "goes out and has a look" (Pring, 2000, p. 33). In my research I wanted to find out how teachers interpreted the curriculum and how they implemented it in their classrooms. I therefore decided to enter the mathematics classrooms to investigate teachers' practice, and to have focus group interviews with the teachers to find out what they said about L97, their own teaching practice and about mathematics teaching and learning.

I conducted an empirical study using research methods fitting largely into an ethnographic style of inquiry. The study was a case study of mathematics teachers' interpretation of the curriculum reform L97, both in terms of what they said about it and in terms of their classroom practice. Focus of the study was how teachers' practices were related to their beliefs about teaching and learning mathematics.

I chose methods of data gathering in line with methods suggested in the literature to carry out research with an ethnographic approach (Bryman, 2001; Eisenhart, 1988; Walford, 2001; Wellington, 2000). I used focus group interviews, individual interviews with the teachers, classroom observations, estimation form and teachers' own writings about ideal teaching. All these research methods provided me with data to analyse with regard to teachers' teaching practice and their beliefs about teaching and learning mathematics. Use of focus group interviews which this paper is about, was thus one of several research methods I used in addressing teachers' beliefs.

I used focus groups both for the purpose of selecting teachers for my study and as a research method. I contacted the school leader of a community outside Oslo. The teachers who participated in the first meeting were selected by her. None of these teachers became part of my further study. The next two focus groups were conducted with teachers from three different schools in another community. They were selected by their headmasters whom I had contacted. Four of these teachers became part of the whole study and participated in the fourth focus group meeting which took place after the classroom observations. The process by which the teachers for my study were selected is beyond the scope of this paper. However, it is outlined in Kleve (2007).

Focus groups contain elements of two research methods: it is a *group interview* and the *interview is focused.* The members of a focus group are invited because they are known to have experience from a particular situation which in this case was teaching mathematics. A focused interview is to ask open questions about a specific situation (Bryman, 2001).

According to Krueger (1994) focus group interviews are useful in obtaining information which is difficult or impossible to obtain by using other methods. Using focus groups generally means that the researcher can intervene into the conversation and pose questions to probe what somebody just has said. According to Bryman (2001) the use of focus groups has not only a potential advantage when a jointly constructed meaning between the members of the group is of particular interest. Participants' perspectives are revealed in different ways in focus groups than in individual interviews, for example through discussion and participants' questions and arguments. However, Bryman pointed out possible problems of group effects in a focus group situation that must not be ignored. I experienced such group effects and I realise the importance of treating group interaction as an issue when analysing data from the focus groups.

## TEACHERS' BELIEFS ABOUT MATHEMATICS TEACHING

In my study I use the term belief, and I look upon teachers' beliefs about teaching and learning mathematics and about L97 as cognitive constructions highly influenced by socio-cultural factors such as teacher's own experience and the school context, and also influenced by the teacher's knowledge in mathematics and about mathematics teaching. The insight I can get in my research into teachers' beliefs is through what the teachers say and write and through my interpretations of what I have observed in their classrooms. I do not look upon beliefs as something that can be directly observed. Through the use of different theoretical lenses, my conceptions about teachers' beliefs have to be inferred from what they say about what they are doing in the classroom; what they say they think about their practice; what they say they think is good mathematics teaching and what they say about L97.

It has been important for me both to study teachers' beliefs about teaching and learning mathematics and also what I observed them doing in their classrooms. Thompson (1992) wrote that in order to understand teachers' teaching practices from the teachers' own perspective, understanding teachers' beliefs with which they understand their own work is important. I do not see a teacher's beliefs and his/her practice as a cause-effect issue, but rather as a reflexive process. A teacher's beliefs are influenced by his/her practice and the interactions in the classroom are again influenced by the teacher's beliefs. A teacher's practice can both act as a reinforcement of his/her beliefs but also as an incitement for change.

One component of the teacher's interpretation of the curriculum is what s/he did in the classroom, the enacted curriculum (which is also influenced by incidents in the classroom, students' interactions, behaviour, and so on). The other component is what the teacher said in focus groups and in conversations, what s/he wrote and his/her responses to an estimation form. It was the relation between these two components I studied. It is the latter I term teachers' beliefs.

## ANALYSING DATA FROM FOCUS GROUPS

A challenge in using focus groups was to what extent I was able to interpret the meanings lying behind and looking through the words the participants were saying and from that make inference about the teachers' interpretation of the curriculum. In analysing the data from my focus groups it was important for me to be aware of the different levels of information the data give. On one level teachers speak from their inner thoughts and meanings, struggling to express what are really inside their heads, they speak from their individual constructions they have perceived viable in their own practice. On another level they speak from what they know as a teacher and what they say is deeply embedded in social practices of being a teacher, and thus socio-culturally rooted. A third level can be rhetoric: The teacher knew who I was, and could try either to express what s/he was thinking I wanted to hear or since s/he knew what the curriculum said, s/he could express that or s/he could challenge that. In such cases the teachers would respond to me and who I am rather than to who they are. When analysing what teachers said in focus groups it is important to be aware that the teachers' views were revealed in different ways than in individual conversations. What they said could be a way of positioning themselves rather than trying to express their inner thoughts. Information revealed that way illuminates other aspects of teachers' beliefs than aspects illuminated through use of other research methods. Krueger & Casey (2000) encourage use of questions leading persons to speak from experience rather than wishes for or what might be done in the future. That increases the reliability since it focuses on particular experience from the past.

What the teachers said in focus groups conducted before classroom observations was not influenced by my presence in their classrooms and individual interviews. In that respect data from these focus groups provided me with information about teachers' beliefs and practice which went beyond what was obtained through the other research methods. On the other hand, data from the focus group meetings were also valuable for the purpose of triangulation and supporting the other sources of data from the teachers' utterances (individual interviews, self-estimation, writings and questionnaire). I audio recorded and transcribed the discussions that took place in these groups. Below I present some findings from these meetings which highlighted issues from perspectives of L97.

## FOCUS GROUPS AND TEACHERS' BELIEFS

I will now present an analysis of the third focus group (FG3), which was conducted before I started the classroom observations.

The teachers participating in this focus group were the four teachers in my study: Alfred, Bent, Cecilie and David. In addition Petter, Kari and Tom, one of my former students, participated. For this focus group I had prepared the following questions for discussion:

- What in your opinion is important competence for mathematics teachers?
- In what way do you relate your work to L97?
- Has L97 inspired you to try out new activities in your mathematics teaching?
- What is the greatest challenge in your work as a mathematics teacher?
    - What have you succeeded with?
    - What do you think you have not yet accomplished?

I started with the first question explicitly, and aspects of other questions were addressed as part of the discussion. However, there was no time to discuss the last two parts of the fourth question. What the teachers felt they had succeeded with and what they found they had not accomplished, were issues explicitly discussed in the fourth focus group meeting later in my study. An analysis of this focus group interview (FG 4) is presented in the final part of this paper.

**Focus groups from a socio-cultural perspective**

How does what participants say reflect meanings of the group or society more widely? How does what they say reflect aspects (including criticism) of the political and cultural society, of dominant groups influencing the official educational discourse (Lerman, 2000), of their own school situation as a teacher or the one they had as a student themselves? Or how does what they say reflect aspects of the curriculum?

To illustrate this I will provide an example from FG3 which shows use of rhetoric. David knew who I was; he knew I was a teacher educator; he knew I had carried out courses for teachers in relation with the curriculum reform. Therefore, I conjecture David thought I wanted to hear nice things about the curriculum. Based on his understanding of what L97 said, he challenged it. This could have been because he wanted to position himself within the group, but it could also have been because he really meant that L97 is not a good curriculum for the mathematics subject. Yet another way to interpret what he said and why can be that he did not really know what the curriculum was saying, and he wanted to react reluctantly to it from the very beginning. In the quotation below, Petter (P) indicated he was sceptical to L97. David (D) then said (sarcastically?): “there are some nice pictures in it”. That illustrated how teachers argued for or against a new curriculum, how they interpreted it. The language (also what was not said) was a mediating tool in the exchanges of meanings. Petter was the most experienced teacher in the group and had a special role here. He indicated something to which David responded and it illustrates how what they said was deeply embedded in the socio-cultural setting in the group and their experience. (I is me)

I: L97, how well do you know it? P, you seem dying to say something…

P: Yes, I feel I am getting hot-headed when you mention L97.

D: There are some nice pictures in it (sarcastic?)

I: Now we have talked very much about how L97 is weighting the mathematical topics. But what about the working methods it initiates? Do you have any opinions about that?

D: Read the newspaper, many interesting writings about it there.

[There had been written many critical articles in the newspaper about L97 recent days]

I: But what do you mean?

D: I am critical to the correct pedagogical view we are served from above. I am not sure if it is right.

I: Can you say some more about it?

D: I believe that maybe pupils learn most if they have a teacher, who knows their things, is enthusiastic, finds teaching being fun, who is a good motivator, and good in making the pupils function together. I really believe that the learning outcome becomes better then than if the students have lessons outdoor, working schedules and so on. I dare being that old fashioned, I think so.

P: One must be allowed to disagree with L97? Or?

D: Disagree, and say it over and over again, everywhere you are

I: I want to know what your disagreement is about. What is the pedagogical view coming from above?

D: I think it implies knowledge's loss of flavour. Projects where pupils find something on the internet print it out and read it with a few replacements of words in front of the whole class.

I: Is that what L97 says?

D: No, but that is what happens.

My experience with Petter and David, and to a certain degree also Alfred (he was not so outspoken as the other two) in this focus group was that they were supporting each other with regard to a kind of ignorance towards L97. They had been teaching mathematics for many years, and they expressed their frustration of how the "old" kind of mathematics, especially algebra, was not in the curriculum any more to the extent they wished. Their mutual support in these views expressed in the focus group can be looked upon as communication of a rhetorical kind.

Next I will provide an example of how what teachers said in the focus groups reflected aspects of their experience as a teacher. Reflecting on the utterance from Bent below, he talked from a socio-culturally related everyday experience. Bent offered us something about the way he operated in the classroom. He spoke from his experience as a teacher, and what he had learned from this experience. From the quotation below it may be hard to understand what he meant, which demonstrates his struggle to express his experience. He said that teaching from the board could start off from a simple level. However, very soon what was presented from the board became too difficult for some students whereas others wanted to proceed even further. This illustrates the challenge of having students with different abilities in the same class. He said:

> I think a typical course, when you shall start with a new topic, is to teach from the board in the beginning and to start with something simple and then build it up to a certain level, and to work on tasks parallel to that. At a certain level you just have to stop the lecturing and separate. Some disappear far up and some remain on that level if they have at all reached the level they should. After that it is almost impossible to deal with teaching.

Below I will discuss how Bent went beyond his experience and offered us some of his reflections on his teaching.

## Aspects of teachers' confidence

When studying the transcripts, which I had imported into NVivo, I noticed how the teachers expressed differing degrees of confidence throughout the discussion. Bent suggested the ability to motivate the students, and the importance of having mathematical knowledge to get an overview of the subject oneself, as competencies for a mathematics teacher. He used the expression "I am trying to …" when relating these competencies to his own practice: "I am trying to relate to practical issues, trying to make a relation to real life in a way, however I don't always manage". He was "trying to" make the students see the relevance in what they worked with; he was "trying to" convey the mathematics' intrinsic value, especially when it was not so easy to relate the mathematics to students' everyday life. He also said that he was trying to be enthusiastic. His use of words when speaking from his classroom practice revealed that he was not sure if he succeeded in doing what he thought was important, but he was trying. Continuing the quotation from Bent above, he went beyond his everyday experience in saying something about the issues that arose for him when he operated in certain ways, and his thoughts about it. Bent also revealed some of the "weaknesses" he perceived in himself as a teacher. He had tried out something but through what he said he demonstrated awareness that this might not have been the right thing.

> Then you have to walk around giving tasks. Last year I optimistically tried MUST tasks, OUGHT tasks and MAY tasks, that they should try to stretch themselves, but I didn't succeed in making it work. It turned out to be that they did what they had to (MUST) (agreement in the focus group), and some just tried OUGHT. But if they had homework in other subjects, they chose the less challenging way. So then it was easier to do as P says, give many tasks and rather reduce for those who need it. It is easier to put pressure on those who need challenges.

By saying this Bent also demonstrated that he had reflected on his own practice as a teacher. Being able to put his weaknesses as a teacher on the spot like this and sharing it with me and the other teachers in the group, I do not interpret as lack of self confidence but rather as reflecting a teacher who had faith in himself and had self confidence enough to be able to see his own teaching from more than one point of view. He had been able to step aside to consider his own teaching.

Bent also offered us his reflections on different levels of students' learning of mathematics, in which the other teachers consented, but without any further discussion. Bent said: "I have a feeling that they learn on different levels". He said that on one level they learn to solve a problem theoretically and perhaps manage to solve a similar problem in a same kind of context: "you have learned it in one setting on one level". He said:

> The next level is being able to carry out what you have learned theoretically for example about symmetries, and applying that when searching for and finding symmetrical patterns in a carpet: Going out looking in math-morning [which was the project work he talked about], having to apply it, then you learn and experience on a higher level.

He called this an "application competence". On yet another level you learn by expressing a problem orally. He said: "Formulating a problem for others is yet one level of learning".

When Tom said he felt that he did not know how to make students understand, especially those with "two"[1] in mathematics, David responded:

> I believe you'll have to live with that as a teacher. It is classical. You can work with some students throughout three years and they do not see /understand /remember the difference between $2x+2x$ and $2x \cdot 2x$. Even if you stand on your head and invent all possible variations you can think about there will still be some I believe [who will never manage], regardless of how clever you are as a teacher.

By saying this David demonstrated confidence as an experienced teacher. He spoke from his own experience as a teacher, an experience he knew that Tom did not have. This utterance also reflects a view that not all mathematics is for everybody, and that you cannot put the responsibility for this (the "two-students" not understanding or remembering) on the teacher. Through his long experience as a teacher, David had learned to accept this and he was now telling that to Tom who was a younger and less experienced teacher.

Cecilie also demonstrated self-confidence when telling about how she was handling the issue that students with different abilities in mathematics were placed in the same class. She had mixed two classes and grouped them according to interest in mathematics. She expressed her disagreement with Tom who had said that clever

[1] He referred to getting the grade (mark) 2 in mathematics which is the lowest passing grade. 6 is the best grade.

students will always manage, and she recommended the other teachers to group the students according to abilities (“interests”) the way she was doing.

The above discussion about aspects of teachers’ confidence demonstrates how such information can be obtained through the use of focus groups. The way in which teachers expressed their confidence in own teaching practices highlighted issues of their teaching practices and informed my investigation of how they responded to a curriculum reform.

### Mathematical focus

To highlight issues of my study of teachers’ mathematics teaching, it was useful to study what aspects of mathematics they talked about in the focus group. One significant aspect throughout the conversation in the focus group was that *algebra* was the mathematical focus teachers mentioned most frequently when expressing their meanings and exemplifying from their teaching. David referred to algebra several times and was very concerned about algebra having been toned down in the new curriculum and said that he put more weight on algebra, equations and functions than L97 suggests. He also said that he would keep doing it because some students would need it for further studies. David said he was not so eager to force all work within mathematics into an everyday context: “I am more concerned that mathematics is a ‘logical and playing subject’. When the students have done a huge algebra task and say ‘YES I have managed’, that makes me happy”.

Bent also referred to algebra when expressing the importance of the mathematics’ intrinsic value. He expressed the value in itself of having the knowledge to solve an algebraic task or equation. Furthermore, Bent talked about having carried out a project work in mathematics which had been very successful. L97 encourages interdisciplinary project work and also project work within each subject. It was one of the latter in mathematics Bent referred to.

Cecilie mentioned algebra together with mathematics history as exciting topics to work with in her teaching of mathematics.

With regard to my study, what the teachers said in this focus group and how they said it gave me information about how the teachers responded to L97 in terms of what they were saying about it and what they were saying about their own classroom practice. The focus groups highlighted key issues and gave me a starting point for working with each of the teachers, Alfred, Bent, Cecilie and David, who became part of my further study.

## FOCUS GROUPS FOR THE PURPOSE OF VALIDATING THE RESEARCH

The last focus group I had with the teachers who had been part of my study took place towards the end of my work with them. I have chosen to comment briefly on my findings from Focus group 4 for the purpose of cross case-analysis and also to illuminate and validate my findings from the rest of my study with the teachers.

I had asked the teachers to prepare two issues to share with the group; first, one issue they felt they had succeeded in carrying out as a mathematics teacher and one issue they felt they not yet had accomplished. They found the task difficult. However, after a few minutes discussing and reflecting on the difficulty of the task, Cecilie volunteered to start with hers. She felt she had succeeded in challenging and motivating the clever students, which is in accordance with what she had expressed in our conversations. The task she felt she had not yet accomplished was enabling the students to copy out their written work in mathematics clearly. Bent responded by expressing that more important for the students than the written presentation of mathematics is for them to understand when to multiply and when to divide in working it out. This emphasises Bent's focus on students' conceptual understanding which I also found through my work with him in the classroom and in our conversations.

Bent chose to present issues from two of the lessons I had been observing with regard to what he felt he had succeeded in and what he not yet had accomplished. His presentation of the issues revealed that he had been reflecting on these lessons. About the fraction lesson he said that he felt he had succeeded to a certain extent. However, he could have done more with it. With regard to the use of concrete materials, he expressed a disappointment that the effect had not been as intended. It had however been better in the other 9th grade class he was teaching. He thus expressed a feeling of having succeeded with the use of concrete materials in that class (in which I did not observe). This suggests that the complexity of the classroom and the classroom discourse often influence the outcome of an activity, and thus the enacted curriculum which is jointly constructed by the teacher and the students and the materials used.

Presenting what he felt he had been successful with, David said: "I have managed to make them cleverer in doing percentage calculations". This emphasises how he looked upon himself as conveying mathematics to the students and that students' learning is dependent on the teacher's ability to explain. When he was asked by the others in the group how he had done it he said: "It is just to explain as well as possible". This emphasises further how he looked upon explaining as the most "efficient" teaching strategy, which also characterised his teaching. However, he also offered an elaboration of how he had done it which revealed that he as a teacher was consciously systematic when presenting mathematics for his students. He said:

> I have been very systematic with percentage types 1, 2, 3, 4, 5. Therefore, when one of the types turns up, I refer to the type. Number 1 is like "3 students absent how many percent?" Then it is in connections with changes, then having to calculate backwards, and then comparing two numbers.

David's systematic way of preparing the mathematics to be taught was a feature in his teaching.

With regard to what he had not yet accomplished, David focused on kinds of errors students made, especially how they used the equal sign wrongly, and he also supported Cecilie in her suggestion: how to enable students to copy out mathematics in a lucid written way which clearly showed how they had solved the task.

What was said in this last focus group emphasises my findings from the analysis of the individual teachers: Cecilie felt she was successful in her work with the clever students, but had difficulties enabling students to present written mathematics with a clear overview; Bent reflected upon both success and not-yet-accomplished aspects of the issues presented; and David felt success in explaining and had not yet found out how students could avoid making errors. For detailed portraits of the three teachers see Kleve (2007).

This last meeting provided me also with information beyond what I had observed in the classroom, and what I had talked with the teachers about in the conversations. Bent offered his reflections around his work with fractions and use of concrete materials. Cecilie shared her difficulties with enabling students copying out their written work clearly, in which David supported her. By challenging David about *what* he had done to make students become good in percentage calculations we were initiated into a systematic way of preparing his teaching. This demonstrates that the use of focus groups provide researchers with information beyond what can be obtained otherwise.

## REFERENCES

Bryman, A. (2001). *Social research methods*. Oxford: Oxford University Press.

Eisenhart, M. (1988). The Ethnographic Research Tradition and Mathematics Education Research. *Journal for Research in Mathematics Education*, *19*(2), 99-114.

Hagness, R., & Veiteberg, J. (1999). *The Curriculum for the 10-year compulsory school in Norway*. [Oslo]: Nasjonalt læremiddelsenter, Kirke- utdannings- og forskningsdepartementet, National Centre for Educational Resources.

Kleve, B. (2007). *Mathematics Teachers' Interpretation of the Curriculum Reform, L97, in Norway*. Doctoral Thesis, Agder University College, Kristiansand.

Krueger, R. A. (1994). *Focus groups: a practical guide for applied research* (2nd ed.). Thousand Oaks, Calif.: Sage.

Krueger, R. A., & Casey, M. A. (2000). *Focus groups: a practical guide for applied research* (3rd ed.). Thousand Oaks, Calif.: Sage.

Lerman, S. (2000). Some Problems of Socio-Cultural research in mathematics teaching and learning. *NOMAD, 8*(3), 55-71.

Pring, R. (2000). *Philosophy of educational research*. London: Continuum.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics learning and teaching* (pp. 127-146). New York: Macmillan.

Walford, G. (2001). *Doing qualitative educational research: a personal guide to the research process*. London: Continuum.

Wellington, J. J. (2000). *Educational research contemporary issues and practical approaches*. London: Continuum.

# ANALYSIS OF INTERACTIONS IN A COLLABORATIVE CONTEXT OF PROFESSIONAL DEVELOPMENT

**M. C. Muñoz-Catalán, J. Carrillo & N. Climent**

*University of Huelva, Spain*

maria.cinta@ddcc.uhu.es; carrillo@uhu.es; climent@uhu.es

*We consider that the processes of interaction in a collaborative context of professional development have a significant influence on the degree of involvement of one of the participating teachers, and modulate the influence the context exerts on her professional development. We present an instrument for the analysis of interactions, which was developed in the course of this research and which aims to capture the dialogical nature of the discourse through three defining features distributed across six columns: the unit of information (utterance); the co-participants (the teacher and Interactant); and the contexts providing the sense of each contribution (Episodes, Action and Nature of the action). We also include a column for Content to complete the analysis with the epistemological input of each contribution to the discourse.*

***Keywords:*** analysis of interactions, collaborative context, professional development, dialogical approach, mathematics education.

## *INTRODUCTION*

This paper is part of a longitudinal study researching the professional development, in terms of mathematics teaching, of new entrant into primary teaching participating in a collaborative research project (PIC) (Muñoz-Catalán et al., 2007).

The collaboration is composed of two experienced primary teachers, three researcher-trainers, and Julia, the subject of the study (from her first year of teaching onwards). The group meets once a fortnight for three hours, during which tasks are carried out with the aim of deepening understanding of our own classroom practice, as well as the learning and the teaching of mathematics from a problem solving perspective. Until now, this project had remained the background to our studies, constituting a privileged source for data gathering (Climent & Carrillo, 2002). In the case of Julia, however, given the relevance that this project has proved to have for understanding her professional development, the analysis of Julia's interactions within the group has emerged as a key element for understanding not just the what, but also the how of said development. We believe that in and through the interaction, Julia goes about constructing her interpretation of the suggestions, critiques and knowledge brought into play, an interpretation which moulds the formative potential of the PIC.

So as to analyse Julia's interactions in the group, we have devised an instrument which is presented in this paper, and which we refer to as IMDEP (the Spanish acronym for Instrument for the analysis of Teacher's Interaction in a context of Professional Development). It has been devised during the research process following our methodological perspective of allowing the data to speak (Strauss & Corbin, 1998), and consonant with our dialogic perspective of the discourse (Linell, 2005).

## *A DIALOGIC APPROACH TO THE ANALYSIS OF THE DISCOURSE*

We consider that knowing implies an interaction with the object of knowledge, through which the subject interprets and reconstructs the meanings in play in the process. Following G. H. Mead and J. Dewey (in Corbin & Strauss, 2008), knowledge is created through action and interaction, for which reason we attribute a relational nature to it. According to this perspective, we can identify cognition with communication in that the interaction is an essential requirement for each to develop. While communication necessarily requires an interpersonal exchange, cognition can occur in solitary activities such as reading, in which the interaction is with the text. Communication and cognition, then, are two aspects of the same phenomenon, and are dialogically interlinked (Linell, 2005).

Our interest in Julia's construction of meaning activities within the group led us to approach the analysis of interactions with a dialogic conception of discourse (Linell & Marková, 1993, Linell, 1998, 2005). We recognise that people's responses to others' actions depend on the meaning they attribute to them. From this perspective, human dialogue is more than the sum of individual discourse acts; it is a sequence of activities with the aim of establishing mutual understanding on the topics under discussion. In this sense it is a question of shared activities, coordinated amongst all the members and mutually interdependent (Linell & Marková, 1993; Marková & Linell, 1996). The semiotic mediation acquires a key place in the communication, which "*may be understood as some kind of abstract third party in the dialogue*" (Linell, 2005, p. 10).

The relation between discourse and its context is one of interdependence: a particular discourse derives a large part of its sense from the specific context, but at the same time "*these contexts would not be what they are in the absence of the (particular) discourse that takes place within them*" (Linell, 2005, p. 7). This interdependence is established at two levels: on one hand, the specific time and place in which the interaction takes place (situation); on the other, the sociocultural praxis governing the specific situation. This is what Linell (2005) refers to as the double dialogicality of discourse.

Following the dialogical approach (Linell, 2005), the principle features we can attribute to conversation are interaction, context and the joint construction of meaning, semiotically mediated.

## *THE INSTRUMENT FOR ANALYSING INTERACTIONS: IMDEP*

We can understand professional development as defined by an increased awareness of the factors bearing upon educational phenomena and contributing to a better understanding of one's own practice (Krainer, 1999). Practice becomes a source for development when the teacher becomes actively involved in the process of questioning their own practice (Jaworski, 1998), and develops a critical, reflexive attitude. In this conceptualisation, reflection becomes medium and referent of the development (Climent, 2005; Llinares & Krainer, 2006).

Analysing Julia's interactions in the PIC allows us to focus on her construction of meaning within the frame of shared construction. Our focus, then, is not on the result of this social construction, but rather the individual processes of construction within the said social construction. We concur with recent studies, such as Llinares & Krainer (2006) point out, in considering contextual and organisational elements as key to accounting for teachers' learning.

This analysis leads to a better understanding of how reflections deriving from the group influence individual understanding and performance. The features of Julia's contributions to the discourse provide clues to the meanings which she attributes to the joint understanding under negotiation at each stage of the conversation.

### *Development, structure and features*

This instrument emerged during the research process in close relation with the data (Strauss & Corbin, 1998). Our focal point was Julia, and hence our analysis of interactions centred on her contributions to the discourse. In the same way that dialogical properties can be attributed to a single contribution to the discourse, without considering previous and subsequent contributions (Linell & Marková, 1993), so can they equally be applied to the set of contributions by a single member, namely Julia.

Audio recordings are made of all the PIC sessions and fully transcribed, recording the contributions of all members. The transcription does not include gestures, but provides a verbatim record of all spoken language, along with all information concerning the discourse relevant to our understanding. The presence of the researcher in the PIC sessions ensures a better interpretation of each contribution, given that the dialogue is constructed in and through the processes of interaction and in relation of interdependence with the contexts.

With respect to analysing Julia's contributions to the discourse, we were interested in recording to whom they were directed, in what moment of the session, the form in which the action was expresses, its nature and the content it conveyed. These concerns became questions which guided the close inspection of the data, and which resulted in the instrument below:

| Utterance | Episodes | Julia | Action | Interactant | Nature of the action | Content |
|---|---|---|---|---|---|---|
| | | | | | | |

Instrument for analysing Julia's interactions in the discourse in a training context

The instrument aims to capture the dialogical nature of the discourse, and covers the three key elements felt to be intrinsic to all the interaction: the unit of information (the column labelled *Utterance*), the co-participants (*Julia* and *Interactant*), and the context which provides the meaning of each contribution (*Episodes*, *Action*, *Nature of the action*). An additional column, *Content*, was added in the interests of linking the sociological aspect of each intervention to its epistemological contribution to the dialogue.

We consider the contribution as the basic unit of interaction, equivalent to the turn with respect to dialogue (Linell, 1998). A numerical code was assigned to each of Julia's contributions, indicating the order in which each appeared in the discourse. This code is the content of the *Utterance* column.

The columns *Julia* and *Interactant* refer to the co-participants in the communicative exchange under analysis at any particular moment. Each contribution must be understood in its sequential environment (Linell & Marková, 1993) as it is dependent on previous and subsequent contributions. As a result, we understand the participant at in each turn to be both emitter of their own contribution and receiver of the previous contributions of others (including those not specifically directed at them). Nevertheless, when we broke the group interactions down into contributions during the analytical process, we identified two types of operational interlocutors for each of them: the person originating the contribution, that is Julia in all cases so far as this study is concerned, and the addressee of the contribution, whom we designate with the generic label interactant (whether the group as a whole or some member(s) of it).

The transcript for each session was also analysed from the point of view of content, with units of information being identified. The code for these units corresponding to each contribution comprises the column Julia. Whilst it might be observed that this column could be substituted for that of *utterance*, given that it is essentially a new way of codifying the same contribution, each column nevertheless fulfils different analytical aims: the *utterance* column focuses on each contribution from a discursive perspective; the *Julia* column locates Julia's contributions with a view to analysing their content and so serves as a bridge between analysis of the interactions and analysis of the content (both at different moments of analysis, but subsequently integrated into a joint interpretation).

We now turn our attention to the third item we have highlighted as key to the processes of interaction – the context (as reflected in the columns *Episodes*, *Action* and *Nature of the action* in the instrument).

We are aware of the variety of factors which influence and interact with each other at each moment of the interaction. Strauss & Corbin (1994) represent this influence as a conditional matrix, formed by concentric circles corresponding to distinct aspects of the world: "*In the outer rings stand those conditional features most distant to action/interaction; while the inner rings pertain to those conditional features bearing most closely upon an action/interaction sequence*" (p. 275). Out of all the circles we are interested in those that are most germane to each session and at each moment of the interaction. This leads us, on one hand, to structure each session into *Episodes*, and on the other, to consider the sequential environment, that is, the simultaneous dependence of each utterance on the adjacent contributions (*Action* and *Nature of the action*). The activity frame (represented in *Episodes*) and the sequential environment together comprise the double contextuality of each contribution (Linell & Marková, 1993; Linell, 1998).

We define *Episode* as any segment the session can be divided into which coincides with a change in activity or in the aim of the work being undertaken. In the case of an episode being particularly long, or involving various self-contained discussions, we then divide it into sub-episodes, consistent with Schoenfeld's (2000) procedure for video analysis.

The *Action* column refers to the kind of response Julia makes to previous utterance, emphasising the responsive nature of each contribution. Given that the actions are defined by their contextual relations, we conceive the action as an inter-action (Linell & Marková, 1993). Four different actions emerged during the course of analysis:

| | |
|---|---|
| <u>Respond</u> | The act of reciprocating appropriately to what has been asked, including those questions expressed in an indirect way. |
| <u>Ask</u> | The act of questioning another in order to ascertain their opinion or knowledge of some topic; indirect questions are also included. |
| <u>Answer</u> | The act of replying to statements directed specifically to her. |
| <u>React</u> | The act of providing a response to a statement which is not specifically directed at her. This category includes both responses which contribute to the overall communicative goal in hand and those which are autonomous. |

Table1. Principle actions deriving from the analysis

Although we consider that all contributions imply an active interpretation on the part of the emitter, this latter can adopt a role which is receptive with respect to others' turns, that is a *responsive role* (when responding or answering), or one which impulses or promotes new turns, that is an *initiatory role* (when asking and reacting). Hence, these inter-actions provide an indication of the degree of initiative and the role adopted by Julia in the unfolding of the discourse.

The column *Nature of the action* seeks to capture the communicative function of each contribution to the discourse. Although we recognise the multifunctionality of these (Linell & Marková, 1993), we have generally chosen the one (or ones) which we consider best capture Julia's role in the discourse dynamics at each specific point.

Unlike the *Action* column, here we realise an interpretative rewriting of each contribution, headed by the verb which better describes its function in the discourse. A list of the verbs which emerged during the course of the analysis was compiled, from the definitions of which we then selected the usage applicable to Julia's contributions (see appendix).

Below is an extract from the table for analysing interactions, corresponding to a PIC session in which a video of Julia's practice is analysed.

| Int. | Episodes | Julia | Action | Interactant | Nature of the action | Content |
|---|---|---|---|---|---|---|
| 62 | Continuing the analysis of G7, begun in the previous session | S8. 78 | Answers | Researcher-trainer (R) 1 | **Agrees** that the activity was difficult and that the pupils were tired and did not yet have the left/right distinction fully assimilated. | Difficulties that she associates with the activity |
| 63 | | S8. 79 | Responds | R2 | **Points out** the objectives of the worksheet | Objectives of the worksheet |
| 64 | | S8. 80 | Reacts | R1 | **Points out** that besides taking the objectives from the book, as other teacher affirms, she also adds her own. | Objectives of the worksheet |
| 65 | | S8. 81 | Asks | R1 | **Understands** what he is asking about. | |
| 66 | | S8. 81 | Responds | R1/Inés (experienced teacher) | **Evades direct answer. Explains** other occasions in previous years when she had tackled the topic. | |

Example of the use of the IMDEP instrument

Given that it is an instrument for analysing interactions in a context of professional development, an analysis of the discursive dynamics of the interactions is insufficient without the addition of the epistemological contribution of each turn to the discourse. For this reason we have included the *content* column, in which we briefly outline what each contribution deals with, like a signpost for later interpretation.

## *THE INFLUENCE OF THE PIC IN PROFESSIONAL DEVELOPMENT THROUGH THE ANALYSIS OF INTERACTIONS*

The PIC, as a collaborative environment structured according to the principles of professional development rather than training (Ponte, 1998), exerts its influence through the joint pursuit of professional activities through means of debate and reflection. In this context, Julia was not required to assimilate the knowledge and information transmitted by others, but rather to participate in the collective construction of meanings which takes place in the interaction – a construction which is assimilated by Julia via a new personal interpretation.

Julia’s processes of assigning meaning are mediated by various factors and are produced in and through the interaction. Some of these factors are inherent in Julia herself, others are characteristic of the PIC and its members, but all of them operate concomitantly with others which arise in and are determined by the interaction. It is in the interaction that the role of Julia within the group is defined, along with the degree of confidence she establishes with each member, the image she has of them and they of her, and so on, aspects which influence how Julia accepts the reflections, opinions, suggestions and critical analyses about her practice. In short, we consider that the processes of interaction determine the extent to which Julia is involved in the group and hence, mediate the role which the PIC has in her reflection and professional development.

Our instrument of analysis provides us with information on:

-At what points in the session Julia tends to contribute and the degree of involvement towards her professional development within the group.

-Whether she tends to act on her own initiative or in response to others’ turns explicitly directed to her; that is, the way in which her role develops during the course of the interaction (initiatory or responsive).

-Whose critical comments she receives best and whose she seems not to accept; likewise, towards whom she shows a greater interest in knowing their thoughts or opinions. What features characterise the contributions of these members such that these reactions happen.

-After or before whom she usually contributes and why.

-Depending on the episode or activity to be done, what functions predominate in Julia’s contributions; in addition, the relation between the function of her actions and the people to whom they are directed.

-The relation between the actions and the nature of the contributions and the episodes framing them. For example, whether there is a difference in Julia’s contributions when a video of herself, or of the other teacher, is analysed.

-The relation between the characteristics of her contributions and the content under discussion at any moment. What kind of content would she seem to give more importance to according to the predominating function or action.

It can be seen from this perspective that the analysis of interactions allows us access to the meanings which Julia constructs and which she attributes to the various contributions at each point in the conversation, providing us with clues as to how the PIC shapes her professional development. Consequently, we feel that the interactions are the means through which Julia develops in the group and in turn the point of reference by which we as researchers gain access to how the PIC exerts its influence.

## *CONCLUSION*

This paper presents the instrument for the analysis of Teacher's Interaction in a context of Professional Development, which has been developed in the course of the research we are conducting. The IMDEP shows itself to be a useful tool for accessing and understanding the meaning that Julia constructs at each point of the interaction, with a view to gathering clues to the role that the PIC plays in her professional development. We have explained the theoretical grounding of the instrument, both from the perspective of our epistemological position and from our dialogical conception of discourse (Linell, 1998, 2005).

The IMDEP represents a contribution in three senses: first, our interest does not lie with the communication between students working on groups or between the teacher and students as is usually the case in the research literature (Bjuland, 2004; Cobb et al., 1997), but rather it lies in the interactions between educational professionals in a context of professional development. Secondly, the adoption of dialogical approach to the analysis of interaction tends to involve an interest in the joint construction of knowledge taking place in the group, in place of the attribution of meaning of one member participating in the group, as is our case. Finally, we aim to establish a relation between the interactions arising at each point of the communicative flow of the PIC and the extent of its influence on professional development, which allows us to gain insights into how social contexts operate upon it.

We intend to continue deepening in the analysis of interactions in contexts of professional development and making improvements to our instrument. In future papers we hope to illustrate and discuss examples of how the IMDEP is helping us to understand how the PIC is having an influence in Julia's professional development.

## *References*

Bjuland, R. (2004). Student teachers' reflections on their learning process through problem solving in geometry. *Educational Studies in Mathematics*, 55(1-3), 199-225.

Climent, N. (2005). *El desarrollo profesional del maestro de Primaria respecto de la enseñanza de la matemática. Un estudio de caso*, Doctoral dissertation. Michigan: Proquest Michigan University. www.proquest.co.uk.

Climent, N. & Carrillo, J. (2002). Developing and researching profesional knowledge with primary teachers. J. Novotná (ed), *European Research in Mathematics Education II, Proceedings of the CERME 2.* Praga: Charles University, 269-280.

Cobb, P.; Boufi, A.; McClain, K. & Whitenack, J. (1997). Reflective Discourse and Collective Reflection. *Journal for Research in Mathematics Education*, 28 (3), 258-277.

Corbin, J. & Strauss, A. (2008). *Basics of qualitative research (3rd edition).* Thousand Oaks: Sage publication.

Jaworski, B. (1998). Mathematics teacher research: Process, practice and the development of teaching. *Journal of Mathematics Teacher Education*, 1(1), 3-31.

Krainer, K. (1999). Teacher education and investigations into teacher education: A conference as a learning environment. K. Krainer; F. Goffree & P. Berger (eds), *European Research in Mathematics Education I.III. On Research in Mathematics Teacher Education*, chap.1. Osnabrück: Forschungsinstitut für Mathematikdidaktik, 13-39.

Linell, P. & Marková, I. (1993). Acts in Discourse: From Monological Speech Acts to Dialogical Inter-Acts. *Journal for the Theory of Social Behaviour*, 23 (2), 173-195.

Linell, P. (1998). Elementary contributions to discourse. *Approaching Dialogue: Talk, interaction and contexts in dialogical perspectives*, John Benjamins Publsihing Co., Amsterdam, 159-180.

Linell, P. (2005, June 23). *Essentials of dialogism. Aspects and elements of a dialogical approach to language, communication and cognition*. [Retrieved June 2007, 11, from http://isis.ku.dk/kurser/blob.aspx?feltid=119154.]

Llinares, S. & Krainer, K. (2006). Mathematics (student) teachers and teacher educators as learners. A. Gutiérrez & P. Boero (Eds), *Handbook of research on the Psychology of Mathematics Education: Past, Present and Future.* Netherlands: Sense Publishers, 429-459.

Marková, I. & Linell, P. (1996). Coding elementary contributions to dialogue: Individual acts versus dialogical interactions. *Journal for the Theory of Social Behaviour*, 26 (4), 353-373.

Muñoz-Catalán, M.C., Carrillo, J. & Climent, N. (2007). The professional development of a novice teacher in a collaborative context: An analysis of classroom practice. D. Pitta-Pantazi & G. Philippou et al (eds), *Proceedings of the V Congress of the European Society for Research in Mathematics Education.* Larnaca, Chipre: University of Cyprus, 1935-1944.

Ponte, J. P. (1998). Da formação ao desenvolvimento profissional. *Actas do ProfMat 98*, Lisboa: APM, 27-44.

Schoenfeld, A. (2000). Models of the Teaching Process. *Journal of Mathematical Behavior*, 18 (3), 243-261.

Strauss, A. & Corbin, J. (1994). Grounded Theory Methodology: An overview. N.K. Denzin & Y. Lincoln, (Eds), *Handbook of qualitative research.* Thousand Oaks: Sage, 273-285.

Strauss, A. & Corbin, J. (1998). *Basics of Qualitative Research: Techniques and Procedures for developing Grounded Theory.* Thousand Oaks: Sage Publications.

## APPENDIX: NATURE OF THE ACTION (ORGANIZED BY ACTIONS)[1]

| RESPOND | | |
|---|---|---|
| **Accept** | **Confirm** | **Inform** |
| **Clarify** | **Disagree** | **Show openness** |
| **Analyse** | **Explain** | **Propose** |
| **Offer idea** | **Express doubt** | **Reaffirm** |
| **Agree** | **(Re)formulate** | **Reject** |
| **Joke** | **Indicate** | **Recognise** |
| **Express lack of knowledge** | | |
| **Deny** | | |
| **Evade response**: Avoid an awkward question or one to which the addressee lacks a reply (assigned together with Offer idea, Agree, Explain and Reaffirm) | | |

| ASK | | |
|---|---|---|
| **Know:** Hear or obtain information about something | | |
| **Understand**: comprehend | | |
| **Question** | **Request confirmation** | **Propose** |
| **Provoke** | | |

| REACT AND ANSWER | |
|---|---|
| **Accept** | **Express doubt** |
| **Clarify** | **Express surprise** |
| **Analyse** | **Reformulate**: Reduce a proposition to clear and simple terms. |
| **Offer idea** | **Point out**: Briefly give information or an opinion. |
| **Agree**: State truth or appropriacy of previous affirmation or proposition. | **Inform** |
| **Joke**: Express own idea humorously, point out nonsensical aspect of some previous utterance, or respond ironically to an utterance. | **Show openness**: Display a favourable attitude towards carrying out a proposed or an assigned action. |
| **Comment on** | **Request confirmation**: Request further proof of veracity of an idea or the acceptance of a suggestion, idea or proposal. |
| **Confirm** | |
| **Correct** | **Propose** |
| **Question**: Challenge the basis of an affirmation, suggesting the reasons and foundations. | **Reaffirm**: Ratify what has been said. Explain one's own response, arguing in favour of a position which appears not to be accepted or shared by the others. |
| **Disagree** | **Reject** |
| **Explain** | **Recognise** |

[1] Only the verbs with a particular nuance in the context of this paper, or which can have several meanings, are defined here.

# ADAPTING THE KNOWLEDGE QUARTET IN THE CYPRIOT MATHEMATICS CLASSROOM

Marilena Petrou

University of Cambridge and The Open University, UK

*This paper builds on the work carried out by colleagues on using an empirically-based conceptual framework, the Knowledge Quartet, as a tool for the analysis of mathematics lessons taught by preservice teachers in the UK. This framework categorises situations from classrooms where mathematical knowledge surfaces in teaching, and was used with the aim of understanding what relationship can be observed between Cypriot preservice teachers' mathematical knowledge and their teaching. In particular, in this paper I suggest that the framework needs to be supplemented in order to incorporate the interpretation of mathematics textbooks by teachers. I illustrate this by giving examples from lessons taught by participants in my study.*

Key-words: Teacher Knowledge, Knowledge-Quartet, Textbook

## INTRODUCTION

The object of the study discussed is based on the classic distinction by Shulman (1986) between two aspects of teachers' mathematical content knowledge, Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK). PCK includes the representations, examples and applications that teachers use in order to make the subject matter comprehensible to students. SMK consists of substantive and syntactic knowledge (Schwab, 1978). Substantive knowledge focuses on the organisation of key facts, theories, and concepts and syntactic knowledge on the processes by which theories and models are generated and established as valid.

From a variety of perspectives, research in the field of preservice teachers' knowledge focuses on their SMK and PCK. Some researchers have investigated preservice teachers' understanding of different topics in mathematics (Ball, 1990; Philippou and Christou, 1994; Rowland, Martyn, Barber and Heal, 2001) and others have focused on investigating the relationship between SMK and PCK and teaching (Rowland, Huckstep and Thwaites, 2004; Hill, Rowan and Ball, 2005) and have suggested that content knowledge might affect the process of teaching. These studies have shown that preservice teachers' substantive knowledge of mathematics was significantly better than their syntactic knowledge, and this was reflected in their teaching.

In Cyprus, concern among policy makers about students' achievement in mathematics has grown recently, and many attempts have been made to improve the instructional practices in public primary schools. Attempts of improving mathematics teaching in Cyprus have focused on learners and the curriculum, rather than focusing on teachers. Research on teacher knowledge has been neglected in the Cypriot literature. The few studies in this field (e.g. Philippou and Christou, 1994) focused on investigating aspects of Cypriot preservice teachers' substantive and syntactic knowledge of mathematics and have shown that the participants were poorly prepared to examine different mathematical concepts and procedures conceptually. However, if we want to understand better what goes into teaching mathematics effectively, the challenge is to identify the ways in which preservice teachers' knowledge of mathematics, or lack of it, is evident in their teaching. No one type of knowledge functions in isolation in teaching and thus, research in the field of teacher knowledge should focus on understanding the relationship between the different kinds of their knowledge. The identification of this relationship will help teacher educators to assess teacher preparation programmes, and to improve them where necessary. The study reported in this paper was carried out in the context of my ongoing doctoral study which is centred on understanding the relationship between Cypriot preservice teachers' SMK and PCK to teaching. In particular, the focus of this paper is on reporting results related to one of my research questions. I discuss whether the original conceptualisation of the Knowledge Quartet was relevant and adequate in the analysis of teaching in the Cypriot primary mathematics classroom.

## THE STUDY

My approach to investigating the relationship between Cypriot preservice teachers' mathematical knowledge and teaching involved a mixed-methods approach. My study entailed four data collection methods. First, a questionnaire was designed to examine Cypriot preservice teachers' SMK of mathematics. 104, final year university students, following a teacher preparation programme, completed the questionnaire. It aimed to collect information about the participants' beliefs about mathematics and its teaching, and their substantive and syntactic knowledge of it. As a part of the questionnaire the participants were asked to respond to ten mathematics items that assessed their SMK. The aim of the interview questions was firstly to clarify the questionnaire data and second to gather some information about the interviewees' PCK of mathematics. The interview questions proposed two hypothetical scenarios that were relevant to teaching mathematics, representing real classroom situations which a teacher might encounter while teaching mathematics. The interview tasks provided information about what teachers know and believe about mathematics, and also about the knowledge and skills that they draw on in making teaching decisions.

While these interview tasks represented real situations in the mathematics classroom, their context remained hypothetical, and did not provide information on what teachers actually do in the classroom and how their knowledge of mathematics influences their teaching decisions in classroom where they interact with their students. This kind of information was provided by observing participants teaching mathematics in the classroom. Five of the interviewees were chosen to be observed while teaching mathematics. In Cyprus a large part of the teacher preparation programme (a four year university course) is spent in teaching in schools under the guidance of a school based mentor.

For the observations I used a framework that emerged from observing several lessons that were taught by preservice teachers in England (Rowland et al, 2004). This framework is called the *Knowledge Quartet* and is a tool that can be used in order to describe the ways in which SMK and PCK are revealed through teaching. As a part of my study I also evaluated the adaptability of the framework in the Cypriot classroom.

Finally, the data from the questionnaire, interview and observations were compared with data from the analysis of mathematics textbooks in Cyprus. Textbook analysis provided information on what policy makers consider desirable knowledge for teachers. However, what is considered desirable knowledge for teachers is often different from the knowledge that teachers use in and reveal through practice. A comparison of these two kinds of knowledge is considered to be helpful in modifying and improving teacher preparation programmes.

The combination of four methods and their integration during the interpretation phase provided strong inferences and produced a more complete understanding of the relationship between participants' content knowledge and their teaching. In the remainder of this paper I will focus on just one aspect of the study described here, and discuss issues related to the adaptability of the framework in the context of the Cypriot classroom.

## THE KNOWLEDGE QUARTET

At the CERME meeting in Spain, Tim Rowland presented a paper (Rowland, Huckstep and Thwaites, 2005) about the Knowledge Quartet and suggested that this can be used as a tool for classifying ways that preservice teachers' knowledge comes into play in the classroom. At the following CERME meeting in Cyprus Fay Turner (Turner, 2007) also presented a paper about the Knowledge Quartet and explained how she is currently using the framework as a tool for professional development with a group of early career teachers.

The Knowledge Quartet consists of four dimensions, namely, Foundation, Transformation, Connection and Contingency. Foundation consists of trainees' knowledge, beliefs and understanding of mathematics. Transformation concerns knowledge-in-action as demonstrated in the act of teaching itself and it includes the kind of representation and examples used by teachers, as well as, teachers' explanations and questions asked to students. Connection includes the links made between different lessons, between different mathematical ideas and between the different parts of a lesson. It also includes the sequencing of activities for instruction, and an awareness of possible students' difficulties and obstacles with different mathematical topics and tasks. Finally, Contingency concerns teachers' readiness to respond to students' questions, to respond appropriately to students' wrong answers and to deviate for their lesson plan. In other words it concerns teachers' readiness to react to situations that are almost impossible to plan for.

Below, I argue that when adapting the framework in the Cypriot mathematics classroom, this needs to be supplemented by consideration of the use and interpretation of mathematics textbooks. I give three examples from lessons taught by participants in my study to illustrate this.

## ADAPTING THE KNOWLEDGE QUARTET IN THE CONTEXT OF THE CYPRIOT CLASSROOM

When adapting the Knowledge Quartet it was not assumed that the knowledge used by Cypriot and English teachers is the same. Therefore, as part of my study I evaluated the adaptability and the validity of the Knowledge Quartet. In this section I describe the appropriateness of the Knowledge Quartet in the context of the Cypriot classroom, and explain that the framework needs to be expanded by adding a new code in the Transformation dimension.

For the most part, I found that the Knowledge Quartet could be used successfully to analyse mathematics lessons in the Cypriot mathematics classroom, in understanding how participants' SMK and PCK were related to their teaching. In particular, the issues raised for attention in lessons observed in the UK were also observed in the Cypriot mathematics classroom.

In my analysis of the lessons, I identified all the situations that I thought were significant with respect to participants' mathematical knowledge. The Knowledge Quartet proved to be comprehensive in describing most of the teaching episodes that were considered important for the purpose of my study. With reference to the 'Foundation', 'Connection' and the 'Contingency' dimensions, the codes proposed in the original study could be used to describe all the situations I thought were significant in understanding the relationship between participants' content knowledge and their teaching. For example, participants' ability to anticipate students' difficulties and obstacles, to hear and respond appropriately to students' thinking, to choose appropriate examples and representations, and to make connections between different mathematics concepts, were significant issues in understanding the ways in which their content knowledge came to play out in their teaching. In addition, issues related to participants' awareness of students' conceptions and misconceptions about a mathematical topic, their decisions about sequencing activities and exercises, or interrupting a classroom discussion to obtain clarification, or their decision to use a student's opinion to make a mathematical remark, were significant in identifying the relationship between participants' knowledge and teaching.

It was also clear from the data that Foundational knowledge underpinned the other three dimensions. In general, the application of teachers' knowledge in the classroom always rested on their Foundational knowledge, which was acquired in the academy in preparation for their role in the classroom.

On the whole the Knowledge Quartet was found to be a valid tool for analysing the lessons observed in the Cypriot classroom. However, an additional issue that proved to be significant in the analysis of my lessons was the use of mathematics textbooks, in particular how activities in the textbooks were adapted. Here, textbooks refer both to students' book and the teachers' guide. In the original study a code 'adherence to textbooks' was classified in the Foundation dimension of the framework. This code was used to describe episodes where teachers accepted textbook as authority for what and how to teach. However, the ways in which teachers adapted textbook activities are not addressed in any of the existing publications about the use of the Knowledge Quartet as a tool for observing mathematics lessons in the UK. This is not surprising, since the use of textbooks is not a common practice in the English primary school mathematics classroom. In contrast, the textbook is central and always present in the mathematics classroom in Cyprus.

All the participants in my study considered the textbook as the main resource both for their planning and teaching. However, they all combined it with other resources, and included their own developed activities. The participants adapted the textbooks in very different ways. For example, there were cases where participants modified the textbook material in ways that made the lesson more meaningful and interesting for their students. However, in some instances participants were not sure how to adapt the textbook activities appropriately, modifying them in ways that altered their focus. This suggested that the ways in which preservice teachers used the textbooks was important in understanding how their knowledge came into play in their teaching.

The above led me to conclude that when adapting the Knowledge Quartet for observing lessons in Cyprus, and indeed in many other countries, there is a need to take careful account of these differences. Thus, issues related to the adaptation, modification, and interpretation of the textbook material are important in analysing a mathematics lesson in Cyprus. Having presented the appropriateness of the dimensions of the Knowledge Quartet in the context of the Cypriot classroom, I provide some examples from the lessons observed to demonstrate how the participants in my study used the textbook activities.

## ADAPTING THE TEXTBOOKS: SOME EXAMPLES FROM THREE PARTICIPANTS

The lessons observed took place during the students' placements in school. These lessons were analysed using the four dimensions of the Knowledge Quartet. In this section, I give some examples related to how three participants (Rita, Elsa and Christiana) used the mathematics textbooks. Christiana chose to do additional courses in mathematics in her undergraduate teacher education course, and was classified in the group with a 'high' SMK score (this was assessed in the questionnaire, see page 2). Elsa was classified in the group with 'low' SMK score and Rita in the group with 'medium' SMK score. Neither of them chose to do additional courses in mathematics during their training. In general, the results showed the positive influence of strong

SMK in the effective use of textbooks. Christiana elaborated upon the textbook in ways that made her lesson more meaningful and interesting for students. She was able to drawn on her own understanding and use appropriately textbook activities and extends them to promote students' conceptual understanding. In contrast, Rita and Elsa seemed to have problems in understanding the textbook suggestions due to their lack of SMK. In many instances they could not understand the mathematics targeted by textbook activities, and so could not make much of them. Therefore, it becomes clear that in order to use textbook activities appropriately, teachers need to understand their content.

### Not understanding the mathematics targeted by the textbook

Rita's lesson on multiplication by four offers an example of how she interpreted one of the activities in the textbook in ways that altered its focus. Figure 1 illustrates this activity.


Mr Michalis has recently opened a new restaurant. He has50 square tables in the restaurant. Each table can seat 4 customers. On Sunday night 36 customers went for dinner. By 23:00 half of them had left. One hour later all the other customers left and the restaurant closed.

1. How many tables does the restaurant have?

2. How many tables remained empty on Sunday night?

3. How many customers were in the restaurant just after 23:00?

4. Show on the clock the time that the restaurant closed.

5. On Monday ten friends went to the restaurant for lunch. Mr. Michalis needed to put tables together so that ten friends could sit next to each other. How many tables were needed?

**Figure 1 Textbook Activity (2nd Grade, Students' Book, Part B, p.87)**

In addition, in the teachers' guide it was clearly stated that:

> intentionally some information is not given […] students should think of all the possible answers to the questions asked, taking into consideration that each table can seat 1,2,3 or 4 customers (Grade B, Teachers' Guide, p. 103)

Rita seemed not to take into consideration what was suggested in the teachers' guide. She used a rather 'traditional' approach in solving the problem. She read the problem to her students, and did not leave them much time to think, before leading them towards the answers. More importantly, when dealing with question two of the problem she seemed to take for granted that exactly four customers were sitting at each table and said:

> 36 customers were in the restaurant. There were four people at a table. Thus, 36 divided by 4 will give us the number of tables that were full.

Rita's approach to solving the problem focused on procedures, required a single answer, and focused on relatively few skills. However, the focus of the problem was meant to provide students with the opportunity to explore a number of possible solutions. Rita showed a desire to develop conceptual understanding in several instances in her lessons, however, it seems that in this case her beliefs about good mathematics teaching could not be implemented because she did not understand the problem solving intention. I can infer from my post-observation discussion with Rita that she changed the focus on the activity due to her lack of understanding. In this discussion I asked Rita if she could think of an alternative way of solving the problem and she was adamant that she could not. Her answer suggested that she might not have read the teachers' guide. However, the aims that were proposed in her lesson plan were exactly the same as those proposed in the teachers' guide, so it seems that she did read the guide, but that her reading was superficial, and for some reason she missed some of the information provided. It could be argued that she followed the teachers' guide rather mechanically, moving through activities without understanding their focus. In this case her problems in understanding the teaching suggestions in the guide might stem from insufficient understanding of the problem.

Another example, of not understanding the suggestions in the textbook occurred in Elsa's lesson on the parts of a circle. In this lesson Elsa tried to define the different parts of a circle. Table 1 shows the definitions that she proposed alongside the definitions that were suggested by the teachers' guide.

The definitions that Elsa gave to her students were mathematically incorrect. Even though she used the activities proposed in the textbooks she did not use the suggested definitions. It seemed that her understanding of the different parts of a circle is limited. Below I provide an extract from our post-observation discussion to support my argument:

Elsa: Generally, I think that everything went well. However, my impression is that students were confused about the chords.

MP: What do you think confused them?

Elsa: Uh, I think that the definition of a chord is confusing itself. To be honest, I am confused myself. On the one hand, according to the definition provided in the textbook, a chord does not pass through the centre. On the other hand, the teachers' guide mentions that the diameter is the biggest chord. I think this is very confusing.

The extract above indicates that Elsa's understanding of the parts of a circle was limited. She seemed not to be aware of the correct definitions of different parts of a circle, and, due to her limited understanding, was unable to follow the suggestions included in the textbook. It was likely that Elsa chose not to use the definitions as suggested in the textbook because she believed that these were too difficult for her students. In trying to make these easier for her students, she made it more difficult.

| | **Elsa’s definitions** | **Definitions suggested by teacher’s guide** |
|---|---|---|
| **Diameter** | Is a straight line that starts from the beginning* of the circle and reaches the end of the circle passing through its centre | Each chord that passes though the centre of the circle. A straight line passing through the centre of a circle and connecting two points on the circumference |
| **Radius** | It is a line that starts from the centre and reaches the end of the circle | A straight line segment connecting the centre of the circle with a point on the circumference |
| **Chord** | Is a line that starts from the beginning of the circle and reaches the end but does not pass through the centre | A straight line segment connecting two point on the circumference |
| **Circumference** | The ‘round -round’ * of a circle | Not included |

* This is the exact translation for Elsa’s definition from Greek, which in effect means the boundary of a circle

**Table 1: Defining the parts of a circle**

In general, in mathematics definitions should be inclusive. However, Elsa’s definition of the chord was exclusive. Her statement ‘does not pass through the centre’ excludes the diameter which indeed is a chord. In contrast the definition of the chord in the teachers’ guide was inclusive. In addition, it was clearly stated that the diameter is the biggest chord. Therefore, it can be argued that her problem in understanding the definition proposed in the textbook stemmed from her limited understanding of the topic. This was indicated by her tendency to refer to the ‘beginning’ and the ‘end’ of a circle, meaning points on the circumference.

## Elaboration upon the textbook: making activities more meaningful and interesting for students.

An example of developing the textbook material is offered by Christiana’s activity illustrated in Figure 2. The version of the activity as proposed in the students’ book is also presented. Both activities have been translated from Greek. It is clear that in her modified version of the textbook activity Christiana put emphasis on developing students’ conceptual understanding. I consider Christiana’s version to be an improvement because she elaborated on the textbook activity in a way that made it more meaningful to her students, by helping them to explore division and multiplication as reverses operations.

**A FACTORY PRODUCING JAM**

The students in Philippos' class visited a factory producing jam. The jam was bottled and then packed into large boxes. Each box could hold 50 bottles. On that day the production was 9250 jars of jam. How many boxes were needed for packing the jars? The table below shows the production of jam for each day of the week. Fill in the information in the table provided

| Days | Jars for each day | Jars in each box | Number of boxes |
|---|---|---|---|
| **Monday** | 24 500 | 50 | |
| **Tuesday** | 18 900 | 50 | |
| **Wednesday** | 11 750 | 50 | |
| **Thursday** | 21 600 | 50 | |
| **Friday** | 12 600 | 50 | |

**Textbook activity (4th grade, Students' book, Part C, page 33)**

Christiana modified the activity and asked her students to fill in the information in the table presented below

| Days | Jars for each day | First filling | | Second filling | |
|---|---|---|---|---|---|
| | | Number of jars in each box | Number of boxes | Number of jars in each box | Number of boxes |
| **Monday** | 24 500 | 50 | | 100 | |
| **Tuesday** | 18 900 | 50 | | 100 | |
| **Wednesday** | 11 750 | 50 | | 100 | |
| **Thursday** | 21 600 | 50 | | 100 | |
| **Friday** | 12 600 | 50 | | 100 | |

Then the students were asked to write down their observations relating to the numbers of boxes needed for the first filling and the second filling

Figure 2: Elaborating textbook activities

## CONCLUSION

In general the Knowledge Quartet was comprehensive in the classification of teaching situations in which participants' mathematical knowledge surfaces in teaching. Issues related to the interpretation of textbooks were not addressed by the framework, however were important in analysing mathematics lessons in a Cypriot classroom. This suggests that when adapting the Knowledge Quartet for observing lessons in Cyprus, and indeed in many other countries, there is a need to take careful account of possible differences between the context in which the framework was originally developed, and the context in which this is applied.

## REFERENCES


Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in M athematics Education, 21*(2), 132-144.

Hill, H. B. & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42 (2), 371-406.

Philippou, G. & Christou, C. (1994). Prospective elementary teachers' conceptual and procedural knowledge of fractions. In J. Pedro & J. Filipe (Eds.), *Proc.18th of the Int. Group for the Psychology of Mathematics Education* (Vol. 4, pp. 33-40). Lisbon, Portugal.

Rowland, T., Huckstep, P. & Thwaites, A. (2004). Reflecting on prospective elementary teachers' mathematics content knowledge: the case of Laura. In M. J. Høines & A. B. Fugelstad (Eds.), *Proc. 28th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 4, pp. 121-128). Bergen, Norway.

Rowland, T., Huckstep, P. & Thwaites, A. (2005). The knowledge quartet: considering Chloe. In M.Bosch, (Ed.), *Proc. 4th Congress of the European Society for Research in Mathematics Education* (pp. 1568-1577). Sant Feliu de Cuixols, Spain.

Rowland, T., Martyn, S., Barber, P. & Heal, C. (2001). Investigating the mathematics subject matter knowledge of preservice elementary school teachers. In M. van den Heuvel-Panhuizen (Ed.), Primary trainees' mathematics subject knowledge: an update report. In E. Bills (Ed.), *Proc. 25th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 4, pp. 121-128). Utrecht, Netherlands.

Schwab, J.J. (1978). Education and the structure of the disciplines. In I. Westbury & N. J. Wilkof (Eds.), *Science Curriculum and Liberal Education* (pp. 229-272). Chicago: University of Chicago Press.

Shulman, L. S. (1986). Those who understand knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.

Turner, F. (2007). The mathematical content knowledge of beginning teachers: The case of Amy. In D. Pitta-Pantazi and G.Philippou (Eds.), *Proc. 5th Congress of the European Society for Research in Mathematics Education* (pp. 2004-2013). Larnaca: University of Cyprus.

# PROFESSIONAL KNOWLEDGE IN AN IMPROVISATION EPISODE: THE IMPORTANCE OF A COGNITIVE MODEL

**C. Miguel Ribeiro[1], Rute Monteiro[1], José Carrillo[2]**

*[1]University of Algarve (Portugal), [2]University of Huelva (Spain)*

*One approach towards improving teacher performance is that of classroom practice. In this paper, taking a cognitive perspective, we present a system for modelling teacher performance. We demonstrate the process of construction of this model with reference to a brief lesson episode involving teacher improvisation, which took place in the first cycle (the first four years) of primary school in Portugal. Included in the model are the cognitions made evident by the teacher as well as the relations between them.*

Keywords: Improvisations, cognitions, modeling the mathematics teaching, practice, primary school

The teaching process can be analysed from various theoretical perspectives and focus on very different aspects, amongst them the teacher and their performance. With respect to classroom practice, the teacher's decisions are influenced not only by the particular context, but also, and we believe fundamentally, by his or her cognitions.

With the aim of understanding what happens in the classroom from the point of view of the teacher, in terms of both their actions and their cognitions, we decided to focus on performance, in particular the relations between the teacher's actions, cognitions and the type of communication used. The teaching-learning process is far too complex to permit a single, all-encompassing analysis, however, and hence we recognise the need for developing a model which allows it to be simplified for a more fruitful analysis. The model we developed to fulfil this aim was based on Monteiro (2006), Monteiro, Carrillo & Aguaded (2008), Schoenfeld (1998a, 2000) and Schoenfeld, Ministrell & Zee (2000). We denominate it a 'cognitive model', because it focuses only on certain of the elements comprising the system it models, in this particular case, the cognitions of the teacher with respect to their classroom practice. With this model we try to study some dimensions of professional knowledge and some relations amongst them. We hope this paper helps consider the common analysis of lessons by focussing on a limited number of variables as beneficial for researchers, trainers and teachers working in collaboration.

In the next sections we are discussing the cognitions and the kinds of communication. For the purpose of this paper, teacher's action should be identified with his/her performance in the classroom when dealing with their students' knowledge building.

## The cognitions

Following Artz & Thomas-Armour (2002), we understand by cognitions all those cognitive constructions – beliefs, knowledge and goals – which each individual carries with them, the study and analysis of which, along with the relations among them, offers valuable contributions for both research and classroom practice, which can be understood as the ultimate aim of research.

As teachers we can have goals over the short, medium and long term. For Schoenfeld (1998b), goals can be simply something which one aims to attain, and can be explicit or latent, and can likewise be pre-determined or emerge during the teaching activity (Aguirre & Speer, 2000). We believe that such emergent goals especially occur in unplanned situations, particularly those which the teacher have not anticipated. We concur with Saxe (1991) that each individual – and specifically here a teacher - has the capacity to construct, adapt, model and remodel such goals in accordance with his or her own personal and professional development.

As was noted in respect of goals, so too does research into beliefs offer great potential for both theory and practice. The more we can learn about the influence of teachers' beliefs on their teaching, the deeper our understanding (Aguirre & Speer, 2000). In this study the instrument used to undertake the analysis of teachers' beliefs was that of Climent (2002). Climent presents a set of indicators of primary school teachers' beliefs (i.e., first six years in Spain) with respect to beliefs on methodology, mathematics, learning, and the roles of pupil and teacher.

Concerning our focus on professional knowledge, of particular relevance is the work, still in progress, of Ball, Thames & Phelps (submitted) which adapts Shulman's (1986) formulation for the components of professional knowledge. Further, some incorporations, namely certain descriptors from Park & Oliver (2008), are also included.

Ball and colleagues (Ball, 2003; Ball, et al., submitted), following Shulman's (1986) classification, introduce the notion of mathematical knowledge for teaching. They divide content knowledge and pedagogical content knowledge each into three categories. Content knowledge, they consider to be formed by horizon knowledge (HK), common content knowledge (CCK) – i.e., typical 'schoolboy' mathematics – and specialised content knowledge (SCK). Pedagogical content knowledge (in Shulman 'curricular knowledge'), they likewise divide into three types, each a variant of content knowledge: teaching (KCT), student (KCS), and the curriculum (KC). Hence, they maintain that teachers should have a specific professional knowledge, so that in addition to a knowledge of 'how to do' – that is, common mathematical knowledge (CCK) – they should also have a knowledge of 'how to teach to do'. Thus, for example, beyond knowing how to calculate the difference between two numbers (CCK), it is necessary for the teacher to possess an understanding which allows him or her to perceive and identify not only the students' mistakes but also the source of these mistakes, which becomes much more complex (SCK). Likewise, they should also be familiar with alternative procedures for dealing with content, so that they can easily meet the needs of their pupils. Equally, a knowledge of how the various mathematical topics relate to one other and the way in which the learning of a particular topic develops as one moves up the school (HK) is essential for the effective teacher.

As an integral part of methodological and curricular content knowledge identified by Shulman (1986), Ball, et al. (submitted) consider that teachers should possess a composite knowledge of teaching and specific content (KCT).This corresponds to the type of knowledge to which the teacher resorts in situations that are related to the organisation of different ways the students explore mathematical contents, such as: determining the sequencing of tasks, choosing examples, and selecting the most appropriate representations for each situation. Park & Oliver (2008) also include the specific strategies for teaching the content in question.

Regarding knowledge of content and students (KCS), Ball et al (submitted) relate this to the need for the teacher to anticipate what the students think, their difficulties and motivations as well as listening to and interpreting their comments. Park & Oliver (2008) include here the knowledge of the possible wrong conceptions, motivations and interests of the students, as well as their needs.

## Kinds of communication

The way in which the teacher communicates with others (their students in this case) provides a great deal of information about him or herself and how they regard the whole process of teaching – including body language, level of anxiety, etc. The type of communication the teacher employs is in direct relation with the cognitions they hold, in that the way the teacher chooses to communicate reflects the way they view the teaching process. With different forms of communication, so the actions are distinct and quite possibly the underlying teaching views themselves.

We adopt the classification of Brendefur & Frykholm (2000), with some adaptations introduced by Carrillo, Climent, Gorgorió, Rojas & Prat (2008). Brendefur & Frykholm (2000) propose four types of mathematical communication: unidirectional, contributive, reflexive and instructive.

Unidirectional communication is associated with a form of teaching in which the teacher takes the principal role, requiring the student to do no more than faithfully repeat what he or she has heard. With respect to contributive communication, the student is afforded some participation in the classroom discourse, although the interactions which take place are by and large of a corrective nature and do not go very deeply into the content. The key feature of reflexive communication is that the interactions between the teacher and students act as triggers for subsequent investigative work. We agree with Carrillo et al. (2008), that development of students' mathematical comprehension is best achieved through such inquiry-based activities. Instructive communication, is similar to reflexive communication, but aims also to shed light on the matter in hand, bringing about an integration of students' ideas – progress and/or difficulties – made explicit or intuited by the teacher or by the students themselves.

## The context and modelling process

The remainder of this paper is dedicated to presenting and discussing the modelling of an episode in which the teacher reviews content through dialogue. This occurs in a 4th year class given by a teacher of 18 years experience. The episode is taken from a wider research project on professional development studying the relationships between teachers' beliefs, knowledge, goals and actions. It combines a case study with an interpretative methodology whereby there is minimal intervention on the part of the researcher. Data collection – audio and video recordings of the teacher – was conducted in situ. Brief informational talks were also used before and after each lesson to gather lesson previews – lesson image – and to clarify some inferences. The video recordings provided a record of the teacher-students interactions, and enabled lessons to be viewed and analysed, as many times as required.[1] That wider research project involves a collaborative work between the researcher (first author of this paper) and two primary teachers. The collaborative work started after the first phase of data collection. It was focused in the teacher's practices mainly by discussing some situations they consider to evidence good practices and others they want to improve their **teaching**.

The first stage of the modelling process involved the transcription of the audio recordings, followed by the video (Illustration 1). Transcription also included an initial division of the lessons into episodes, defined by triggering and terminating events and associated with specific goals. Subsequently, when all the lessons pertaining to the same phase (of three in total) had gone through this procedure, there began the process of identifying the indicators of beliefs (Climent 2002), content, specific goals, type of episode, type of communication, means of working, resources used, and the teacher knowledge required for implementing the episode (Ball, et al., submitted; Park & Oliver, 2008). Also determined at this point, was whether or not the episode formed part of the lesson image (cf. Table 1).

The action sequences identified correspond to routines, scripts or action guides, and improvisations (Monteiro, 2006; Monteiro et al., 2008; Schank & Abelson, 1977; Schoenfeld, 2000; Schoenfeld et al., 2000; Sherin, Sherin & Madanes, 2000). A routine is any kind of action independent of context, executed routinely; scripts, or action guides, are specialisations of routines, but conceptually dependent. Improvisations correspond to all those actions undertaken by the teacher in response to an unexpectedly arising event.

In this study the definition of improvisation has a wider sense than that of the researchers mentioned above[2], and distinguishes two types that can arise in class. The distinction concerns the relation pertaining (or not) between the events/actions and the contents. Thus, either the action is related to the content under consideration at that moment (or which has been, or is to be, dealt with), or the action has no relation

[1] The recordings also allowed the teacher to prepare reports and to reflect more fruitfully on the various interactions between the participants through repeated viewings.

[2] They only consider situations in which the actions are unconnected to the contents. We consider that improvisations correspond to the set of teacher's actions in response to all unexpected events.

with the teaching contents, focusing only on administrative questions, student conflicts or general management issues. We call the first type (concerned with the teaching activity) ‘content improvisations’, and those of the second (concerned with classroom management) ‘management improvisations’.

It should be noted that content improvisations constitute episodes which do not form a part of the lesson image and which necessarily have emergent goals. Because such episodes have not received prior consideration, the teacher’s cognitions come very much more to the fore since their response is so much more intuitive. Content improvisations are consequently one of the points in which cognitions are most in evidence.

## A teaching episode and its analysis

In this section we present a transcript of an episode from the first of a series of four lessons aimed at introducing the concept of ‘a thousandth’. Given that the transcript illustrates a goal in emergence, the episode cannot be considered to form part of the lesson image. The extract shows the teacher taking the opportunity presented by a student doubt to revise, via a whole-class dialogue, the difference between squares and rectangles through reference to the lengths of the sides.

| | | |
|---|---|---|
| 246 | S | This isn’t a rectangle, it’s a square . . . |
| 247 | T | Is this shape a rectangle or not? |
| 248 | S | No! |
| 249 | T | So, why isn’t it a square, Tiago Luís? |
| 250 | S | Because the sides aren’t the same length. |
| 251 | | I thought it was a square, Miss. |
| 252 | | (Inaudible) |
| 253 | T | Paulo quiet. |
| 254 | | What features does it have it have to be a square? |
| 255 | S | It has to have the sides the same length. |
| 256 | T | The sides all the same. |
| 257 | Ss | (Inaudible, everybody speaking at the same time) |
| 258 | T | (Puts hand up) |
| 259 | | Quiet, quiet, put your hands up. |
| 260 | | (T points to one of the sides of the square) |
| 261 | | Paulo, if this side is twenty-five squares long, and this side is … how many? |
| 262 | Ss | Forty! |
| 263 | T | Forty … so, is it a square? |
| 264 | S | No! |
| 265 | T | Why not, Paulo? |
| 266 | S | Because the sides aren’t the same length. |
| 267 | T | Exactly. |
| 268 | S | To be a square it has to be twenty-five by twenty-five. |

**Illustration 1 – Transcript of an excerpt from the first, of a series of four, classes aimed at introducing the concept of ‘a thousandth’, corresponding to an improvised content revision dialogue (T: teacher; S(s): student(s))**

This excerpt corresponds to the ninth episode in the first lesson of the first phase of work [I.1.9]. The triggering and terminating events coincide with the start end of the transcript. The teacher's emerging goal is to revise the difference between squares and rectangles in terms of the lengths of their sides. The communication type she employs is contributive, with the students working in a large group (the whole class).

The coding within the square brackets indicates that the lesson takes place during the first phase (pre-collaborative work) and corresponds to the ninth episode of the first lesson [I.1.9]. The left-hand box provides information on the specific category to which each indicator of beliefs belongs (in brackets) in addition to the goal and knowledge which have been identified, the triggering and terminating events, the type of episode and whether or not it forms part of the lesson image. The right-hand boxes record the sub-episodes ([I.1.9.1], [I.1.9.2]) along with their specific goals and the kind of dialogue involved.

[I.1.9] Dialogical revision of content - difference between squares and rectangles - in a contributive way, with the whole class (246-268)[3]
**Forms part of the lesson image?** No.
**Triggering event**: T asks whether shape is a rectangle or not.
**Indicators of beliefs**[4]:
TT30 (**Teacher's role**) – The teacher is the one who validates ideas raised in class, questioning students, whose replies lead to self-correction (in reality veiled correction, stage-managed by the teacher).
TR16/TT16 (**Learning**) – The student interacts with the material and the teacher, the latter being the mediator between material and student. The interaction produced between teacher and student is unequal, with the flow teacher-student being stronger than the contrary.
**Goal**: Revise difference between squares and rectangles (length of sides).
**Knowledge**:
CCK (**Common Content Knowledge**) – Knowing the difference between squares and rectangles (in terms of the length of the sides).
SCK (**Specialized Content Knowledge**) – The teacher gives evidence of an incorrect use of the classification of polygons (using a disjunctive classification implying that the set of squares is separate from that of rectangles)
KCT (**Knowledge of Content and Teaching**) – The teacher considers contributive dialogue appropriate for the revision of the difference between the length of the sides of squares and rectangles.
KCS (**Knowledge of Content and Students**) – The teacher considers that the students show difficulties in considering squares as specific cases of rectangles (246-250), (254-256)
(GAP: the teacher does not perceive this difficulty of considering squares as rectangles as she uses disjunctive classifications and an incomplete definition of squares focused exclusively on the properties of the sides (forgetting the rhombus), which could generate erroneous conceptions (256).)
**Type of episode**: Content improvisation.
**Terminating event**: T considers that the students' doubt has been clarified.

[I.1.9.1] T holds a dialogue with the group, and contributively revises the difference between the relative lengths of the sides of squares and rectangles (246-260)

Interactive dialogues (246-260)
Specific goal: Revise the difference between the relative lengths of the sides of squares and rectangles.

[I.1.9.2] T holds a dialogue with the group, and contributively clarifies that, by virtue of its sides not all being the same length, the shape cannot be a square (261-268)

Interactive dialogues (261-268)
Specific goal: Clarify that a rectangle is different from a square as one has sides of all the same length and the other does not.

[3] Line numbering of transcript.
[4] This episode reveals beliefs concerning methodology (TR3, TR5), the role of the teacher (TT26/29, TT30) and learning (TR16/TT16, TT14), where TR denotes Traditional Tendency and TT Technological Tendency.

**Table 1 – Modellisation of an episode corresponding to the ninth episode in the first of four lessons introducing the concept of a thousandth**

This episode did not form part of the teacher's lesson image as it arose from a student comment. In the course of enacting the episode the teacher employs two actions which, from the analysis we have carried out until now, form the basis of all the revision episodes, independently of the resource(s) used, the form of work and the type of communication. It should be noted that, for this type of episode, these actions do not have to occur in the same order as in this specific case and that these are the only two kinds of actions the teacher does when she wants to implement this specific type of episode in this particular manner.

## Relations between cognitions

The evidence for the teacher's cognitions is obtained from their actions, the kind of communication which occurs, the form of work of the students and the resources used. The table below illustrates the relations observed between the actions and cognitions in respect of the specific goal in this case. Some of the teacher's knowledge (to the right of the table) are relevant to the whole episode while others are specific to particular actions.

<table>
<tr><th>Indicators of beliefs/contributive language</th><th>Actions</th><th colspan="2">Knowledge/contributive communication</th></tr>
<tr><td>TT30 (Teacher's role) – The teacher is the one who validates ideas raised in class, questioning students, whose replies lead to self-correction (in reality veiled correction, stage-managed by the teacher).</td><td>T holds a dialogue with the group, and contributively revises the difference between the relative lengths of the sides of squares and rectangles (246-260).</td><td>KCS (Knowledge of Content and Students) The teacher considers that the students would have difficulties in considering squares as specific cases of rectangles (246-250), (254-256).</td><td rowspan="2">CCK (Common Content Knowledge) – Knowing the difference between squares and rectangles (in terms of the length of the sides).<br>KCT (Knowledge of Content and Teaching) – The teacher considers contributive dialogue appropriate for the revision of the difference between the length of the sides of squares and rectangles.</td></tr>
<tr><td>TR16/TT16 (Learning) – The student interacts with the material and the teacher, the latter being the mediator between material and student. The interaction produced between teacher and student is unequal, with the flow teacher-student being stronger than the contrary.</td><td>T holds a dialogue with the group, and contributively clarifies that, by virtue of its sides not all being the same length, the shape cannot be a square (261-268).</td><td>SCK (Specialized Content Knowledge) – The teacher gives evidence of an incorrect use of the classification of polygons (using a disjunctive classification implying that the set of squares is separate from that of rectangles).</td></tr>
</table>

**Table 2 – Relations between actions and cognitions with respect to the revision of the difference between squares and rectangles, in terms of the lengths of their sides, via a contibutibutive whole class dialogue.**

The actions of revising and clarifying the content are underpinned by beliefs related to the role of the teacher (TT30) and to the learning process (TT16). The cognitions identified show that the teacher regards herself as the only person with the capacity/ability to validate the information mobilised in class. In viewing her role in this way, she conditions the interactions between other elements of the process of learning, thus preventing a balance being reached among them and making it impossible to achieve a triangle of learning, as advocated by Pinto & Santos (2006). These actions/beliefs are linked to each other in such a way that together they form the basis of all revision episodes.

The knowledge identified, as well as the gaps in knowledge, are specific to the situation and the context, and so cannot be generalised, not even for this teacher.

**Possibilities for initial and in-service teacher training**

This type of analysis may also be of use in initial teacher training as the starting point for an approximation between theory and practice. It would mean that researchers and teachers "speak the same language", using the same codifications; in doing so, a great degree of collaboration is needed.

This type of analysis (by student teachers in their teaching practice), although based on the experience of others, may lead to an awareness of their own cognitions, of the way they relate and influence one another. This awareness would help the development of a critical, as opposed to submissive, attitude during their teaching practice; merely observing the mentors does not necessarily lead to learning (Brophy, 2004). It is important, then, that the time spent in schools by trainee teachers as observers or assistants should be given careful consideration and attention.

In the sphere of in-service training, this type of analysis can be effected by the teacher him or herself, who, in watching recordings of their lessons, will be able to reflect upon their own practice (Schön, 1983, 1987). This reflection, accompanied by discussion and critical exchanges with colleagues and researchers, can be considered a first step towards sustained professional development (Climent & Carrillo, 2003; Jaworski, 2006) aimed at improving professional competence through qualified professional reflection (Hospesová, Tichá & Machácková, 2007).

We selected content improvisations as the focus of our analysis because, when they occur, the teacher "is working without a safety-net". They are unforeseen situations not subject to advanced planning, and consequently all the teacher's cognitions come into play in their purest form, faithfully reflecting their mode of acting and their position with respect to the process and intervening elements. It will be in these situations that, in the initial stages of training programmes as in professional development, significant information can be obtained which can contribute to the development process, enriching discussion and leading to a self-awareness of one's professional attitude. These situations can permit access to what Tomás Ferreira (2005) terms 'teaching modes', underlining the relationships between their dominant classroom interaction, teacher's key beliefs and in this case, also their professional knowledge.

This analysis and understanding are very important now that there exists in Portugal a Programme of In-service Training in Mathematics for teachers of the 1st and 2nd cycles of Basic Education with a supervision component (Serrazina et al., 2005). One of the ways of achieving some of the goals of this programme – deepening teachers' mathematical, pedagogical and curricular knowledge and encouraging a positive attitude in teachers towards mathematics and the capabilities of the students – could involve the analysis and discussion of teachers' classes, through the use of this cognitive perspective and of the model.

## References

Aguirre, J. & Speer, N. (2000). Examining the relationship between beliefs and goals in teacher practice. *Journal of Mathematical Behaviour*, *18(3)*, 327-356.

Artz, A. & Thomas-Armour, E. (2002). *Becoming a reflective mathematics teacher: A guide for observations and self-assessment.* New Jersey: Lawrence Erlbaum Associates.

Ball, D. L. (2003). *What mathematical knowledge is needed for teaching mathematics (Adobe PDF).* Paper presented at the U.S. Department of Education, Secretary's Mathematics Summit, Washington, DC, February 6, 2003 (http://www-personal.umich.edu/~dball/presentations/index.html).

Ball, D. L., Thames, M. H. & Phelps, G. (submitted). Content knowledge for teaching: what makes it special? *Journal of Teacher Education*.

Brendefur, J. & Frykholm, J. (2000). Promoting mathematical communication in the classroom: two preservice teachers' conceptions and practices. *Journal of Mathematics Teacher Education*, *3*, 125-153.

Brophy, J. (2004). *Using Video in teacher education*. Amsterdam: Elsevier Ltd.

Carrillo, J., Climent, N., Gorgorió, N., Rojas, F. & Prat, M. (2008). Análisis de secuencias de aprendizaje matemático desde la perspectiva de la gestión de la participación. *Enseñanza de las Ciencias*, *26*(1), 67-76.

Climent, N. (2002). *El desarrollo profesional del maestro de Primaria respecto de la enseñanza de la matemática. Un estudio de caso.* Unpub.doctoral dissertation, University of Huelva (2005.Proquest Michigan University. www.proquest.co.uk).

Climent, N. & Carrillo, J. (2003). El dominio compartido de la investigación y el desarrollo profesional. Una experiencia en matematicas con maestras. *Enseñanza de las Ciencias, 21 (3)*, 387-404.

Hospesová, A., Tichá, M. & Macháčková, J. (2007). Differences and similarities in (qualified) pedagogical reflections. In D. Pitta-Pantazi & G. Philippou & e. al (Eds.), *Proceedings of the V Congress of the European Society for Research in Mathematics Education (CERME 5)*. Larnaca, Cyprus: University of Cyprus.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education, 9*(2), 187-211.

Monteiro, R. (2006). *La enseñanza de las ciências naturales desde el análisis cognitivo de la acción.* Unpub.Doctoral Dissertation, University of Huelva.

Monteiro, R., Carrillo, J. & Aguaded, S. (2008). Emergent theorisations in Modelling the Teaching of Two Science Teachers. *Research in Science Education, 38*(3), 301-319.

Park, S. & Oliver, J. S. (2008). Revisiting the Conceptualisation of Pedagogical Content Knowledge (PCK): PCK as a Conceptual Tool to Understand Teachers as Professionals. *Research in Sciences Education, 38*, 261–284.

Pinto, J. & Santos, L. (2006). *Modelos de avaliação das aprendizagens*. Lisboa: Universidade Aberta.

Saxe, G. (1991). *Culture and cognitive development: Studies in mathematical understanding*. Hillsdale: Lawrence Erlbaum Associates.

Schank, R. & Abelson, R. (1977). *Scripts, plans, goals, and understanding: An inquiry into the human knowledge structures*. Hillsdale: Lawrence Erlbaum Associates.

Schoenfeld, A. (1998a). On modelling teaching. *Issues in Education, 4(1),* 149 - 162.

Schoenfeld, A. (1998b). Toward a theory of teaching-in-context. *Issues in Education, 4(1)*, 1-94.

Schoenfeld, A. (2000). Models of the teaching process. *Journal of Mathematical Behaviour, 18(3)*, 243 - 261.

Schoenfeld, A., Ministrell, J. & Zee, E. V. (2000). The detailed analysis of an established teacher's non-traditional lesson. *Journal of Mathematical Behaviour, 18(3)*, 281 - 325.

Schön, D. (1983). *The reflective practitioner: How professionals think in action*. Nova York: Basic Books, Inc., Publishers.

Schön, D. (1987). *Educating the reflective practitioner: Toward a new design for teaching and learning in the professions*. San Francisco, CA: Jossey Bass.

Serrazina, L., Canavarro, A., Guerreiro, A., Rocha, I., Portela, J. & Saramago, M. J. (2005). Programa de Formação Contínua em Matemática para Professores do 1.º e 2.º Ciclos. *(unpub. document).*

Sherin, M., Sherin, B. & Madanes, R. (2000). Exploring Diverse Accounts of Teacher Knowledge. *Journal of Mathematical Behaviour, 18(3)*, 357 - 375.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15 (2)*, 4-14.

Tomás Ferreira, R. A. (2005). *Portuguese student teacher's evolving teaching modes: A modified teacher development experience.* Unpub. Doctoral Dissertation, Illinois State University, IL, USA.