

# **DEVELOPING MATHEMATICS TEACHERS' EDUCATION THROUGH PERSONAL REFLECTION AND COLLABORATIVE INQUIRY: WHICH KINDS OF TASKS?**

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*Abstract. After the reprise of a model of intervention for the training of mathematics teachers (both initial and in-service) developed after experiences carried out in a cooperative modality (Pesci, 2007a), several tasks are presented for encouraging the development of disciplinary, didactic, and relational competences of the teachers. The theoretical framework related to these tasks puts in evidence the reasons of their choice: the importance, for teachers, of collaboration in sharing personal experiences, difficulties, and resource, the importance of autobiographical reflection, of reflection on one's own classroom practices, and of epistemological reflection on the disciplinary contents. The connection to the debate about tasks which is developing considerably in relation to the education of teachers (Javorski, 2007) is underlined.*

Key-words: mathematics, teachers, cooperation, collaboration, tasks.

## **INTRODUCTION**

This paper has two goals, that of developing and specifying the model of intervention on teachers delineated in the contribution at Cerme5 (Pesci, 2007a) and that of explicitly connecting the model to some crucial ideas for the education of mathematics teachers which the literature is highlighting with growing intensity. How do I intend to reach the two goals? By supplying examples of tasks for teachers which, on the basis of the mathematical contents proposed, on the didactic modalities adopted and on the requested personal reflections, make evident their theoretical motivations and their connection to the debate delineated by Jaworski (2007) and synthesized by Watson and Mason, in the same special issue of JMTE. More specifically, this paper foresees a brief look back at the model of intervention on mathematics teachers already outlined (Pesci, 2007a) and the description of some tasks for teachers which have the goal of promoting personal reflection on their own relationship with mathematics and the encouraging of epistemological reflection on specific mathematical contents. Then there are the synthesis of the theoretical background for the choice of such tasks, with reference to related literature, and some suggestions for future research.

## **A MODEL OF INTERVENTION ON MATHEMATICS TEACHERS**

The main issues of the model described in Pesci (2007a) are summarized shortly in this section, with the aim to make evident the frame in which the following tasks

should be placed. The model was developed in the framework of situated cognition and distributed cognition:

The frame of reference is that of social constructivism, which emphasises discussion, negotiation of meanings, collaboration, and development of positive personal relationships (Ernest, 1995, Bauersfeld, 1995) and the concept of cognition is that formulated both as “situated cognition” (Nunez, 1999) with relevance to the context, and as “distributed cognition” (Crawford, 1997) with relevance to interrelationship and to sharing. (Pesci, 2007a, p. 1946).

The model was based also on cooperative modality, which gives special importance to relational and social aspects: in their different interpretations, all cooperative models share their explicit attention to both disciplinary dimension and social one. The goals to be reached along the educational process are not placed only at the disciplinary level but also at personal and social ones, with a special attention to the quality of the relationships established amongst people (Johnson, Johnson & Holubec, 1994, Cohen, 1984). At the base of the model (interpreted both for students and for teachers), there was, therefore, the idea of a co-construction of knowledge, a social construction, with the principles that for several decades, even with different accents, pervade the most diffuse teaching-learning models. At the centre of the learning process, managed by an expert, there are the learners and the inter-relationships (between learners and with the expert) with the consequent emphasis on the role of language and on the phases of discussion, argumentation, confutation, comparison, and sharing. What is suggested by teaching-learning cooperative models is also coherent to what is underlined by neuroscience (Damasio, 1999) and by epistemology (Polanyi, 1958): in each process of building or revisiting knowledge it is necessary, as a matter of fact, to keep track of the close connection between emotion, sentiment and cognition. This is valid not only for the students, in class, but also for the teachers, in their training meetings. In each training intervention, therefore, there was a special attention to the affective-relational aspects.

With reference to relational and social aspects, I consider essential that a meaningful intervention on mathematics teachers (a) could give time and space to their reality as teachers in that precise moment of their professional history through the autobiographical discourse; (b) could constitute a direct experience of what is proposed, with wide possibility of dialogue with the other participants; (c) could be, in each case, attentive to the modalities of communication. (Pesci, 2007a, p. 1952)

The main goal, in planning meetings for teachers, was to promote their personal reflection, taking account of disciplinary, didactic and relational aspects:

The basic idea is that of creating, in each encounter, occasions for personal reflection and for dialogic inquiry, with the same spirit stressed in the project *Learning Communities in Mathematics* (Jaworski, 2004), where the main objective is that both researchers and practitioners are engaged in action and reflection for mutual growing. (Pesci, 2007a, p. 1952).

The following tasks for mathematics teachers are examples of how it could be possible to foster their reflection and inquiry on the three different and essential aspects of their competence: *disciplinary*, *didactic* and *relational*.

## EXAMPLES OF TASKS FOR MATHEMATICS TEACHERS

**Autobiographical reflection.** Every time that it is possible, in particular when the training meeting foresees more than one session, I organize the initial phase with the teachers starting with their personal relationship with mathematics, both with reference to their own history as *student* and to their own history as *teacher*.

In the first case, I propose answering several written questions, which have to do with their recollection of a pleasant episode (and respectively an unpleasant one) during a mathematics lesson, referring to all of their pre-university scholastic life. Sometimes I turn to the request for an opportune metaphor, such as “to do mathematics was like entering a jungle, or a challenging game, or a long marathon, etc.”, described in Pesci (2006).

In the second case, the activity of reflection on one’s own “history” as a mathematics teacher can come about through a choice of metaphor or with the request to complete a questionnaire of this kind:

### *From my “history” as a teacher*

*An episode to remember*

*An episode to forget*

*A moment of change*

*A wish that came true*

*A wish that didn’t come true*

In both cases, the task, by its nature, is individual, but I usually invite the participants to share within their own group (of 4-5 people), if they want, the interpretation of the task or some experiences, both before writing and at the conclusion of the writing. The only recommendation is that, in each case, there is a period of silence, during which each person can collect his own thoughts and write calmly. To this aim, it is essential that, right from the beginning, each commits to observing the others attentively, being aware of when it is opportune to intervene with their own contribution or give space to the intervention of another person or remain silent.

The personal reflections which are asked for are of various natures and, obviously, depend a lot on the characteristics of the group itself. For example, in a group of teachers who have been in-service for several years, but were not yet confirmed, it came out that more than a third of the participants (there were about 60) highlighted, as a ‘wish not yet come true’ that of didactic continuity. It is clear that the same kind of wish does not appear anymore with teachers in regular service for years. It is not important, in this context, to list the different kinds of responses collected. Instead, it seems interesting to observe, at least, these two facts:

- the tasks of an autobiographical nature, followed by the sharing of personal experiences, have as a consequence to immediately orientate teachers' attention toward the other members of the group, reducing the attention which, at the beginning of the activity, everyone has toward the presenter of the training, and encouraging the perception of the others' resources, at the level of disciplinary competence and interpersonal qualities. When the activities are carried out together, it is, without a doubt, the most productive starting point;
- to put into play the one's own memories and one's own history is unusual but manages to capture the participants in an absorbing way: the result is a sort of requalification of the way of being present at the training event. Often I perceive in the teachers, also during the following activities, a less superficial, more meaningful, and more profound, involvement, as if the autobiographical connotation were able to give greater strength and authenticity to the actions that they share.

**Reflection on one's own classroom practice.** Amongst the tasks proposed to the teachers to encourage their reflection on their own classroom practice, I'll quickly cite two examples connected to two different kinds of experiences. During a cycle of seminars on how to confront the difficulties in mathematics, in the secondary school, it came out that all the participants (about 20) had already adopted specific strategies to help students overcome difficulties in mathematics. Therefore, I held it to be opportune to dedicate an entire meeting to the specific reflection on such strategies, inviting each one to respond to some questions, amongst which were the following:

*You have already adopted specific strategies to help your pupils overcome difficulties in mathematics: choose, in the case of several strategies, the one which you hold to have been the most effective and describe how you realized it in class, according to the following chart:*

- a) Strategy used
- b) With what frequency?
- c) With pupils of which classes?
- d) Briefly describe how you develop such strategy in class
- e) For which mathematical contents did you turn to such strategy?
- f) Which are, in your opinion, the strong points of such a strategy?
- g) Which are, in your opinion, the weak points of such a strategy?

Naturally, it was only the beginning of a longer path, certainly not exhausted in one meeting. Still, I noted that the participants were not used to reflecting on the methodology of their own practices, but they were almost exclusively worried about the mathematical content to develop in class. For example, it came out that whoever had tried to make the young people work in groups, had not structured the activity in any way, not foreseeing specific roles for the pupils and not planning sufficient time and adequate space for the activity. Even the mathematical questions were chosen without specific motivations. Analogously, whoever had proposed a learning experience of peer tutoring, had not programmed any form of collection of the work

carried out, neither for the pupil in the role of the teacher, nor for the one in the role of the pupil. Not having a clear idea that a key element for success, in these cases, is precisely the awareness of the importance of *setting*, they did not share with the pupils the methodology of the activity to be carried out and they did not put the right emphasis on it. The results, in fact, were not satisfactory.

In another case, following experiences conducted in classes with the cooperative learning modality, after a rather long period (more than a year), I had foreseen with the teachers specific instances of reflection on the perceived effects (positive or negative at the disciplinary or relational level) on the pupils and on themselves. Several questions and several results, which are not necessary to take up here, are described in detail in Pesci (2007a). Here, I would like to put in evidence some general observations, also in relation to what I noted during the seminars on the difficulty of learning cited before.

The modality that I put into effect with the teachers is usually that of sharing and discussion in small groups (4-5 people) before the general discussion and debate. I noted that this encourages, in a decisive way, the participation of everyone. Each one, in the small group, feels more welcome, safer, and freer therefore to express their own difficulties, their own fears, their own experiences and desires. Realizing that a fear (for example, that of not being up to maintaining control of the class) or a difficulty (for example, that of managing the time in class well) is common to others, gives greater strength to each one in the search for and sharing of the best strategies for confronting them. The requested reflections on the practices of the teachers go on to involve their acting in class and out of class and the sharing with colleagues shapes itself as an important occasion of comparison and growth. The relational competences of the teachers, specifically the ability to communicate with their colleagues, to share resources, and to confront together the obstacles has, without a doubt, a central role in the building of a team of prepared, reflective and able to change teachers (Dozza, 2006). In other words, it seems necessary to give time and space to such activity of personal reflection.

**Reflection on specific mathematical contents.** I will describe briefly two different situations as examples of the tasks proposed to the mathematics teachers for reflection on their teaching discipline. The first is more appropriate for a single intervention, which can be completed in one meeting and the second is more appropriate for starting a longer activity, which can be developed in successive meetings. Both tasks have the characteristic of a simple enough presentation, which makes the teachers curious and therefore easily involves them, but continuing on they become more complex. These tasks are therefore right to be confronted in collaboration, in the direction of the discovery of their didactic values and of the variety of the mathematical themes from which to choose possible developments. Besides, both the tasks could be proposed to the students, the first example starting

from the upper classes of the primary school and the second starting from the secondary school.

The first problem is placed in  $ZxZ$  (the “pointed” plane) and proposes a search for isosceles triangles with the oblique side assigned  $AB$ , limited to those with all three vertices in points of  $ZxZ$ . The investigation, apparently very simple, proves to be quite demanding, both for the geometric questions and the arithmetic questions involved. Besides, it can be developed with questions of isoperimetry, of equiextension, and of congruence between the triangles found, going on to weave together, in a single context, the use of arithmetic and geometric competences and of argumentative and demonstrative procedures. It is evident, therefore, that also the discussion about the didactic value of the problem turns out to be quite full and interesting.

The second problematic context looks at Euler’s formula and its validity; to be explored in several models proposed concretely or drawn on the blackboard. It is well known that in simpler cases, for example for regular polyhedra or for convex polyhedra, it is easy to count *faces*, *corners*, and *vertices* and immediately to verify the validity of the well known numerical relationship  $V - S + F = 2$ . In more complex situations, instead, one encounters some difficulties. It is necessary to clarify, on the one hand, which are the figures in the space that can be considered “polyhedra”, and on the other hand, which are the elements in the space that can be considered “faces” or “vertices” or “corners”. The two questions are obviously connected and it is well known how much they are not banal, as is highlighted by the historic reconstruction of the attempts to demonstrate Euler’s formula described in Lakatos’ book (1976). When this activity is proposed to the teachers, it usually turns out to be evident how it is right for encouraging collaboration and the sharing of resources. It has to do, in fact, with an investigation that is not taken for granted, with an obligatory end, but rather open to further reflections, of a theoretical type or also an epistemological one (Pesci, 2007b).

## **THEORETICAL FRAMEWORK FOR THE CHOICE OF TASKS**

In this section there are the basic ideas which constitute the theoretical framework for the choice of the kinds of tasks described.

a) On the cooperative methodology to put into effect with the teachers, I have already described the theoretical references in the second section of this presentation. Here, I would add some reflections which could clarify better the features of the model proposed. It is important to remember that, in general, when one speaks of the shared principles of the models of social construction of knowledge, one has not yet arrived at outlining a standard didactical procedure, because for this it is necessary to choose the fundamental values which one intends to promote. As Ernest (1995) observes, standard didactical procedure is defined in each case on the basis of the *values* which one intends to promote. To define better the model of intervention experimented with

mathematics teachers, I would like to stress that the fundamental *value* that I chose to promote is the *collaboration* amongst all the participants (teachers and didacticians) at the educational moment. The goal is that of more easily arriving together at a higher result than that which each one could reach alone, whatever the proposed task could be. The term *collaboration*, here, could be interpreted as a synonymous of *cooperation* in reference to the fact of sharing the urgency to develop, in a symmetric way, both the cognitive-disciplinary and the affective-relational competences of the subjects. But here the term *collaboration* has a more general meaning: a positive inter-relationship amongst the people involved, not necessarily connected to a specific modality of acting in groups. The *collaboration* amongst the participants (teachers and didacticians) has the following goals: to encourage the sharing of personal experiences, of resources, of difficulties, and to encourage reflection on the mathematical contents, on their epistemological meaning, on their classroom practice, and on their own professional history. In short, the *collaboration* with peers, interpreted at the level of teachers, seems the most efficient road for covering the role of teacher, which lies within the competence in projecting the educational path and the reflection-evaluation of the processes activated.

I would like to add one last characteristic of this model. The interaction between equals, in a climate of positive *collaboration*, implies a particular *setting*, that is the organization of time, space, and modes of interaction which allow the progressive evolution of the disciplinary and relational competences. All that is a privileged environment also for the well-being and for the mental health of the participants (Dozza, 2006). Trust in oneself, generosity in the welcoming and helping of the others and the recognition of oneself in the others, contribute to affirming and enriching one's own identity in the community to which one belongs, supporting the development of personal potentialities.

b) Autobiographical reflection, by means of the use of metaphors or narrations of meaningful episodes from one's life, turns out to be a preferred tool for accessing the deepest parts of self, allowing that decentralization which is necessary to be able to tell about oneself (Barker, 1987; Darrault-Harris & Klein, 1993). The narration of self was rediscovered in the last 10-15 years as an educational modality which is important for both students and teachers (the first direct references to the autobiographical practice in adults' education can be found in French studies, i.e. Pineau, 1983, the Italian studies have been developed mostly starting with Demetrio, 1996). Amongst the objectives that can be pursued, there is fundamentally the reflection on one's own experience, in particular, on its attributive implications and on the causal links to the events of one's history. This allows the recognition that the narration of oneself is not a simple report of events, but rather a reinterpretation of them, in the light of the present. Telling about self means giving meaning, coherence, and continuity to one's various experiences and also encourages the definition or the reformulation of one's identity. Autobiographical reflection, elaborated for oneself, but also communicated to and shared with others, encourages a positive development

of interpersonal communication, the recognition and re-evaluation of personal facts and characteristics, the ability to listen to oneself and understand oneself, and a consequent openness to listening to and welcoming of others. So, it seems that autobiographical activity emerges as a fundamental tool in the work with teachers, a work which has at its centre the teachers in their totality, personal and professional at the same time.

c) The tasks of the disciplinary type proposed in the preceding section are, on the basis of the experiences carried out, particularly appropriate for developing epistemological reflection on mathematics in an *inquiry* style (Javorski, 2004), in a climate of investigation of mathematics which could be transferred to the class. With reference to this I would like to link to a question proposed by Watson and Mason (2007, p. 213).

We question whether tasks need to be structured in ways which require ‘inquiry’ or whether instead ‘inquiry’ is the mindset with which teachers, and ultimately their students, need to approach all tasks.

I would say that both things are necessary. A task must be interesting enough to stimulate involvement and action. It must be open enough, that is, appropriate to being developable in several ways and therefore with personalized in-depth study. In other words, the task has to be *generative* of several different possibilities of development (as Borasi well described in the 21 examples showed in detail in her book, 1996). Besides, the structuring of the environment in which the task is proposed must be adequate, in the sense that it must foresee times, materials, and attitudes which can fully support the investigative activity. In other words, the *milieu* (Brousseau, 1997), in which a task and the following activity take place, has to be suitable for the intended work. It is still evident that also the attitudes of the participants in the investigation must be appropriate, that is, ready to participate in the activity, allowing themselves to be involved in the problem and putting into play their own time and their own resources. The two aspects (the characteristics of the task and the attitude of the one who confronts it) turn out to be, in my opinion, strongly intertwined and they influence each other in turn. A task which does not have the characteristics cited cannot give rise to *inquiry* and on the other hand an appropriate task, proposed in an unprepared *milieu* for the inquiry, will not be developed and unlikely will not become object of research.

d) The last observation that I would like to propose is relative to the general sense of a training experience proposed to the teachers, with the modalities and by means of the tasks described. As shown also by the analysis conducted by Watson and Mason (2007, p. 208):

Tasks are often designed so that teachers can experience for themselves at their own level something of what their learners might experience and hence become more sensitive to their learners. The fundamental issue in working with teachers is to resonate with their experience so that they can imagine themselves ‘doing something’ in their own situation,

through having particularised a general strategy for themselves ... their professional choices of actions are the manifestation of what they have learned or are learning.

It is precisely in this direction that I develop each intervention on the teachers. I am convinced that a training meeting can be effective in the measure in which it can be set up, for the participants, as *metaphor* of experiences of living in class; a *metaphor* therefore understood not as verbal construction, but as life experience (Pesci, 2003, 2005, 2006, Fabbri & Munari, 2000).

## CONCLUSION

The model of intervention on teachers and the tasks here described put an explicit accent on the necessity to intertwine *disciplinary*, *methodological* and *relational* aspects for teachers' professional preparation, without leaving out a special care for structuring an adequate *setting* for the intervention itself. A theoretical frame for this complexity can not be simple and, of course, it could be different from that here described. It could be the occasion for further investigation and analyses, for instance in the direction: a) to formulate different models which could describe the same complex "scenario" of mathematics teachers' professional education; b) to elaborate specific and adequate instruments of analysis of teachers' interaction, at the different levels of competences involved by the model proposed. A final observation refers to the importance the model puts on the necessity to take account of teachers' personal biographies (their personal stories, their preferences, their expectations). I believe this is a feature not yet explored in depth for teacher education (see for instance the review about the common assumptions related to mathematical tasks in teacher education in Watson & Mason, 2007). Such orientation could be of interest for research, with possible fruitful resonance from perspectives of teachers' educators.

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