PROBLEM POSING AND DEVELOPMENT OF PEDAGOGICAL CONTENT KNOWLEDGE IN PRE-SERVICE TEACHER TRAINING

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The paper focuses on problem posing as the possible method leading to development of pedagogical content knowledge of mathematics education in pre-service training of primary school teachers. In the background there is our belief that this knowledge is of utter importance for quality of the education process. Using samples of (a) problems posed by teacher students, (b) students' assessment of the problems posed, (c) students' opinions on the significance of "problem posing" in teacher training, we will demonstrate how we employed problem posing in pre-service teacher training. We start from the belief (proved in our previous work) that an analysis of the posed problems is a good diagnostic tool; it gives the opportunity to discover the level of understanding as well as the causes of misconceptions and errors.

Keywords: mathematics education, teacher training, content knowledge, problem posing

INTRODUCTORY REMARKS: MATHEMATISATION OF THE SOCIETY AND MATHEMATICAL LITERACY

On many different occasions we come across the signs of an increasing importance of mathematics in contemporary life, the opinion that the society is being "mathematised". We must understand mathematics if we are to be able to understand the world that surrounds us. That is why the need of mathematical literacy is more and more emphasized. These trends also impact the focus of the research in the field of didactics of mathematics (e.g. the central topic of PME 30 conference in 2006 was "Mathematics in the centre").

We understand mathematical literacy as functional. It begins with the ability to understand a mathematical text, the ability to recall mathematical terms, procedures and theory, to master the necessary mathematical apparatus and with the ability to apply it, to solve problems. However, in our view to be mathematically literate also means to "understand mathematics", to perceive it as an abstract discipline. Development of mathematical literacy triggers perfection of the ability to reason, of critical thinking, it teaches how to apply mathematics efficiently. To be functionally mathematically literate means to see the mathematics that surrounds us; to see the questions and problems arising both from real and mathematical situations. In order to educate mathematically literate pupils we need professionally competent teachers.

In our previous work we have been focusing on the potential of a qualified pedagogical reflection and we have showed that it is one of the possible ways of development of professional competence of primary school teachers (Tichá, Hošpesová, 2006). In this paper we show that problem posing represents another possible way. We also show the potential of problem posing in diagnosis of the teacher-students' subject didactic knowledge.

THEORETICAL FRAMEWORK

Professional competence and content knowledge

The calls for development of mathematical literacy make demands on professional competences of the teacher. In our previous research, especially the need for a good level of *subject didactic competence* appeared very strongly, i.e. the knowledge of mathematical content and its didactic elaboration as well as its realization in school practice (Tichá, Hošpesová, 2006). It corresponds with the following generally accepted Shulman's idea: if teaching should become a profession, it is necessary to aim at creating a *knowledge base for teaching* which encapsulates, in particular, *subject-matter content knowledge, pedagogical content knowledge, and curriculum knowledge* (Shulman, 1986). It is the knowledge of mathematical content that most authors place in prominent positions on their lists of items of knowledge required from teachers (e.g. Bromme, 1994; Harel, Kien, 2004). The need of solid niveau of subject didactic competence is extremely demanding for primary school teachers. Especially if we realize that the content of mathematical education at primary school level is a system of propaedeutic to many fields (arithmetic, algebra, geometry, …, functions, statistics, …). Yet these teachers are not specialists in the subject – on the contrary, they must master many more subjects than mathematics.

What is often emphasized is the need to create an "amalgam" of the components of the teacher's education. "The two basic elements of teacher knowledge are mathematics and pedagogical knowledge. When these two elements are separated and remain at a general level, mathematics teaching does not share the characteristics of ... a good teaching. The blending of mathematics and pedagogy is necessary for developing mathematics knowledge for teaching." (Potari et al., 2007, p. 1962). In other words "... mathematical experiences and pedagogical experiences cannot be two distinct forms of knowledge in teacher education." (Potari et al., 2007, p. 1963).

Problem posing as a way to refinement of competences

Opinions on the employment of problem posing

Our existing experience indicates that one of the beneficial ways of improving subject didactic competences of pre-service teachers of mathematics is development of the ability to pose problems (and the related activities). Already Freudenthal and Polya emphasize the significance of activities aiming at problem posing as a part of mathematics training. The same need is referred to by many others (Silver, Cai, 1996; English, 1997; Pittalis et al., 2004 etc.). Apart from "problem solving" (in the sense of "learning mathematics on the basis and through problem solving") they emphasize the need and significance of development of the ability to pose problems. There is an agreement among many authors that "problem formulating should be viewed not only as a goal of instruction but also as a means of instruction. The experience of discovering and creating one's own mathematics problems ought to be a part of every student's mathematics education" (Kilpatrick, 1987, p. 123).

Teacher educators show and stress links between problem posing and problem solving, and problem posing and mathematical literacy (competence). That is why stress is on the inclusion of activities in which students generate their own problems in mathematics education. At the same time most literature points out that the treatment of issues regarding problem posing has by no means been satisfactory so far. For example, Christou *et al*. (2005) bring forward the fact that "little is known about the nature of the underlying thinking processes that constitute problem posing and schemes through which students' mathematical problem posing can be analysed and assessed" (p. 150). And Crespo (2003, p. 267) adds "... while a lot of attention has been focused on teacher candidate's own ability to solve mathematical problems, little attention has been paid to their ability to construct and pose mathematical problems to their pupils."

Problem posing in the frame of grasping of situations

We started to pursue the issue of problem posing while studying the process of grasping situations (Koman & Tichá, 1998). What we understand by grasping situations is the search for questions and problems growing from a mathematical or "non-mathematical" situation, i.e. also problem posing. We define problem posing similarly to a number of other teacher educators as (a) creation of new problems or (b) re-formulation of a given problem, e.g. by "loosening the parameters of the problem" (by modifying the input conditions), by generalization, on the basis of the question "What if (not)?", etc. The problem may be worded or re-worded either before its solution or during the solving process or after it. We perceive the process of problem solving as a dialogue of the solver with the problem, we ask: How to begin? How to continue at the point reached? The solver reacts to the "behaviour, response of the problem", chooses a particular strategy, creates an easier problem, changes the conditions of the assignment to be able to continue.

Our experience from work with teacher students (and also with 10-15 year old students) confirms that their effort to pose problems guides them to deeper understanding of mathematical concepts and development of their mathematical and general literacy. Problem posing enriches both the teaching and the learning.

TEACHER STUDENTS AND PROBLEM POSING (INVESTIGATION)

The focus of the investigation: goals and questions

In our ongoing research we look for the ways leading to development and refinement of professional competences of both teacher students and in-service teachers. We try to show if and to what extent “problem posing” and “the level of subject didactical competence” and also “mathematical knowledge” influence each other, i.e. in presently research we look for answers to the following questions: How rich knowledge base (general as well as specific, mathematical) is needed for proficiency in problem posing? How does systematic application of problem posing contribute to development of subject didactical competence / mathematical knowledge?

The topic of the investigation: translation between representations of fractions

We believe that problem posing can be regarded as a translation between representations e.g. as posing problems that correspond to a given calculation (Silver, 1994). The incentive to this focus was investigations that confirm the great significance of utilization of various modes of representation for the development and deepening of the level of understanding. Many authors (see e.g. Janvier, 1987; Tichá, 2003) stress that the level of understanding is related to the continuous enrichment of a set of representations and emphasize the development of the student’s capability of translation between various modes of representation.

One of the key topics of mathematical education in primary school is the foundation of the base for understanding the relations between a part and the whole. In the process of division of the whole into equal parts, the preconception of the concept of fraction is formed. The concept of fraction is one of the most difficult concepts in mathematics education at primary school level. The subject mater is difficult not only for pupils and teacher students but often also for in-service teachers who face problems regarding both the mathematical content and its didactic treatment. That is why we paid so much attention to this topic in teacher training. The core of our work lay in the construction of the concept of fractions and in posing problems with fractions. We focused on formation of preconceptions and intuitive perception of fractions, on problem solving and the potential of problem posing.

The procedure and findings of the investigation

The stress in the course of didactics of mathematics for primary school teacher students was continuously on problem posing, thus on the development of the students’ proficiency in problem posing (the seminar was attended by 24 teacher students). One of the components of the work in the course was realization of an investigation whose aim was to show the students that problem posing can also be employed as a diagnostic tool, thus which on the basis of the problems posed it is possible to investigate the level of understanding as well as the obstacles in understanding and misconceptions.

The investigation was carried out in several steps: posing problems corresponding to a given calculation; individual reflection on the posed problems; joint reflection on a chosen set of the posed problems; evaluation of the activity “problem posing”.

Posing problems corresponding to a given calculation

The students were assigned the task**:** *Pose and record such three word-problems to whose solution it is sufficient to calculate* 1/4 • 2/3.

The problems were posed during work within one of the last seminars. What is satisfactory is the immediate finding that problem posing competence can be developed in appropriate conditions; teacher students who attended the course in which stress was put on the development of proficiency in problem posing were able to pose several problems. On the contrary students who came in contact with problem posing more or less haphazardly were not able to pose any problems if asked to do so. Some of the latter even did not understand what the point of the activity was and refused to pose any problems – in their opinion they should only solve such problems that were assigned to them and had been formulated by somebody else. The same can be observed in mathematics education at schools.

Reflection on the posed problems

A database of the posed problems was formed (without giving the author's name); each of the participants had access to the database. The participants of the course assessed the suitability and correctness of the posed problems that they had chosen themselves.

Then the lecturer selected a triplet of problems posed by one student. This triplet was then assessed and analyzed by all participants (the lecturer found this triplet of problems very interesting and asked their author for permission to use them in the subsequent work). The following step was joint reflection; joint assessment of individual problems, comparison and justification of opinions.

The following triplet of problems was chosen

1. *There was 2/3 of the cake on the table. David ate 1/4 of the 2/3 of the cake. How much cake was left?*
2. *There was 2/3 kg of oranges on the table. Veronika ate 1/4 kg. How many oranges remained (kg)?*
3. *The glass was full to 2/3. Gabriel drank 1/4. What part of the glass remained full?*

In advance, the lecturer went through the problems with their author. It was only in this dialogue that the student began to consider correctness of the posed problems. (It is interesting that all students began to ponder over correctness of the posed problems only after being asked to do so. However, to our gratification the students generally found and corrected their mistakes themselves.) Let us quote an extract from the dialogue between the student (S) and the lecturer (L).

S: Here (*she points at problems 2 and 3*) I don't count a part of something, I reduce, take away. ... Actually I don't know what I meant by it.

L: What could you have meant?

S: Something like this (*she sketches an illustration – a circle*) – I divide in into quarters and take away one. But, somebody could understand it that he drank a quarter of the glass. Well, I posed only one correctly. ... I should have checked.

L: How would you have checked?

S: Well, it seems I should have calculated it somehow. Or have somebody else to calculate it. Somebody who is better at it.

Samples of student assessment of the triplet of the posed problems

The **third problem** can be, according to some students, accepted on the condition that its wording is modified / supplemented; the given wording is regarded by many as confusing. However, the students only stated that it was confusing, they did not specify why or where.

The **first problem** was evaluated by a majority of the students positively. But the arguments of some of the evaluators reveal misconceptions: *If we have 2/3 of a cake, we can eat ¼, but the denominators do not equate. If he ate 1/3 out of the 2/3, then they would. It would be possible in real life but it is not correct mathematically.*

This statement was illustrated by a picture (Fig. 1) and by the word problem: *There are 1/4 of all pupils present in class A today and 2/3 of all pupils present in class B. If we multiply the number of pupils from both classes present today, what will the result be?*

Another student wrote and claimed: *The problem is correct. David ate 1/4 out of 2/3 of a cake ... = 1/4 • 2/3 = 1/6 of the cake.*
However, the student supplemented his statement with a picture (Fig. 2) that testifies his wrong interpretation of the whole (1/4 and 2/3 out of the same whole).

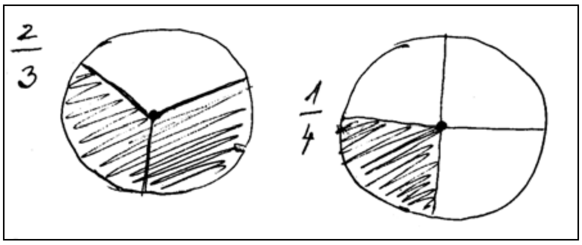

Fig. 1

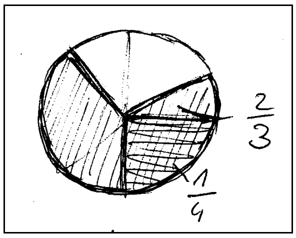

Fig. 2

When assessing the **second problem**, the students stated that this problem did not meet the condition from the assignment. However, their justification reveals that the conceptions of the evaluators themselves are also incorrect. Several illustrating examples of such evaluation follow.

- *Problem 2 is incorrect. There was 2/3 kg of oranges = 2/3 out of one (out of 3/3). Veronika ate 1/4 kg – but out of what? Out of 2/3? of 1/3?*

- *Number two is incorrect. From the total 2/3 kg of oranges, she ate 1/4 kg. She ate 1/4 but it does not say out of what.*

- *The second word problem isn't correct; it's not a suitable problem. I am not* *interested in the number of oranges but their weight. This wording would require that the oranges should be cut to pieces.*

What does the students' production show?

The subsequent joint reflection on the posed problems was of utmost benefit both to the participants and the lecturers. It enabled the students to become aware of their own weaknesses and it pointed to the teacher educators what they should focus on. Some of the findings follow.

The individual assessment and especially the following joint reflection show that many students do not have any idea of "what is in the background" of a particular simple calculation that they perform mechanically. They are not able to place it into a specific real life context. They did not pose problems in accord with the given calculation (what become transparent here are obstacles as far as multiplicative structure is concerned). A considerable proportion of the students posed additive problems corresponding with the calculation 1/4 + 2/3.

What comes to surface is the students' difficulty as far as interpretation of fractions is concerned. The offered formulations show that when assessing the second problem they for example do not realize that they understand and interpret the fraction alternately as an operator and as quantity *"she ate 1/4 kg"* vs. *"She ate 1/4 but it does not say out of what."*).

If the students were asked to pose more than one problem, we could observe stereotypical nature of these problems. Students often set their problems either only into discreet space (sets consisting of isolated elements) or only into continuous space. We could also observe a monotony of the motives: marbles and cakes (those are the models most often used in our textbooks).

What do the students think of problem posing?

The students were also asked to express their opinion on these, for them often unusual, activities. Let us present here several statements from individual reflections which illustrate how the students perceive "problem posing".

- *I have problems with word problems. To pose a word problem on my own ,…, was extremely difficult. The difficulty is not in posing a problem, but in being able to solve it myself. It was toil and moil for me.*
- *What I personally found most difficult was to ask the question correctly, when I posed the third one, I could think of no further questions and that's why I only managed to pose the most banal ones.*
- *As soon as I came to understand the assignment of this task, I was immediately full of various ideas ... I was delighted because I love discovery … that there were no limits.*
- *My first reaction was that of fear. However, I started from what first came to my mind – a simple problem and then I began to toy with it. It was very pleasant to look*

for and discover various combinations...

In the discussion the students indicated that it was easy to formulate a great number of problems of the same type but it was difficult to formulate a sequence of problems (cascade) of a growing difficulty or a problem for whose solution it was necessary to connect various pieces of knowledge or problems in which the role of the fraction alternates (i.e. various sub-constructs of fractions, …, Behr et al., 1983).

CONCLUDING REMARKS ON THE BENEFIT OF PROBLEM POSING AND ON THE PERFORMED INVESTIGATION

Our experience from work with teacher students (and also from our long-term cooperation with in-service teachers) proves that poor level of pre-service mathematical training is pervasive and the flaws are difficult to overcome (Hošpesová & Tichá, 2005; Hošpesová *et al*., 2007). Problem posing is in our opinion one of the beneficial possibilities.

The detection of a change in the “nature, climate” of work in the seminar

It seems to us that problem posing contributed to a change in approach to work in the seminar – the students gradually overcame their fears or anxiety and many of them gained self-confidence.

The character of the problems posed by the participants also changed. Before their participation in the seminar they posed simple, “textbook-like” problems, predominantly drill. The wording of the problems was often erroneous and the problems were uninteresting and demotivating from mathematical point of view. Many of the problems had no solution, despite the author’s intention.

After the course finished, a great variety of assignments of the problems could be observed (including charts, graphs etc.). There were also problems enabling different ways of solution and problems demanding explanation, reasoning, argumentation, allowing different answers with respect to the solver’s interests.

It turned out that it is not enough to demand from the students to pose a problem if one is to detect the quality of their understanding. It is crucial that it should be possible to assess the posed problems individually and/or collectively. This certifies the need to carry out joint reflection. If the authors are given the chance to assess the problems of each other, their insight into the situation deepens and the ability to handle reality, i.e. to “see mathematics in the world surrounding us” develops.

The benefit for students

The analysis of the posed problems makes the participants map the level of their own notions and concepts, understanding, various interpretations and makes them realize possible misconceptions and erroneous reasoning. It is an impulse for work on themselves (reeducation).

It was confirmed that the result of inclusion of problem posing into the curricula is better approach to problem solving. It stimulates the use of various representations, construction of knowledge nets, development of creative thinking, improvement of attitude to mathematics and increase in self-confidence.

The benefit for teacher educators and researchers

From the point of view of teacher trainers and researchers problem posing provides an opportunity to get an insight into natural differentiation of students' understanding of mathematical concepts and processes and to find obstacles in understanding and misunderstandings that already exist.

Our belief that problem posing supplemented with reflection is the path to development and enhancement of subject didactical competence, i.e. of pedagogical content knowledge was confirmed.

Open questions

There still exist many questions which ask for deeper investigations, e. g. How can be the benefit that problem posing brings to its authors and the shift in their (pedagogical) content knowledge detected and measured? Which teacher's and/or student's competences are developed? What conditions are essential for introduction of problem posing? What help and guidance can be offered when incorporating problem posing?

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REFERENCES

Behr, M., Lesh, R., Post, T. & Silver, E.A. (1983). Rational number concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of Mathematics Concepts and* processes (pp. 91-125). New York: Academic Press.

Bromme, R. (1994). Beyond subject matter: A psychological topology of teachers' professional knowledge. In R. Biehler *et al*. (Eds.). *Didactics of mathematics as a scientific discipline* (pp. 73-88). Dordrecht: Kluwer Academic Publishers.

Crespo, S. (2003). Learning to pose mathematical problems: exploring changes in preservice teachers' practices. *Educational Studies in Mathematics, 52*, 243-270.

Christou, C., Mousolides, N., Pittalis, M., Pitta-Pantazi, D. & Shiraman, B. (2005). An empirical taxonomy of problem posing processes. *ZDM, 37(3*), 149-158.

English, L. D. (1997). The development of fifth-grade children's problem-posing abilities. *Educational Studies in Mathematics, 34*, 183-217.

Harel, G. & Kien, H. L. (2004). Mathematics teachers' knowledge base: preliminary results. In M. J. Hoines & A. J. Bishop (Eds.), *Proceedings of 28th PME* (pp. 25-32). Bergen, Norway.

Hošpesová, A. & Tichá, M. (2005). Developing mathematics teacher's competence. In M. Bosch (Ed), *Proceedings CERME 4* (CD ROM) (pp. 1483-1493). Barcelona: FundEmi IQS.

Hošpesová, A., Tichá, M. & Macháčková, J. (2007). Differences and similarities in (qualified) pedagogical reflection. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings CERME* 5 (CD ROM) (pp. 1906-1915). Larnaca: University of Cyprus.

Janvier, C. (1987). Representation and understanding: the notion of function as an example. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 67-71). Hillsdalle: Lawrence Erlbaum Associates.

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123-147). Hillsdale: Lawrence Erlbaum Associates.

Koman, M. & Tichá, M. (1998). On travelling together and sharing expenses. In D. Burghes (Ed.), *Teaching mathematics and its applications* (pp. 117-122). Oxford University Press.

Pittalis, M., Christou, C., Mousolides, N. & Pitta-Pantazi, D. (2004). A structural model for problem posing. In M. J. Hoines & A. J. Bishop (Eds.), *Proceedings of 28th PME* (pp. 49-56). Bergen, Norway.

Potari, D., Zacharides, T., Christou, C., Kyriazis, G. & Pitta-Pantazi, D. (2007). Teachers' mathematical knowledge and pedagogical practices in the teaching of derivative. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings CERME* 5 (pp. 1955-1964). Larnaca: University of Cyprus.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*, 2, 4-14.

Silver, E.A. & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27(5*), 521-539.

Silver, E.A. (1994). On mathematical problem posing. *For the Learning of Mathematics*, *14(1*), 19-28.

Tichá, M. (2003). Following the path of discovering fractions. In J. Novotná (Ed.), *Proceedings of SEMT '03* (pp. 17-27). Praha : UK PedF.

Tichá, M. & Hošpesová, A. (2006). Qualified Pedagogical Reflection as a Way to Improve Mathematics Education. *Journal for Mathematics Teachers Education. Special Issue: Inter-Relating Theory and Practice in Mathematics Teacher Education.* Springer Netherlands, 9, 2, 129-156.