# MATHEMATICS TEACHER EDUCATION RESEARCH AND PRACTICE: RESEARCHING INSIDE THE *MICA* PROGRAM

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*The paper describes an ongoing collaborative work between department of mathematics and department of pre-service teacher education, aimed at connecting research and practice in the development and study of mathematics teacher education. The work draws from learning experiences of future teachers through the designing and implementing Learning Objects in department of mathematics. The focus of research is to address the need for a better understanding of how future teachers of secondary school mathematics are shaped by didactic-sensitive activities during their undergraduate mathematics education.*

Keywords: *future teachers; mathematics needed for teaching; research and practice; innovative undergraduate mathematics program, mathematics teacher education*

## Introduction

In their preface to a special issue of *Educational Studies in Mathematics*, titled "Connecting Research, Practice and Theory in the Development and Study of Mathematics Education," Even and Ball (2003) highlighted the need for addressing the gap between theory and practice, the divide between mathematics and mathematics education, and the divide between mathematicians and mathematics educators in the study of mathematics education. As they noted, there are emerging efforts to build collaborations and connections focused on the issues of practice in order to develop and study mathematics education. It is this sort of sensitivity to building connections and collaboration in addressing issues of practice and research that underpins our research. The central focus of our research is to address the need for a better understanding of how future teachers of secondary school mathematics are shaped by didactic-sensitive learning experiences during their undergraduate mathematics education (Mgombelo & Buteau, 2008a, 2008b). The research draws from learning experiences of future teachers in a non-traditional core undergraduate mathematics program called "Mathematics Integrated with Computers and Applications" (MICA) (Ben-el-Mechaiekh, Buteau, & Ralph, 2007; Ralph 2001). Among other things, MICA, launched at our institution in 2001, integrates computer, applications and modeling where students make extensive use of technology in ways that support their growth in mathematics (Ralph & Pead, 2006). Previous work describing MICA student learning experiences is reported in Muller and Buteau (2006); Buteau and Muller (2006); and Muller et al. (in press). Our focus in this paper is to describe our ongoing collaborative work aimed at connecting research and practice in the development and study of mathematics teacher education.

The rationale for our research is based on epistemological and practical grounds.

Mathematics teacher education is premised on the assumption that one has to be educated in mathematics in order to be able to teach it. This assumption highlights the well know problem of divide in mathematics teacher education between mathematics and teaching. From an epistemological perspective, the question is how mathematics and teaching could be integrated in mathematics teacher education. An initial characterization of this integration comes from Shulman's (1986) work on pedagogical content knowledge. Recently, Ball and Bass (2002) elaborated on pedagogical content knowledge and used the term *mathematics knowledge for teaching* to capture the complex relationship between mathematics content knowledge and teaching. This is the epistemological ground for our research.

In practice, any mathematics teacher education program has to contend with questions of how much mathematics and how much method or educational study should comprise such programs, and then whether and how these programs should integrate or separate out opportunities to learn mathematics and teaching (Adler & Davis, 2006). Answers to these questions are reflected in a wide spectrum of variations of programs, opportunities, and learning activities for future teachers (Mgombelo et al. 2006). In addition, there are also lessons from mathematics teacher education research and practice. With regard to secondary school teacher education, many teachers still struggle with teaching school mathematics for understanding even though their knowledge of mathematics may be adequate (Kinach, 2002). This points to mathematics needed for teaching.

Following Ball and Bass's (2002) work on mathematics for teaching there has been recognition that mathematics teacher education is an important area of study in departments of mathematics (Conference Board of Mathematical Sciences [CBMS], 2001; Davis & Simmt, 2005). For example, the 2001 report from the CBMS on "The Mathematical Education of Teachers" has two main recommendations for ways in which mathematics departments can attain these goals:

> First, the content and teaching of core mathematics major courses can be redesigned to help future teachers make insightful connections between the advanced mathematics they are learning and the high school mathematics they will be teaching. Second, mathematics departments can support the design, development, and offering of a capstone course sequence for teachers in which conceptual difficulties, fundamental ideas, and techniques of high school mathematics are examined from an advanced standpoint (p.123).

It is with this sort of understanding that some departments of mathematics have made ongoing and emerging attempts to reform their programs to provide meaningful experiences for future teachers (Bednarz 2001; CMS 2003; Muller & Buteau 2006; Pesonen & Malvera 2000). This points to the need for research to investigate whether and how these attempts impact future teachers' learning of mathematics needed for teaching (Bednarz 2001). More importantly, as we noted earlier, for this research to be meaningful and productive, collaboration among mathematicians and mathematics educators is crucial (Even & Ball, 2003; Mgombelo & Buteau, 2006). We are addressing this need for research and collaboration in our research. We, a mathematician and a mathematics educator, are interested in collaboratively extending our understanding of how future teachers of secondary school mathematics are shaped by their experience of designing so-called *Learning Objects* in the MICA program. In the following section we describe the MICA program and what we learned from reflections on practice regarding the students' learning experiences.

**Learning from Practice: The MICA experience**

In 2001, our institution launched its innovative core undergraduate MICA program based on guiding principles (a) to encourage student's creativity and intellectual independence, and (b) to develop mathematical concepts hand in hand with computers and applications. MICA also strives to strengthen the concurrent mathematics teacher education program. It exposes future teachers to a broad range of mathematical experiences rather than to a deep concentration in one or two areas. Future teachers also make extensive use of different software programs such as Maple, *Journey Through Calculus* (Ralph, 1999), Geometer's SketchPad, and Minitab, all of which nurture the understanding of mathematics.

In addition to a revision of all the traditional courses under the above-mentioned guiding principles, three innovative, core project-based courses, called MICA I - III, were introduced in which all students learn to investigate mathematics concepts by designing and implementing interactive computer programs, so-called *Exploratory Objects* (Muller et al., in press), from year one. As their final projects in MICA courses, students individually (or in groups of two) complete an original interactive computer program on a topic *of their own choosing*. These projects can be (a) exploratory (e.g., testing his/her own conjecture; see *Structure of the Hailstone Sequence* Exploratory Object, (MICA Student Projects, n.d.); (b) an application (e.g., modeling or simulation; see *Running in the Rain* Exploratory Object, MICA Student Projects); or (c) didactic, i.e., so-called *Learning Object*s (LO). The latter, generally designed by future teachers, are innovative, interactive, highly engaging, and user-friendly computer environments that teach one or two mathematical concepts at the school level. For example, a 9-task adventure with Herculus covering (Grade 4) perimeter and area; a journey through MathVille for learning the (Grade 9) exponent laws; or a fourfold Pythagorean Theorem plate-form offering (i) a review of right angles and triangles, (ii) an exploration of the theorem, (iii) a game to practice, and (iv) a five question test with applications, are all projects designed by first-year future teachers (see respectively *Hercules and Area* LO, *Exponent Laws* LO, and *Exploring the Pythagorean Theorem* LO, MICA Student Projects).

Overall, observations and reflections on students' experiences of designing LOs and Exploratory Objects indicated that the experiences promoted positive student learning experiences. Muller et al. (2008) summarize these experiences:

> We suggest that the students develop the following skills: (a) to express their mathematical ideas in an exact way; (b) to self-assess their mathematics; (c) to

> realize their creativity in mathematics and in communicating their understanding of mathematics; and (d) to become independent in mathematical thinking. We also suggest that students are exposed to the opportunity (a) to concretize personalized original mathematics work, and (b) to identify with their future profession. Finally, our observations lead us to suggest that students develop a personal relationship with the activity of designing and implementing an ELO; indeed, students seem to demonstrate a strong engagement and ownership in the activity, and exhibit much pride of their ELO (p.4).

These reflections prompted a pragmatic collaborative project between the Department of Mathematics and the Department of Pre-Service Education which involved LOs designed by MICA students and teacher candidates enrolled in pre-service education elementary mathematics methods course (Grades 4 to 8) (Muller et al., in press). Pre-service teacher candidates were asked to use LOs to learn or review the involved mathematics in the Object and to write their reflections on their experience. Their overall experience was positive as they appreciated the LOs and commented on their high regard for the first-year MICA student LO designers. Some teacher candidates who self-identified as having math anxiety, thought that the LOs provided a safe environment for them to re-learn mathematics.

Reflecting on MICA student learning experiences as well as pre-service teacher candidates' experiences of using the LOs, we started to focus on the MICA future teachers' experiences of designing and implementing LOs. It was clear to us designing and implementing LOs involves mathematical didactics work. Interesting empirical questions started to emerge: In what ways do future teachers experiences of designing and implementing LOs promote their learning of mathematics needed for teaching? What aspects of designing and implementing LOs prompt such a positive experience? How do these future teachers' learning experiences through designing and implementing LOs differ from their learning experiences in other traditional activities? These questions led us to focus on the suggested future teachers' development of a "personal relationship with the activity of designing and implementing [a] Learning Object" (Muller et al. 2008). We postulated that future teachers' behaviour, in terms of dedication, pride, ownership, and engagement with the activity could be a key to the future teachers' positive experiences and their learning of mathematics needed for teaching. This pointed to an in-depth investigation to explore the impact of future teachers experiences of designing and implementing LOs on their learning (Mgombelo & Buteau, 2008a).

**Researching inside MICA: Learning Mathematics Needed for Teaching through the Designing and Implementing of LOs**

The purpose of our research is to explore how future teachers of secondary school mathematics are shaped by their didactic-sensitive learning experiences during their undergraduate mathematics education. Our research is guided by the following questions: (a) Does the experience of designing and implementing LOs promote future teachers' learning of mathematics needed for teaching? (b) In what ways do designing and implementing LOs provoke future teachers' awareness of their own learning of mathematics as well as what does it mean for students to learn mathematics? Guided by previously mentioned postulate (that ownership, dedication, engagement of the activity, and pride are key for the positive learning experience) we are interested in probing deeper into these future teachers' experiences in order to capture the qualitative aspects of their learning of the mathematics needed for teaching. The goal in our research is not to measure this impact in terms of how much do future teachers know mathematics needed for teaching. Our focus in the research is on future teachers' "knowing." Given the complexity of this kind of research we initially conducted a pilot –small scale study (2006-07). The goal of the pilot study was to gather first evidence of future teachers' experiences as well as to inform the design of a large scale study.

Guided by the above postulate our pilot study was framed by Mason and Spence's (1999) work on "knowing-to act" as a kind of knowing that requires awareness. Building on Gattegno's (1970) work on awareness, Mason (1998) further elaborates on the relationship of "knowing-to act" and awareness in mathematics teacher education. Mason developed three forms of awareness: "awareness-in-action," which involves a human being's powers of construal and of acting in the material world; "awareness-in-discipline," which is awareness of awareness-in-action emerging when awareness-in-action is brought into explicit awareness and formalized; and finally, "awareness in counsel," which is awareness of awareness-in-discipline involving becoming able to let others work on their awareness-in-discipline. To put this into a mathematics perspective, awareness-in-action might be exemplified by an act of counting numbers (1, 2, 3) without being aware of the underlying notions such as one to one correspondence. Awareness-in-discipline emerges when one becomes aware of this one to one correspondence in counting. Finally, awareness-in-counsel emerges when one is able to support others develop their awareness of counting as well as develop their awareness of the notion of one to one correspondence. Mason's levels of awareness served as analytical/interpretive tool for analyzing data

Data were collected from detailed questionnaires, journals, and focus group discussions that involved 4 future teachers enrolled in the MICA program, 4 teacher candidates in the Department of Pre-Service, and 1 practicing teacher. In order to probe MICA future teachers' experiences deeply in terms of awareness, questions and prompts in the questionnaires and journals were open-ended. The roles of the Pre-service teacher candidates and the practicing teacher in the research were to facilitate data collection through focus group discussion and not to act as research subjects.

All data from questionnaires, LOs, and transcripts from videos were analysed according to the interpretation of themes guided by the postulate that ownership, engagement in the activity and pride were key for positive learning experiences and by using Mason's three forms of awareness as outlined in the conceptual framework..

Using Mason's levels of awareness we identified which levels were engaged as well as ways in which they related to experiences of ownership, engagement and pride. Our analysis of data further elaborated on three prospective teacher behaviour aspects, ownership, engagement, and pride. We briefly elaborate these aspects.

*Ownership*

As noted earlier in this paper, prospective secondary school teachers can perform a number of school mathematics tasks without problem. Using Mason's (1998) forms of awareness, we could say these future teachers have awareness-in-action of mathematics needed for the tasks. Yet (as noted) if you ask future teachers how they would explain a mathematics concept or skill to someone who is learning for the first time, most of them would respond by rule-based explanation (e.g., negative times negative is positive in case of integers multiplication). These future teachers would be attending to content of their awareness-in-action and not their awareness of their awareness-in-action. As Mason notes, the behaviours to which awareness-in-action play a role can somewhat be trained without explicit allusion to awareness. We found a different scenario with the experience of designing and implementing LOs. This experience seems to prompt future teachers to take into account their own experience of learning the mathematics in order to generate ideas on how to design their LOs in ways that will make sense for the user's learning of mathematics in question. It is this future teachers' attention to their learning in order to bring to awareness their awareness-in-action that we refer to as *ownership*. This is exemplified by the following prospective teacher's response to the questionnaire question on why she chose the topic for her LO.

> My MICA I Learning Object [...] dealt with explaining and practicing multiplication…. I chose this topic because in Grade four I was very, very behind on my multiplication. I could not do the calculations in my head, and I was stuck on the first sheet of questions my teacher would give us… Since it is something I struggled with and something that I have to overcome to become a Math major, I thought it would be a great idea to develop a program that could allow students to practice without just doing the same questions over and over. I also included different ways of thinking about what multiplication means (Mgombelo & Buteau, 2008a)

It underlines that this prospective teacher attended to her own learning of multiplication or own awareness in action of multiplication. The prospective teacher in the above response did not want to design a program based on multiplication routines and rules but instead wanted to include the different ways of thinking about what multiplication means – this involves awareness.

*Engagement*

Awareness-in-discipline arises when we become aware of awareness-in-action. According to Mason (1998), the term "discipline" means encountering both facts and techniques as well as habits of thought, types of meaningful questions, and methods of resolving those questions. Our analysis of the data indicates that through the designing and implementation of LOs, future teachers engage with mathematics in terms of both aspects outlined above by Mason. Our analysis further indicated that future teachers' experiences of designing and implementing LOs tend to elicit the need to explain and attend to different representations and meanings of mathematics concepts, a very important aspect of mathematics for teaching (Ball & Bass, 2002; Davis & Simmt, 2005). We distinguish *engagement* as another aspect of learning mathematics needed for teaching. Engagement with mathematics is recognized in the way future teachers use games, graphics, and colors in their LOs in order to engage students in a meaningful way. These future teachers attended to different representations or meanings of mathematics concepts such as grid or area models of multiplication as revealed in a response from a prospective teacher questionnaire below.

> I learned how to keep instructions short and simple, and how to gear a lesson towards your audience. I learned to think about the audience I was trying to reach and what would be engaging to them. I added in Bart Simpson and made it as bright and colorful as I could. I learned multiple ways of explaining multiplication. (Mgombelo & Buteau, 2008a)

We see from the above response from the prospective teacher questionnaire, that she "learned to think about the audience …and what would be engaging to them." It is through this experience that she learned multiple ways of explaining multiplication. It is worth to note that this experience involves both future teachers' own engagement with mathematics as well as their audience's (students') engagement as revealed in the above response.

*Pride*

In order to sustain ownership and engagement in mathematics activities in the way we have described here, future teachers have to invest themselves in the activity (in terms of energy, emotion, interest, etc.). In addition to investing themselves, they need to have a sense of purpose and accomplishment. We have identified this investment as *pride*, the third aspect of future teachers' learning of mathematics needed for teaching. Here is an example from a prospective teacher's response that supports our claim.

> You're always thinking about ideas and ways to improve your project while you are in class, watching television [...] (Mgombelo & Buteau, 2008a)

We can see clearly from the above quote how much personal energy, or in other words, dedication, this prospective teacher invested in the project. Our small scale study addressed the need to know about the impact of designing and implementing LOs on the learning of mathematics needed for teaching. It strongly suggests that the experience of designing and implementing LOs promotes future teachers' learning.

## Conclusions: Further Research and Practice Collaborations

Our work underscores the importance of collaboration between mathematicians and mathematics educators in connecting practice and research in mathematics teacher education. From our pilot study further empirical questions emerged: What aspects of the designing and implementing LOs prompt such a positive experience? In what ways do prospective teachers' learning provoked by designing and implementing LOs differ from other traditional learning tasks? These questions have led to a larger-scale, collaborative research project (involving some 30 MICA future teachers candidates each followed over two years) that will thoroughly investigate the students' "repositioning" in terms of engagement, ownership, and pride, with respect to mathematics and mathematics didactics when realizing their MICA final projects (the LOs) compared to more traditional mathematics activities. We are also interested in exploring the characteristics or features of the learning activity (of designing and implementing a LO on a topic of their own choosing) that promote learning. A theoretical framework has been thereafter developed to guide this comprehensive study (Mgombelo & Buteau, 2008b). It mainly relies on Brousseau's (1997) work on theory of didactic situations; Mason's (1998) work on knowing-to act as previously discussed; and on positioning theory.

Our work has been extending on the connection between research and practice in many different ways. First, a collaborative Learning Object project building on Grade 5 students' ideas from a local school (Buteau et al. 2008) has been completed. The project involved the principal, 2 teachers, and Grade 5 students from the elementary school, as well as a mathematics student, pre-service teacher candidates, and both co-authors from our institution. The principal commented,

> From day one, our Grade 5 students were extremely motivated and engaged in developing this tool that will be used by students from other schools. (Buteau et al., 2008, p.28)

A second connection yielded in the ongoing integration of MICA Learning Object use for didactical assignments in the Methods course at our institution. In addition, Mgombelo's informal observations about MICA pre-service students with stronger dispositions towards learning versus non-MICA pre-service students led her to reflect on the design of the course. This naturally leads to asking what is it exactly in the MICA education program that seems to promote this disposition - a question that points to our long-term research program. Thirdly, the research has been guiding Buteau's reflections on her teaching practices of the MICA I course and on the MICA activities (e.g., the description of the student development process of designing and implementing Exploratory and Learning Objects, (Buteau & Muller 2008), thus pointing back to the LO activity attributes that might promote learning mathematics for teaching.

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