PRE-SERVICE TEACHER-GENERATED ANALOGIES FOR FUNCTION CONCEPTS

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The purpose of the present study was to determine pre-service teacher-generated analogies in teaching function concepts and then to discuss them in terms of the content validity – whether analogies used are epistemologically appropriate to illustrate the essence and the properties of the functions as well as the structural relations between the analogues and the targeted concepts. The videotaped data of five pre-service teachers' were collected from their microteaching during "Practice Teaching in Secondary Education" course. Results revealed that pre-service teachers did not consider too much on their analogical models. So they generally failed to make effective transformations between the analogies and the target concepts.

Keywords: Function, analogy, pre-service teacher, content validity, teacher training

INTRODUCTION

What distinguishes a mathematics teacher from mathematics major is "the capacity of a teacher to transform the content knowledge he or she posses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (Shulman, 1987, p. 15). In order to move from the personal comprehension to preparing comprehension of others, some combination of the following processes: preparation, representation, instructional selections, adaptation and tailoring to students' characteristics are proposed (Shulman, 1987). For representation of the selected sequence, teacher makes use of appropriate analogies, metaphors, examples, demonstrations, explanations, etc.

Analogies constitute one crucial component of the teachers' pedagogical content knowledge that they need most to transform subject matter into forms that could be grasped by the students of different ability and social background. Analogies are heuristic tools that enhance imagination and creativity in terms of making causal relations between the unknown and the well-known concepts (Gentner, 1998). By developing mental models students have the opportunity to access to a wide range of conceptual explanations and transformations that facilitate capturing similarities and making parallels between the concepts in areas other than mathematics and the concepts in different contexts within mathematics itself. Therefore, this article focuses on pre-service teacher-generated analogies in teaching function concepts. Function concept is central for secondary school curriculum and advanced mathematical topics taught at school and university level. Further, the function concept is considered to have a unifying role in mathematics that provides meaningful representations of real-life situations (Lloyd & Wilson, 1998). Hence, the use of analogies is very common in the teaching of functions.

Pedagogical content knowledge (PCK) refers to “the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction” (Shulman, 1987, p. 8). That is, PCK is a key aspect to address in the study of teaching. To use an example in our context, pedagogical content knowledge refers not only to knowledge about functions, but also to knowledge about the teaching of functions with analogies. To teach functions with analogies teachers should transform the subject matter for the purpose of teaching and give arguments about it. That is, they should consider the characteristics of the function concept, choose or construct well constructed analogies, and consider the similarities and differences between the different aspects of the function concepts and the analog concepts. Therefore, the study reported here is related to pre-service teacher pedagogical content knowledge. Since the process of learning is influenced by the teacher, it is therefore important to understand how teachers explain what a function is to students, what they emphasize and what they do not; and what ways they choose to help students understand.

The present study contributes to a growing body of research in the field of function by examining pre-service teacher generated analogies to determine the analogies and the target concepts and then to discuss them in terms of the content validity – whether source analogies used are epistemologically appropriate to illustrate the essence and the properties of the functions as well as the structural relations between the analogues and the targeted concepts. More specifically, we posed two main research questions for this study: (1) How do the pre-service teachers manage with the analogies they introduce? and (2) Are these analogies relevant?

Task analysis of the lessons of the pre-service teachers provides less experienced mathematics textbook authors and teachers with guidelines on how to form and use analogies effectively in teaching functions. A careful examination of an analogy is a prerequisite to using it effectively in instruction. When teachers and authors use an analogy, they should anticipate analogy-caused misconceptions and eliminate them by forming epistemologically appropriate analogies and by mapping the similarities and differences between the different aspects of the function concepts and the analogies constructed. The present study directly responds to a need among mathematics educators for insight into the nature of analogies in function concepts and guidance on how to construct ones that are pedagogically effective.

THE STUDY

Context and Participants

The study was conducted with all pre-service teachers (PT1, PT2, PT3, PT4 and PT5) taking "Practice Teaching in Secondary Education" course that was offered in Master of Science without Thesis Program at Middle East Technical University during 2005-2006 fall semester. One was male and four were female. Three of the participants (PT2, PT3, and PT5) had experience in teaching mathematics at an institution where additional courses out of school were offered and other two had experience in teaching mathematics as a private tutor. Three graduated from mathematics department (PT2, PT3, and PT4), and attending to the Master of Science without Thesis Program and rest were continuing previous mathematics teacher education program to get their bachelor degree. Master of Science without Thesis Program is a certificate program to teach mathematics at secondary school level (grades 9-12). All these students were the total number of the students in their second term.

"Practice Teaching in Secondary Education" course involves practice teaching in classroom environment for acquiring required skills in becoming an effective mathematics teacher. In this course pre-service teachers spend their six class hours in real classroom environment at an arranged public secondary school, and two class hours at the university. In that two hours period at the university, pre-service teachers presented sample lessons one by one to their colleagues and the instructor.

At the beginning of the course, function topics covered at the 9th grade and triangles topics covered at the 10th grade were assigned to each participant to be presented in a 30 minutes period at the university, to provide an effective flow of lesson and to cover all topics relevant to functions and triangles. Each participant prepared three lesson plans about assigned topics to be presented at the classroom. Two of those presentations were on functions and one on triangles. Additionally, they also did teaching two times at the school with presence of the instructor (the first researcher) and the classroom teacher. At other times they did teaching at the school when the classroom teacher allowed them to do. Teaching at the university and the school constituted 30 percent of the course grade. Lesson plans constituted 15 percent of the course grade.

While preparing the lesson plans, they mainly focused on objectives, materials, teaching techniques and the development process in the lesson.

The Design and the Analysis

The study used a case study approach with naturalistic observation. The data were drawn from the observation of five pre-service teachers' microteaching on functions conducted in two hours period at the University Class. Topics about functions involved function concepts, operation on functions, composite functions, and types of functions (constant, identity, greatest value, partial, and signum functions). In order to provide flexibility, they were not restricted to use any specific method in their presentations. During some presentations, the use of analogy method aroused. The use of analogy, however, mostly did not appear in the lesson plans. The courses were presented in three different sequences: 1) analogies, definition or rules, and solving examples, 2) definition or rules, analogies, solving examples, and 3) definition or rules, and solving examples. This indicates that analogies appeared either while exemplifying definition or rules or making introductions to the topics. In the Methods of Science and Mathematics Teaching courses the history of and some misconceptions about functions had been included but not theories and applications of analogy. All presentations and discussions were video-taped and transcribed.

Literature about epistemology of the functions (e.g. Cooney & Wilson, 1993; & Harel & Dubinsky, 1992) and the guidelines in the Teaching with Analogies Model developed from task analyses (Glynn, Duit, & Thiele, 1995) provided a conceptual base for the data analysis. Content analysis (Philips & Hardy, 2002) was conducted to discern meaning in the teacher's written and spoken expressions. Lessons were fully transcribed and considered line by line whilst annotated field notes were used as supplementary sources. The first phase of data analysis included detecting analogy-based teaching instances and identifying source analogies and the target concepts. The subsequent phases embraced in-depth examinations of spotted cases in accord with 'content validity – whether analogies used are epistemologically appropriate to illustrate the essence and the properties of the functions as well as the structural relations between the analogues and the targeted concepts. The validity of the analysis was achieved by utilizing multiple classifiers to arrive at an agreed upon classification of analogies and their target concepts as well as their epistemologically appropriateness.

FINDINGS

Data indicate the key analogical models used in teaching function, composite function and types of function concepts particularly while defining or explaining them. The analog and target concept matching was summarized in Table 1.

Analog (Familiar Situation)	Target (Mathematics Concept)
1. Function machine	Function concept, Composite function
2. Posting a letter	Composite function
3. Packing-Unpacking a present to a friend	Inverse function
4. A perforated pail	Identity function
5. Age	Partial functions, Greatest value function
6. Watering a tree	Greatest value function
7. The shelters in the apartment	Greatest value function
8. Eating a cake	Greatest value function

Table 1: Analog and the target concept relations

Here three analogies are presented and discussed because of the space restriction.

Posting a Letter Analogy

"Posting a letter" analogy was given by one of the participants [PT5] during the composite function lesson provided by [PT2]. This analogy was provided to make clear the definition and the explanations. As seen in the dialog, [PT5], however, did not focus on what the inputs and outputs for f and g are. As a result of that, one of the participants [PT1] got confused and then asked "But I can write letters to two different people?". This question reveals the importance of developing relationships among analogies and target concepts. Thereupon, the instructor posed questions as such "What is the domain in each case?", Is it people or letters?, etc. If we consider the "writing a letter" analogy, then the function $f: A \rightarrow B$ is composed by f (writing) to an argument x (people) with an output (letters). This analogy could be given for not being a function because the univalence or single-valued requirement, that for each element in the domain there be only one element in the range, is not supplied in this analogy.

> I think about posting a letter example. Let's take the action of taking the letter to the post office as f function and the letter to be posted as x. Different people's letters may arrive to the same address. For example my siblings' letters would arrive to my family's address too. There occur two actions here. The first operation is "I take the letter to the post office." And the second operation is "The postman takes the letter to my family." We name the first action as f and the second action as g. The composite of the actions is g o f. In the end the arrival of the letter requires the composite of two actions. [PT5]

"Posting a letter" analogy could be an example for composite function provided that the functions $f: A \rightarrow B$ and $g: B \rightarrow C$ are composed by first applying f (posting a letter to the post office) to an argument x (letters) and then applying g (posting letters from the post office to their arrival points) to the result (letters at the post office). Thus $g \circ f$ is the arrival of the letters to their addresses. It must, however, be mentioned that every letter written must have been posted as for each x in A, there exists some y in B such that x is related to y. Otherwise, a binary relation could not be met.

A Perforated Pail Analogy

"A Perforated Pail" analogy was constructed to remind identity function. When someone put something into the bore pail, it will fall dawn as it is. For all input, the output will be the same again. As seen below, [PT2] brought up some examples such as putting a pencil or shoe in the bore pail. She mentioned that the pail does not make any operation on the material. However, the size of the hole on the pail must be big enough for the materials to pass through. If it is not, then this could violate the total condition of being function. Furthermore, the hole on the pail should not give any damage to the material while passing through since identity function is a function that always returns the same things used as its argument. She, however, did not mention the breakdown point of this analogy.

> Think about a bore pail…. We put a pencil in it and then we get a pencil again. Or, we put a shoe in it and then we get the same shoe. The pail does not make any operations on them. You get what you put. Then what we called that function: The identity function. [PT2]

The identity function of f on A is defined to be that function with domain and range A which satisfies f(x) = x for all elements x in A. In the case of “Perforated Pail” analogy, while the function f: A→ A is composed by applying f (putting materials to the bore pail) to an argument x (materials) with an output f(x) (materials).

Function Machine Analogy

[PT2] used “Function Machine” Analogy to remind function concept and to introduce Composite function. First, she drew a function machine figure together with the explanation as such “You have a raw material named x *[began to draw Figure 1]* and you have a machine that gives output. You put x to this machine and this machine gives you the output as *f(x)”*.

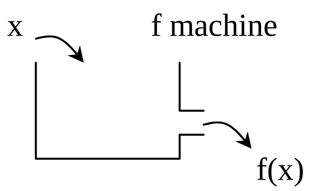

Figure 1: Pictorial analogy for function concept

To exemplify this further “Mixer” analogy - where banana and milk are input and the milkshake is output - was constructed. This, however, is not an appropriate analogy for functions of one variable. “Mixer” analogy can be an example of functions of several variables. When she was asked to make clear what the domain of the function mentioned in the analogy is, she could not make a connection to the function with two variables. One possible explanation for this inappropriate analogy is not considering the function as *mixer(milk, banana) = milkshake*. Further, the instructor expressed that “washing machine” analogy is appropriate for functions of one variable. In this analogy, inputs are dirty clothes, process is cleaning and the outputs are clean clothes.

While introducing the composite function, she first stated that “composite” is a kind of operation like addition and subtraction but operation with different rules. Taking into account the previous function machine figure, she extended the figure to be a pictorial analogy (see Figure 2) for composite function by pointing out that “In the f machine x turns out to be f(x) and then we put f(x) in the g machine. So we get (gof)(x) composite function”.

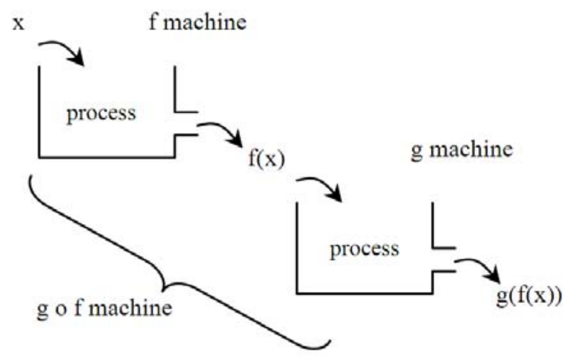

Figure 2: Pictorial analogy for composite function

However, the "washing machine" analogy that was given for functions could have been extended to composite functions. In the case of "washing machine" analogy the functions $f: A \rightarrow B$ and $g: B \rightarrow C$ can be composed by first applying f (washing process in washing machine) to an argument x (dirty clothes) and then applying g (drying process in a dryer) to the result. Thus one obtains a function $g \circ f: A \rightarrow C$ defined by $(g \circ f)(x) = g(f(x))$ for all x.

CONCLUSION

The present findings suggest that analogies need to be carefully thought out to be effective in order not to cause any confusion. The analogical models constructed by the pre-service teachers in the present study were analyzed in terms of whether the analogies constructed are epistemologically appropriate to illustrate the essence and the properties of the functions as well as the structural relations between the analogues and the targeted concepts. While mapping the analogies to the target concepts, the important things are the similarities as well as the break down points between them. The way the pre-service teachers used analogies could fall short of contributing to the students to develop epistemologically correct and conceptually rich knowledge of function due to two reasons. First, the source analogues were epistemologically inappropriate to illustrate the essence and the properties of the functions. Second, the analogies were epistemologically appropriate to illuminate the function concept, yet the teacher did not establish the mappings between the two.

In general they spontaneously followed the three steps: i) selecting an analogy (ii) mapping the analogy to the target (iii) evaluating the analogical inferences. Even the analogical models help students to visualize the newly learned symbols, concepts, and procedures, pre-service teachers need to know and show where the analogy breaks down and carefully negotiate the conceptual outcome. PTs should articulate the similarities and differences between the analogy and the target concept while they are presenting an analogy, and also should be aware of the limitations of the constructed analogy.

In the sense of these findings, it can be concluded that pre-service teachers' knowledge about the use of analogies were insufficient, and participants of the study were weak in transforming knowledge and developing sophisticated ideas in the process of teaching functions. In line with that, pre-service teachers did not consider too much on their analogical mappings and they were not able to construct the adequate relationships between the analogies and the target concepts along with the processes of mapping the analogical features onto target concept features. The difficulty appeared while developing sophisticated ideas in the process of teaching did not occur in giving mathematical definitions, rules, and procedures. For example, function was defined correctly as "f is a relation from set A to set B. If each element in set A correspond only one element in set B, then this relation is a function."

One of the limitations of the present study was that pre-service teachers were restricted to present function concept. May be if they were more flexible in the topic selection they would choose another mathematics topic in which they are more capable, thus they would generate more productive analogical models.

IMPLICATIONS

In teacher preparation courses, student teachers should be asked to generate their own analogies in different contexts of mathematics. This kind of courses could provide them an opportunity to constitute an available repertoire of analogies (Thiele & Treagust, 1994) and to create analogy-enhanced teaching materials. In addition, this array of experiences could allow them to discuss, model, and justify their interpretations of the concepts and to provide different approaches to the teaching of the concepts. The analogies discussed here will help pre-service and in-service teachers develop a sound relational knowledge of the function concepts as well as consider carefully on their analogical mappings to construct epistemologically appropriate ones and to map the similarities and differences between the analogies and target concepts. Discussing the analogies reported here with pre-service and in-service teachers could deepen their understanding of function concept as well as functions pedagogy to offer perspectives on a sound generation of analogies.

In the light of the discussions of the teacher generated analogies, mathematics textbook authors and teachers can develop productive analogies for various mathematical concepts. Carefully crafted analogies can serve as initial mental models for the introduction and presentation of newly learned mathematical concepts.

As a result of this investigation, a further study was planned to describe the multiple analogical models used to introduce and teach grade 9 function concepts. We examine the pre-service teacher's reasons for using models, explain each model's development during the lessons, and analyze the understandings they derived from the models.

Teachers should engage their students in a discussion in which the limitations of the analogy are identified.

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