THE ROLE OF SUBJECT KNOWLEDGE IN PRIMARY STUDENT TEACHERS' APPROACHES TO TEACHING THE TOPIC OF AREA

Carol Murphy

University of Exeter

This study reviews the relationship between student teachers' subject knowledge in the topic of area and their approaches to teaching that topic. The research was carried out with four primary student teachers and examines the similarities and differences between the nature of their subject knowledge and their plans to teach the topic. In this paper results of two of the four student teachers are focused on to illustrate the contrasts in planning and subject knowledge. The intention is not to generalise relationships but to examine the phenomena presented. It raises questions related to the variables in connecting student teachers' subject knowledge and their knowledge of how to teach.

Key-words: subject knowledge; area; student teacher; approaches to teaching; understanding

INTRODUCTION

The importance of subject knowledge in the preparation of teaching activities is clearly recognised (Ball, Lubienski & Mewborn, 2001). If we see teaching fundamentally as an exchange of ideas it would seem evident that a teacher's understanding of a topic will impact on how the idea is 'shaped' or 'tailored' when presented in a classroom. As such "teaching necessarily begins with a teacher's understanding of what is to be learned and how it is to be taught" (Shulman, 1987, p.7). Shulman emphasised the transformation of a teacher's knowledge of a subject into 'pedagogical content knowledge' and consequent pedagogical actions by "taking what he or she understands and making it ready for effective instruction" (p.14). In this way mathematical content knowledge is 'intertwined' with knowledge of teaching and learning (Ball & Bass, 2003).

It is generally accepted that mathematics should be taught with understanding (Hiebert & Lefevre, 1986; Skemp, 1976). In the topic of area it would seem that children often rely on the use of formulae with little understanding of the mathematical concepts involved (Dickson, Brown & Gibson, 1984). They are unable to see the reasonableness of their answers and so are unable to monitor their use of these formulae. There is also evidence that student teachers have a similar reliance on formulae (Baturo & Nason, 1996; Tierney, Boyd & Davis, 1990).

It would seem that a student teacher with limited understanding of the mathematical topic such as area would not be effective in developing children's understanding. This study aims to investigate the impact of primary student teachers' subject knowledge on approaches to teaching the topic of area. As an interpretive study the intention is not to generalise any relationship but to examine phenomena related to differences and similarities in the student teachers' understanding of the topic and in how they plan activities to teach the topic.

DEVELOPING UNDERSTANDING IN THE TOPIC OF AREA

Measuring area is based on the notion of 'space filling' (Nitabach & Lehrer, 1996). However, unlike children's other common experiences of measure such as length, the use of a ruler in measuring area is indirect. In this way instruction that focuses on procedural competence with measuring tools such as rulers "falls short in helping children develop an understanding of space" (p.473) and it is not surprising that many children confuse area and perimeter (Dickson *et al*., 1984). Instruction that models the counting of squares on grids provides more success and may represent the notion of 'space filling'. However this does not represent the full nature of area. As Dickson *et al*. (1984) commented the possible restriction to a discrete rather than a continuous view of area measure might not lead to the notion of Π and the formula of the area of a circle.

Further to this, figures used as representations in the classroom often provide a static view rather than a dynamic view. That is, as a boundary approaches a line, the area approaches zero (Baturo & Nason, 1996). This may lead to misconceptions about the conservation of perimeter and area. The recognition of such a misconception goes back at least to the 1960s with Lunzer's (1968) notion of 'false conservation'. This false notion has more recently been cited by Stavy and Tirosh (1996) as an example of the intuitive rule 'more A, more B', in that as the perimeter increases so the area will increase. Alternatively the intuitive rule can be manifested as 'same A, Same B' in that the same perimeter will mean the same area.

It would seem that once introduced to the formulae, children have a tendency to use these regardless of the success of their answers (Dickson, 1989). Studies such as Pesek and Kirshner (2000) and Zacharos (2006) suggested that, where instruction involved procedural competence and use of formulae, children would insist on repeating strategies that caused errors and they often had difficulty in "interpreting the physical meaning of the numerical representation of area" (Zacharos, p. 229). Where instruction was based on measuring tools such as dividing rectangles into squares children demonstrated flexible methods of constructing solutions and often achieved more success. The studies suggested that the early teaching of formulae presented 'interference of prior learning' (Pesek and Kirshner) or 'instructive obstacles' (Zacharos).

Such 'interference' or 'obstacles' could explain why many children at the beginning of secondary school take algorithmic approaches to the solution of area measurement problems (Lehrer & Chazan, 1998). It follows that student teachers are likely to have a similar reliance on algorithms. If we refer back to Shulman's model of transformation and Ball and Bass's idea of 'intertwining' content and teaching knowledge, then a student teacher's understanding of the nature of area would seem key to the way they would teach it. Studies that have examined student teachers' subject knowledge in the topic of area (Baturo and Nason, 1996; Tierney et al, 1990) found that student teachers often demonstrated a lack of understanding of how practical concrete experiences could relate to the use of formulae and how area measure evolves from linear measure. They were often uncertain about the reasonableness of their answers and were unable to explain how formulae were related. A study that has examined student teachers' lesson plans for teaching the topic of area (Berenson, Van der Valk, Oldham, Runesson, Moreira, and Brockman's, 1997) found that many student teachers represented the topic of area through the demonstration of procedures and use of formulae rather than focusing on the activities that would support understanding. What we do not know from these studies is whether the student teachers that planned to teach the topic through the demonstration of procedures were the students who demonstrated a lack of understanding of the topic.

THE STUDY

The four student teachers involved in this study had varied backgrounds in mathematics. At the time of the study they had completed the taught university based element of a one year Post Graduate Certificate in Education (PGCE) and they were about to start their final teaching practice. The student teachers had attended workshop seminars on the teaching of primary mathematics. All four student teachers had the same course tutor so would have followed the same content in their mathematics seminars. The student teachers were also reassured that the work for this project would not be used as part of their course assessment.

Clinical interviews were carried out with each of the student teachers to reveal underlying processes in their understanding (Swanson, Schwatz, Ginsburg and Kossan, 1981; Ginsburg, 1997). The first part of the interview examined the development of the student teacher's lesson plan and the second part of the interview involved the use of mathematical tasks to investigate the nature of their understanding in the topic of area. The mathematical tasks were equivalent with some standardisation of probing questions but further interrogation was managed flexibly in order to be contingent with the student teachers' responses. The interviews were audio taped and transcribed.

The use of lesson plans

Planning is central to teaching and the development of lesson plans is a key aspect of teacher training. Lesson plans provide a source of data in assessing student teachers' professional development. They can also provide useful cues in follow up interviews when the activities, explanations and questions used by the student teachers help to generate further descriptions (John, 1991, Berenson et al, 1997). Although lesson plans are limited to demonstrating the student teacher's 'espoused' theory of action

(Argyris and Schon, 1974) they can be seen as effective in indicating the student teacher's perceptions of teaching.

The student teachers were asked to plan a lesson to introduce the topic of area to a Y4 class (8 to 9 year olds). The student teachers were advised that they could use any sources they normally would to help plan the lesson. The only restriction being the ideas would be their own or their own interpretation of teaching ideas from other sources. The student teachers were questioned about the following:

1. How they had developed the activities
2. How they felt the activities would facilitate the children's learning
3. The instructions or explanations they intended to give
4. The questions they intended to ask the children
5. The difficulties that they felt the children would encounter

Area Tasks

The second part of the interview involved four tasks adapted from Baturo and Nason's (1996) and Tierney *et al*. (1990) studies to ascertain the subject knowledge of the student teachers.

Task 1 (Baturo and Nason, p.245) includes both open and closed shapes to test student teachers' understanding of the notion of area (see fig 1). Shapes G and F were included to test the ability to differentiate between area and volume, shapes J and K test the notion of area as the amount of surface that is enclosed within a boundary and shapes E, H and L test the understanding of area from a dynamic perspective.

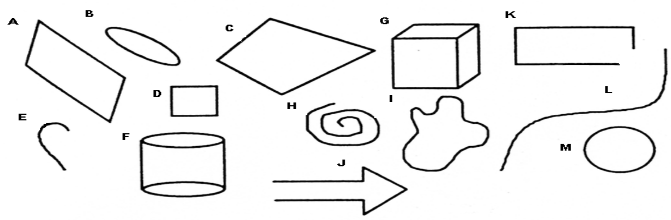

Fig 1: Task 1

Task 2 (adapted from Baturo and Nason) was designed to test the ability to compare areas, initially without the use of formulae (see fig 2). The student teachers were presented with two pairs of cardboard shapes. Dimensions were not given. Comparison by visual inspection alone would be inconclusive so the student teachers were asked to consider ways to compare area. This was used to determine if the student teacher was able to use measuring processes other than external measures and use of the formulae.

Pair A: Pair B:

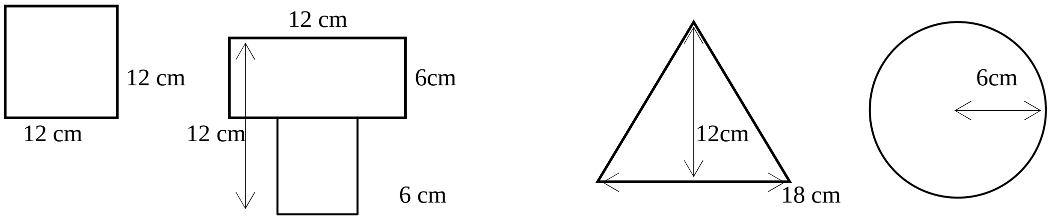

Fig 2: Task 2 (shapes not drawn to scale)

Task 3 (adapted from Tierney *et al.*) was intended to determine a dynamic view of area and the ability to consider changes in area and perimeter (see fig 3). The student teachers were given three cardboard shapes. Dimensions were not given.

1. a rectangle 9cm by 4cm
2. a parallelogram where the area is the same as the rectangle but the perimeter has changed (base 9 cm and height 4 cm)
3. a parallelogram where the perimeter is the same as the rectangle but the area has changed

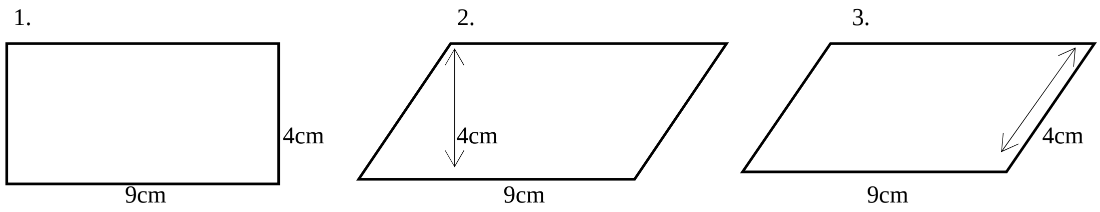

Fig 3: Task 3 (shapes not drawn to scale)

Task 4 (adapted from Baturo and Nason) aimed to test the correct use of formulae. It also tested for an understanding of the relationship with non-rectangular figures, including the use of the ratio Π (see fig 4).

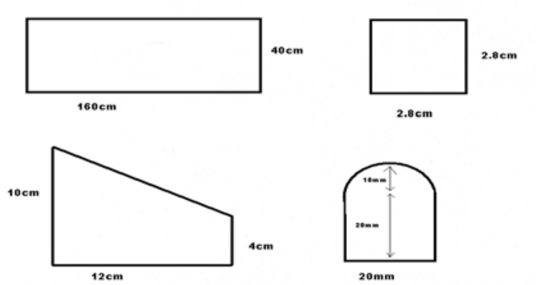

Fig 4: Task 4 (shapes not drawn to scale)

RESULTS AND ANALYSIS

In this paper it is presented the results of two of the four student teachers, Alan and Charlotte, are focused on to illustrate the contrasts in planning and subject knowledge.

Alan

Alan's highest qualification in mathematics was an 'A' level taken over 5 years ago. He felt that his confidence level was moderate to high. In his lesson plan he intended to model the use of the formula using a transparent grid over a rectangle and by, "thinking out loud", would state, "Find this side, this side and multiply together". He would then show the children how to check by counting the squares. He was concerned that the children might confuse area and perimeter and that they might add the lengths rather than multiply. In order to overcome this he would show how to use a ruler to measure the lengths and repeat the instructions from the introduction. He felt that he would have to tell the children what units to use and that the '2' means squared. Alan would continue the lesson with further practice of the formula with other rectangles and with shapes composed of rectangles. He suggested using a 'real-life' context by extending the use of units to square metres and finding the area of the classroom.

Alan's use of formulae and calculations in Tasks 2, 3 and 4 were quick and accurate. He used the formulae as a first resort in comparing areas of shapes in Task 2 and Task 3 rather than reasoning or comparing by placing the shapes on top of each other. Alan gave a clear definition of area related to the covering of surfaces. He was also aware of the relationships between formulae and the notion of Π as a ratio in finding the area of circles. He was able to consider the dynamic view of area with the parallelograms in Task 3 but did not identify the area of the open shapes as zero in Task 1.

Charlotte

Charlotte had obtained a grade C GCSE qualification in mathematics, the minimum entry requirement for a primary PGCE course, and she spoke of lacking confidence in mathematics. Charlotte stated that she found the lesson difficult to plan and had researched pedagogy based texts. Charlotte intended to introduce the topic with a large paper rectangle and ask, "How many children can fit onto this shape?" She would use these arbitrary units to determine the area of other shapes and then draw rectangles on the board and pretend that each child is a centimetre square. Charlotte felt that the activities would "lead naturally" to a definition of area as the "amount of space within a shape" and she intended to note the strategies that the children used. She also intended to set an activity to investigate the area of rectangles and changes in perimeter. She would encourage the children to talk together about the patterns they had found. Charlotte would ask, "What do you notice about the perimeter and area of the two classrooms?" (sketches on the board) and "Can you draw different shaped rectangles with an area of 12 squares?".

Charlotte's notion of area from Task 1 seemed inconsistent. Although she stated that the area was the amount of space inside a shape she attempted to include some of the open shapes as those that had an area. She was uncertain as to whether the three-dimensional shapes would have an area, and if so, how to measure it. She was, however, secure in the relationships between the formula for the area of a rectangle and the area of a triangle and was aware of an activity to determine Π as a ratio. Charlotte was aware of the dynamic view of area from Task 3 and was able to compare the areas of the parallelograms with little difficulty. Charlotte made errors in using the dimensions and formulae for calculating areas in Task 4. She was also not aware of the correct units and confessed that she never knew when to use cm^2 or cm^3.

ANALYSIS AND DISCUSSION

Performances on the mathematical tasks suggested that Alan had a good understanding of the nature of the topic of area. In particular Alan demonstrated quick and accurate use of formulae. In contrast Charlotte's performance on the tasks demonstrated limited knowledge in the use of formulae and units. Her understanding of the nature of the topic of area appeared to be inconsistent.

Charlotte based her intended introduction to the topic of area on the counting of regions. Charlotte initially started with arbitrary units that would be used later to introduce the square unit. Charlotte was aiming to provide children with activities and problems that would help them realise the notion of area 'naturally'. On the other hand, Alan's lesson was focused on teaching the use of the formula. He was concerned that the children would not use the correct formula for area and he would articulate explicitly how to do this. There was an attempt to relate the use of the formula to 'real-life' by finding the area of the classroom.

According to the review of research above, Alan's intended focus on the use of the formula from the start of his lesson might suggest a premature introduction that would create 'interference' or 'obstacles'. However Alan was a confident mathematician who demonstrated accurate use of formulae and secure understanding of the nature of the topic. In contrast, the activities that Charlotte planned to use would be more likely to support children in developing a notion of area as 'space filling'. This might reduce the children's reliance on the use of formulae and consequently support their understanding. However Charlotte was less confident in mathematics and she demonstrated weaker subject knowledge.

Ambrose (2004) has suggested that student teachers may often believe that teaching mathematics is straightforward. They assume that, if they know the mathematics they need to teach, and then all that is needed is to give clear explanations of this knowledge. Further to this the student teacher may believe that the aim of teaching mathematics is to explain useful facts and skills to children to help them become skilful and efficient in their use and to know when to apply them. The analysis of Alan's lesson plan indicates that he may have this belief of teaching. Stipek, Givvin, Salmon and MacGyvers's (2001) referred to this belief as a traditional 'knowing' orientation. They suggested that a shift away from such a traditional orientation towards an 'enquiry' orientation where mathematics is seen as a tool for problem solving, would be more effective. Analysis of Charlotte's lesson plan suggests that she may have been more inclined towards an 'enquiry' orientation.

In order to avoid the 'interference' or 'obstructions' that might become apparent by focusing on the procedures of area measurement we would want student teachers to move towards this 'enquiry' orientation. Stipek et al.'s empirical study indicated that teachers' beliefs about mathematics predicted their instruction. However they also suggested that less confident teachers were more likely to be oriented towards mathematics as 'knowing' due to lack of confidence in dealing with the questions that might be asked through an enquiry based approach. If we interpret Alan's orientation as 'knowing' and Charlotte's approach as moving towards 'enquiry' then this suggests an anomaly as Charlotte was less secure and lacked confidence in her knowledge of the content.

It could be said that as Alan used the formulae with particular ease and accuracy his aim was to support the children in developing such a use. Although he was able to realise relationships he did not see this as an important aspect of mathematics and hence he did not focus on this pedagogically. Charlotte's emphasis was not on ensuring clear explanations were given but that the children arrived at an understanding through the activities. She suggested that the children would use their own strategies and she intended to employ activities that would 'lead naturally' to their understanding. Could it be that Charlotte's lack of confidence and knowledge meant that she was uncertain of how to explain the mathematical ideas to the children? In this way she may have researched pedagogical approaches further. Or could it be that Charlotte's beliefs in the teaching of mathematics differed from that of Alan? Despite a lack of knowledge in mathematics, Charlotte's pedagogical approach may have been based on a belief that children develop understanding through active engagement in activities and that this belief has been carried over from her view of what is important in mathematics.

This is not to suggest that Charlotte would be more effective in teaching the topic. This study has not investigated how the student teachers responded to the children's learning in the classroom and Charlotte's misunderstandings are likely to inhibit her ability to develop the children's learning at some point.

CONCLUSION

Hill, Rowan and Ball (2005) have suggested that it is not knowledge of content but knowledge of 'how to teach' the content that is influential in considering teacher effectiveness. What remains a question is how this knowledge of 'how to teach' is arrived at? Although this research does not provide any generalisable evidence it does raise questions regarding the nature of subject knowledge in relation to the knowledge of ‘how to teach’, and whether there may be other variables at play, such as orientations and beliefs about what is important in mathematics.

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