CATEGORIES OF AFFECT – SOME REMARKS

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Cognitive concepts were insufficient to explain some effects observed in mathematics learning, particularly differences in performance. So researchers began investigating the influence of affect on the learning process, using the concepts of beliefs, attitudes, emotions and values. This paper discusses questions connected with the theoretical status of these concepts.

Introduction

McLeod (1992) wrote in his survey paper, "Research on Affect in Mathematics Education: Reconceptualization", that beliefs, attitudes and emotions are used in mathematics education research to describe a wide range of affective responses to mathematics. Although terms and concepts are often transferred from psychology to mathematics education, McLeod points out why such a transfer to the affective domain can be problematic:

> Terms sometimes have different meanings in psychology than they do in mathematics education and even within a given field, studies that use the same terminology are often not studying the same phenomenon.... Clarification of terminology for the affective domain remains a major task for researchers in both psychology and mathematics education. (McLeod, 1992; 576)

There have been efforts to clarify the meanings of these concepts, particularly with respect to beliefs and attitudes. In a paper appearing in the collection, "Beliefs: A Hidden Variable in Mathematics Education", Furinghetti and Pehkonen (2002) describe a process that clarifies some shared core elements commonly mentioned in characterizations of beliefs:

> Using an international panel we looked for common background suitable in describing the characteristics of the concept of beliefs and the mutual relationship in the critical triad "beliefs – conceptions – knowledge". (Furinghetti and Pehkonen, 2002; 46)

Even if it were not possible to reach a common shared definition of beliefs, the paper clarifies some of the common and contrasting meanings of this concept.

With respect to the problem of definition in the case of "attitude toward mathematics", we find a situation analogous to the one described by Di Martino and Zan (Di Martino and Zan, 2001; Zan and Di Martino, 2008); namely, a

> …lack of clarity that characterizes research on attitude and the inadequacy of most measurement. (Di Martino and Zan, 2008; 197)

In their analysis of academic papers, Di Martino and Zan found three types of definition of attitude toward mathematics: a "simple" definition where attitude toward mathematics is seen as being either a positive or negative emotional disposition toward mathematics; a multidimensional definition where three components constitute attitude – emotional response, beliefs regarding the subject and behaviour related to the subject; and a bi-dimensional definition where attitude toward mathematics is seen as a pattern of beliefs and emotions associated with mathematics.

The lack of clarity in what “beliefs” or “attitude toward mathematics” means also has implications for research in the affective field. Thus Sfard writes:

> Finally, the self-sustained “essences” implied in reifying terms such as *knowledge*, *beliefs*, and *attitudes* constitute a rather shaky ground for either empirical research or pedagogical practices – a fact of which neither research nor teachers seem fully aware. (Sfard, 2008; 56)

Hart, too, referred to this problem and wrote that

> research on the affective domain in mathematics education is in need of a strong theoretical basis that will be developed only through sustained, systematic efforts over time. (Hart, 1989; 38)

All of this suggests we have to rethink the concepts used in research on affect, and, moreover, it seems necessary to consider the problem in a more general way: “Wherein lies the problem of defining concepts and, in relation to this, what is the status of research methods?” “Can results from other fields help us better understand the categories of affect?”

General aspects of concepts

In his paper, “Aspects of the Nature and State of Research in Mathematics Education”, Niss (1999) refers to a crucial fact permeating all research:

> It is important to realise a peculiar but essential aspect of the didactics of mathematics: its *dual nature*. As in the case with any academic field, the didactics of mathematics addresses, not surprisingly, what we may call *descriptive/explanatory* issues, in which the generic questions are ‘what *is* (the case)?’ (aiming at description) and ‘*why* is this so?’ (aiming at explanation). Objective, neutral answers are sought to such questions by means of empirical and theoretical data collection and analysis without any explicit involvement of values (norms). (Niss, 1999; 5)

We use terms and concepts to describe and explain phenomena: therefore we have to see if this duality can be discerned in our terms and concepts.

In the literature on mathematics education numerous accounts exist of deep considerations of mathematical concepts (see, for instance, the Special Issue “Semiotic Perspectives in Mathematics Education” in Educational Studies in Mathematics Education, Saenz-Ludlow and Presmeg, 2006). In these papers, the focus is on the process of construction of the meaning of mathematical concepts. We therefore need to consider the process of constructing the meaning of concepts used in mathematics education research, with a special focus on affective concepts.

Let us discuss the meaning-construction-problem as encountered in the study of affect from a more general viewpoint; i.e. one that considers the ontological and other status of the concepts in the scientific research process, particularly in the way the latter’s relationship to a concept’s meaning.

In semiotics researchers analyse the relationship between symbols and referents. Frege discussed this in his important paper, “Zeichen, Sinn und Bedeutung (Sign, Sense and Meaning)”. Here, “meaning” represents the objective idea of a thing; “sense” contains the subjective interpretation made by a person relating to this thing; and “sign” designates the objective idea (Kilpatrick, Hoyles, Skovsmose, & Valero, 2005; Steinbring, 2005). In modelling the process of meaning construction, Steinbring (2005) uses the scheme of an “epistemological triangle”, in which sign/symbol, object/reference context and concept form the triangle’s corners:

> Mathematics requires *certain sign* or *symbol systems* to record and codify knowledge… these signs do not immediately have a meaning of their own. The meaning has to be produced by the student or the teacher by establishing a mediation between *signs/symbols* and suitable *reference contexts*. (Steinbring, 2005; 22)

Sfard stresses the discourse aspect of a concept definition:

> A concept is a symbol with its use. (Sfard, 2008; 111)

Within this concept definition, the term “symbol” includes more signifiers than words; and “use” refers to the use of a symbol in a discourse (Sfard, 2008; 236). This extension of the term “meaning of a symbol” to its use in a discourse process allows attention to be directed toward more perspectives (such as that of emotional reaction) than was possible in Frege’s classical concept of meaning. Otte refers to the important fact that all our perceptions include elements of interpretation as well as of generalization and therefore all knowledge is in a certain sense indirect knowledge and a function of symbols and representations (Otte, 2005; 231). Thus understanding concepts is a cognitive activity that is connected with intuition:

> Thom, and Bruner as well, intend to draw attention to the fact that we cannot develop our cognitive activities if we do not believe in the reality of our intuitions, and that these intuitions or mental states nevertheless may be treacherous and without objective validity or reference. Subjective meaningfulness and objective validity may not coincide. (Otte, 2005; 231)

Reading this quotation, moreover, raises the question of how an individual acquires a concept. Two answers may be found in mathematics education research, depending on how the problem is viewed. Following the ideas of Piaget, intellectual growth results from a direct interaction between the individual and the world; on the other hand, according to social constructivism,

> …whatever name is given to what is being learned by an individual – *knowledge*, *concept*, or *higher mental* function – all these terms refer to culturally produced and constantly modified outcomes of collective human efforts. (Sfard, 2008; 77)

We should probably accept that knowledge and concepts are outcomes of a cultural process and neither can be learned outside a discourse community. For instance, a learner needs help from an experienced person (Lave and Wenger describe this learning process as “legitimate peripheral participation” (Lave and Wenger, 1991)). Furthermore, we ought to consider the individual parts comprising the acquisition process. Lakoff and Nunez refer to the important role of metaphors:

> One of the principal results in cognitive science is that abstract concepts are typically understood, via metaphor, in terms of more concrete concepts. This phenomenon has

> been studied scientifically for more than two decades and is in general as well established as any result in cognitive science (although particular details of the analysis are open to further investigation). One of the major results is that metaphorical mappings are systematic and nor arbitrary. (Lakoff and Nunez, 2000; 40 – 41)

This role of metaphors is important to keep in mind – especially if we transfer concepts, such as attitude, from other fields– because the borrowed concepts are combined with metaphors in our field to understand the concepts already present in our field. We must specify the metaphors required for using the concepts in our field, mathematics education.

A second crucial point is strongly connected to our use of language. We use words or symbols that are the endpoints of a process of objectification; and these words or symbols produce the illusion that they are in the same category as things, yet they can have no empirical manifestation:

> After objectification, we often interpret metastatements, that is, statements about discourse, as statements about the extradiscursive world (…) This *ontological collapse* (a) may produce an *illusory dilemma*, (b) can result in *phony dichotomies* leading to tautologies disguised as causal explanations, and (c) is likely to lead us to *consequential omissions*; blinding us to potentially significant phenomena that cannot be described in ontologically “flattered” terms. (Sfard, 2008; 57)

In the light of this, we ought to keep in mind that concepts used in mathematics education research that are formulated in words have no empirical manifestation – and therefore no reference objects – and they get their meaning through the metaphors and associations that we imagine in connection with the symbol for the concept. In mathematics one can use a “realization tree” (Sfard, 2008; 165) to overcome, in a certain sense, the lack of a reference context; however, for concepts encountered in mathematics education we have no such realization tree.

The problem of meaning construction for affective categories

Research into affect was motivated by the fact that cognitive concepts were insufficient to explain some of the effects observed in mathematics learning (McLeod, 1992), such as differences in the outcomes of mathematics learning. To explain these differences, researchers used affective concepts such as attitudes and beliefs. Thus differences in mathematical performance were also viewed as a consequence of differences in attitudes or beliefs.

With reference to the general remarks on concepts in the previous chapter of the paper, in our context three components are important: the concept definition (independent of the formal state of this definition (see McLeod and McLeod (2002) for the case of beliefs); the associations and metaphors that combine with the concept definition; and the research methods that are used to investigate and measure the concept. It shall be argued below that with respect to the meaning-construction problem in mathematics education research, the components “concept definition” and “concept images” (or concept trees (Sfard, 2008)) are helpful, but the ontological status of “research methods” is problematic, and the reason for this ought to be made widely understood.

Let us start with a definition of the affective categories, after Goldin (2002); also, in the following, we shall use the concept of beliefs to demonstrate the meaning-construction problem:

> (1) *emotions* (rapidly changing states of feeling, mild to very intense, that are usually local or embedded in context);
> (2) *attitudes* (moderately stable predispositions toward ways of feeling in classes of situations, involving a balance of affect and cognition);
> (3) *beliefs* (internal representations to which the holder attributes truth, validity, or applicability, usually stable and highly cognitive, may be highly structured);
> (4) *values, ethics, and morals* (deeply-held preferences, possibly characterized as “personal truth,” stable, highly affective as well as cognitive, may also be highly structured). (Goldin, 2002; 61)

In the following I also refer to the definitions of beliefs formulated by Op’t Eynde, De Corte and Verschaffel (2002) and Törner (2002; Goldin, Rösken and Törner, 2009):

> Students’ mathematics-related beliefs are the implicitly or explicitly held subjective conceptions students hold to be true about mathematics education, about themselves as mathematicians, and about mathematics class context. These beliefs determine in close interaction with each other and with students’ prior knowledge their mathematical learning and problem solving in class. (Op’t Eynde, De Corte and Verschaffel, 2002; 27)

Törner uses constitutive elements (ontological, enumerative, normative and affective aspects) to define beliefs *B as a quadruple B* = (O, C_0, μ_i e_j), whereby O is the belief object, C_0 the content set of mental associations, μ_i the membership degree function and e_j the evaluation map (Törner, 2002; Goldin, Rösken and Törner, 2009).

It is important to note that each of these definitions refers to descriptions of mental systems. These mental systems are activated in all situations in which mathematics is involved and these systems influence the thoughts and acts of a person in these situations (Furinghetti and Pehkonen, 2000; Hannula, 1998). The lack of reference objects for the concepts (all of which are discourse objects (Sfard, 2008)) leads to a problematic situation when attempting to give the concepts a meaning in the discourse process.

In the definitions we find certain keywords – “intensity”, “stability”, “structure” and “truth”. These keywords are supposed to lead to a meaning for the concepts: we therefore need to analyze them. Intensity is often described as “hot” or “cool” (McLeod, 1992), metaphors that are also used to describe affective states:

> Affection, for example, is understood in terms of physical warmth. (Lakoff and Nunez, 2000; 41)

The terms “stability” and “balance” refer to a metaphor originating from physics and describing a state of equilibrium. In our case this term is used to evoke a twofold meaning. On the one hand, it is meant to capture the notion that some mental system always leads to the same endpoint that persists for an extended period; on the other hand, it describes an equilibrium between the affective and cognitive systems. "Structure" refers to an ordering in the mental system that is clearly distinct from other systems. "Truth" is a metaphor borrowed from logic and used here in the singular sense that all utterances made by an individual are subjectively seen as true.

However, all these keywords are also discourse objects and are therefore at the same level as the concepts that they are intended to give meaning to.

How do we proceed? Another opportunity to construct meaning for a concept is afforded by using insights from other scientific fields striving to understand the same phenomena. In our case we could use insights from neuroscience.

With respect to cognition and affect, neuroscience distinguishes two different systems: cognition and emotion. Both exist as a result of biological evolution, with the aim of aiding the individual's survival (Wimmer and Ciompi, 1996; Damasio, 1999; LeDoux, 1998; Roth, 2001). Although located in different parts of the brain (Damasio, 1999; LeDoux, 1998; Roth, 2001), there are connections between the two systems that allow interactions. A very important consequence of the existence of these two systems is that we have to distinguish between "feeling" and "knowing that we have a feeling" (Damasio, 1999; 26); or "emotional reactions" and "conscious emotional experience" (LeDoux, 1998; 296).

For our problem we should note that although all processes on the neuronal level are not conscious, some of these processes lead to conscious results. We are aware only of these conscious parts of the processes. For remembrances, too, two memory systems exist with respect to emotions: an implicit emotional memory and an explicit memory of emotions (LeDoux, 1998). The implicit emotional memory operates unconsciously, is strongly connected to arousal systems and may often lead to bodily reactions. The explicit memory of emotional situations contains all the conscious knowledge of emotional situations, emotional reactions to objects, persons and ideas etc.. The most important consequence of this is that this memory system is part of the cognitive memory and there is no distinction between a remembrance of an emotion and a remembrance of a cognitive content (LeDoux, 1998). The fact that memory of emotions is cognitive has important consequences (Schlöglmann, 2002):

1) We have knowledge about our feelings, their origin and their effect. This knowledge is stored in memory systems as cognitive knowledge.
2) Memory of emotions is open to "rational" manipulation. That means we are able to think about our emotional remembrances, and that all verbal statements about emotional facts are controlled by cognition.
3) Knowledge of our affect with respect to objects and situations allows us to handle our affect at least in controlled situations (see Goldin's example of the roller coaster experience (Goldin, 2002; 62)).
4) Humans are able to "construct" their remembrances in a way that they are able to live with this memory. Part of this process is forgetting unpleasant facts more easily than pleasant ones: our memory has suppression mechanisms to handle unpleasant remembrances (Roth, 2001).

Assimilation and accommodation processes lead to affective-cognitive schemata (Ciompi, 1999). The affective component is stored in two memories: in the implicit memory that works unconsciously but influences our actions and thoughts (Damasio developed the concept of "somatic marker" to explain this (Damasio, 2004; Brown and Reid, 2004)); and in the explicit memory that stores all the knowledge of affect with respect to people, objects and situations. Affective-cognitive schemata always contain both the unconscious and the conscious components. Repeated assimilation and accommodation processes in relation to a special problem leads to consolidation of the unconscious reactions, as well as to more and more conscious knowledge of feelings and emotional reactions. It provides information on the outbreak of emotional reactions and allows the development of strategies for handling such situations (Goldin, 2002; Schlöglmann, 2006)).

Neuroscientific research suggests that we ought to distinguish between reactions occurring within the two memory systems; however, according to neuroscience, we have no criteria to distinguish between knowledge and knowledge of our affective relationship to mathematics. This underscores the problem that a distinction is also difficult to formulate in philosophy (Österholm, 2009; Pehkonen and Pietilä, 2003), and helps us appreciate that the problem of defining affective categories, especially beliefs, must be considered at the discourse level. Yet we have seen that descriptions of affective categories as "discourse objects" themselves also use discourse objects (e.g. intensity, stability, structure, truth) together with some metaphors. We are in a circle situation: we are bound to define our concepts in terms that contain no reference objects.

On top of these considerations, in order to measure the categories, we need an operationalization of them, usually in terms of items of a questionnaire. The items are formulated by the researchers with the aim of grasping all of the important aspects of the definition, and are formulated as questions or simple statements. The attention of the responder is directed towards finding an appropriate answer or value on a scale. However, the items are more concrete than the definition, and we have a situation where the measurement methods are derived from theoretical concepts, while they themselves become an important part of the concept. This problem is inherent in all discourse objects.

Conclusion

The analysis of the problem of defining affective concepts shows that these concepts are objects of a discourse with no reference objects. To give these concepts a meaning we use discourse to clarify the meaning: in particular, by employing other terms and metaphors. However, these terms are often also objects of a discourse at the same level as the terms they are intended to give meaning to. In a discourse this obstacle can be successfully surmounted. In contrast, if we want to measure a concept, we must formulate the description of it mostly in the form of items of a questionnaire, and these items are a consequence of our definition – yet for the purposes of the measurement they are the realization of the definition. The problem is that we cannot escape this situation. Therefore it is important to be aware of the problem. As a consequence of this state of affairs, researchers have developed numerous methods whose appropriateness depends on the complexity of the phenomenon at hand (for the case of beliefs research see (Leder and Forgasz, 2002)); indeed, in extending the basis of information about some phenomenon, more than one research method is often used to overcome, in a certain sense, the problem of defining a concept.

On the whole we can see three groups of methods: quantitative, qualitative and observational methods. The basis for quantitative methods is the questionnaire, together with the statistical methods used to handle the responses. Qualitative methods are mostly based on texts (protocols of interviews, essays, protocols of narratives and protocols of observations), and are used to look for keywords expressing affective or emotional reactions (see, for instance, Tsamir and Tirosh, 2009; Evans, 2002). Observations can also be used to look for keywords as well as other signs indicating emotional state, such as body language. (A small number of studies exist in which physiological facts are utilized.) All these efforts can help clarify the meaning of a concept, and, in a certain sense, overcome the theoretical obstacle in a discursive way.

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