MATHEMATICAL MODELING, SELF-REPRESENTATION AND SELF-REGULATION

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The aim of the present study was to investigate the improvement of students' self-representation about their self-regulatory performance in mathematics by using mathematical modeling. Three materials were developed and administered at 255 11th years old students, for mathematical performance, self-representation and the use of self-regulatory strategies for problem solving. A web page with the proposed model (the model of Verschffel, Greer & De Corte, 2000) was constructed and used individually by students. Results indicated that the program created a powerful learning environment in which students were inspired in their own experiences. Although the program improved their cognitive and self-regulatory performance, it reproduced the differences among students in respect to their cognitive and metacognitive performance.

Keywords: self-regulation, self-representation, mathematical modeling

In the last decades, children's early understanding of their own as well as others mental states has been intensively investigated, reflecting growing interest for the concept of metacognition (Bartsch & Estes, 1996). In psychological literature, the term metacognition refers to two distinct areas of research: knowledge about cognition and self-regulation (Boekaerts, 1997). Self-regulation refers to the processes that coordinate cognition. It reflects the ability to use metacognitive knowledge strategically to achieve cognitive goals, especially in cases where someone has to overcome cognitive obstacles.

As regards the relationship between academic self-concept and academic achievement, extant literature supports both direct and indirect relationships between them; however, the range of correlations reported is a function of several factors (Guay, Marsh & Boivin, 2003). Age is a factor that affects this relationship since young students, academic self-concept is usually very positive and not highly correlated with external indicators, such as skills and achievement (Guay et al., 2003). Veenman and Spaans (2005) assumed that metacognitive skills initially develop on separate islands of tasks and domains. Beyond the age of 12, these skills will gradually merge into a more general repertoire that is applicable and transferable across tasks and domains. The present work is concentrated on the improvement of metacognitive performance on the domain of mathematics and more specifically on the improvement of self-regulatory behavior.

Learning mathematics, as an active and constructive process, implies that the learner assumes control and agency over his/her own learning and problem solving activities (De Corte, Verschaffel & Op´t Eynde, 2000). Knowing when and how to use cognitive strategies is an important factor to successful word problem solving (Teong, 2002). Metacognitive behavior can be applied in every stage of the problem solving activity (Lerch,2004). For example before starting solving a particular problem, students can ask themselves questions like what prior knowledge can help them develop a solution plan for the particular task; during the application of the solution plan the students monitor their cognitive activities and compare progress against expected goals. Finally, after reaching a solution, the students may need to look back, to check for the reasonableness of outcomes and integrate newly acquired knowledge to existing.

Problem solving procedure and the use of mathematical modeling

Studies on solving mathematical word problems refer to various conditions that cause transfer to occur, for example, providing solved examples (e.g. Bassok & Holyoak, 1989), having a scheme (Nesher & Hershkovitz, 1994), and providing feedback (Hoch & Loewenstein, 1992). The first step in solving a problem is to encode the given elements (Davidson & Sternberg, 1998). Encoding involves identifying the most informative features of a problem, storing them in working memory and retrieving from long-term memory the information that is relevant to these features. Incomplete or inaccurate metacognitive knowledge about problems often leads to inaccurate encoding and could generate learning obstacles.

A specific strategy frequently taught in math classes in order to enhance problem solving ability, is to use analogy in order to create a mental model of similar problems. In this regard, the students are expected to extract the relevant facts from the statement of the problem, compare it to their knowledge base, relevant to the problem domain, and recognize similarities between the new problem and problems they have previously encountered, and abstract the proper entities and principles. Empirical findings show that students fail to see the underlying principles unless they are explicitly pointed out (Panaoura & Philippou, 2005).

The modeling of open-ended problems have been of interest to mathematics educators for decades. Mathematical modeling of problem solving is a complicated procedure which is divided into different stages (Mason, 2001). When a mathematical modeling task is offered in a school the goal generally is not that students learn to tackle only that particular task. Rather, students are expected to recognize classes of situations that can be modeled by means of a certain mathematical concept, relation or formula, and to develop some degree of routine and fluency in mapping problem data to the underlying mathematical model and in working though this model to obtain a solution (Van Dooren, Verschaffel, Greer & De Bock, 2006).

A characteristic is that the modeling process is not a straightforwardly sequential activity consisting of several clearly distinguishable phases. Modellers do not move sequentially through the different phases of the modeling process, but rather run through several modeling cycles wherein they gradually refine, revise or even reject the original model. The present paper discusses the impact of the use of the mathematical model proposed by Verschaffel et al. (2000) on the development of students' self-representation about their self-regulatory behavior in mathematics. The main stages of the model are: 1) Understanding the phenomenon under investigation, leading to a model of the relevant elements, relations and conditions that are embedded in the situation (situation model), 2) Constructing a mathematical model of the relevant elements, relations and conditions available in the situation model, 3) Working through the mathematical model using disciplinary methods in order to derive some mathematical results, 4) Interpreting the outcome of the computational work to arrive at a solution to the real – word problem situation that gave rise to the mathematical model, 5) Evaluating the model by checking if the interpreted mathematical outcome is appropriate and reasonable for the original problem situation, and 6) Communicating the solution of the original real – word problem.

At the first phase of the problem solving procedure by the use of the mathematical model students have to consider and decide what elements are essential and what elements are less important to include in the situation model. In the next phase, the situation model needs to be mathematised i.e. translated into mathematical form by expressing mathematical equations involving the key quantities and relations. Students need to rely on another part of their knowledge base, namely mathematical concepts, formulas, techniques and heuristics. After the mathematical model is constructed and results are obtained by manipulating the model, numerical result needs to be interpreted in relation to the situation model. At this point, the results also need to be evaluated against the situation model to check for reasonableness. As a final step, the interpreted and validated result needs to be communicated in a way that is consistent with the goal or the circumstances in which the problem arose.

Nowadays problem solving skills have become a prominent instructional objective, but teachers often experience difficulties in teaching students how to approach problems and how to make use of proper mathematical tools. Many teachers of mathematics teach students to solve mathematical problems by having them copy standard solution methods. It comes as no surprise, therefore, that many students find it difficult to solve new problems, especially problems within a context (Harskamp & Suhre, 2006). Attempts to improve problem solving should focus on episodes students neglect when solving problems. The aim of the present study was to develop students' (5^{th} grade) problem solving ability and to enhance their ability to self-regulate their cognitive performance in order to overcome cognitive obstacles when they encounter difficulties while trying to solve mathematical problems. One of the main emphases was to oblige students reflect on their cognitive processes while trying to solve the problems and encounter difficulties in order to self-regulate their behavior. We hypothesized that the development of self-representation in order to be more accurate regarding the students' strengths and limitations would improve their self-regulatory behavior in mathematics. Especially for the problem solving procedure we hypothesized that the better distinction of problems and the clustering of those problems according to their similarities and differences would have as a consequence the better transfer of knowledge and strategies from the one domain to the others and from general situation to the specific ones.

METHODOLOGY

Participants: Data were collected from 255 children (107 experimental group and 148 control group), in Grade 5 (11 years old) from five different urban elementary schools. The participation at the program were voluntary because we had used the extra time students stayed at school for the program of the Ministry of Education, called "day-long school".

Procedure: The main emphasis was on the development of the program for the use of the proposed mathematical model, the training of students on the model and the evaluation of its results. At the first phase of the study three materials were constructed for pre and post test. The first one was about students' self-representation, the second for mathematical performance and the third one for their behavior while trying to solve mathematical problems. The first one comprised of 40 Likert type items of five points (1 = never, 2 = seldom, 3= sometimes, 4= often, 5= always), reflecting students' self-representation about mathematical learning (e.g."I can better explain my solution for a problem when I use a diagram", "I can easily compare two pictures in order to find their similarities". The reliability was very high (Cronbach's alpha was .87).

The second questionnaire comprised of 20 mathematical tasks on counting, geometry, statistics and problem solving (e.g. "How the area of a square, side 4cm, will be changed if the side is doubled", "Construct the bigger four digit number with the digits 9 and 3", "In our neighborhood every year since 2000 we organize a celebration, For the three following years, after the first one it did not organize. At what date (chronology) did it start again?") All items in the mathematical performance questionnaire were scored on a pass-fail basis (0 and 1). The reliability was high (Cronbach's alpha was 0.85).

The third questionnaire comprised of ten couples of sentences and students had to choose which one expressed better their cognitive behavior while they were encountering a difficulty in problem solving (a. When I explain to my friend how to solve a problem, I prefer to use a diagram, b. When I explain to my friend how to solve a problem I prefer to do it verbally). All the questionnaires were first used at a pilot study in order to examine their construct validity.

Then an intervention program was developed in order to propose the use of the mathematical model (Figure 1) for problem solving, proposed by Verschaffel et al. (2000). The emphasis was on the understanding that different stages of problem solving would have as a consequence the use of different cognitive procedures and that the cognitive obstacles could be encountered by realizing the cognitive interruptions at one or more of those stages and mainly by self-regulating the cognitive performance. For example a self-regulatory strategy is the ability to recognize the "inner" mathematical similarities and differences of mathematical problems in order to transfer cognitive and metacognitive strategies among different domains. For the purpose of the project we had constructed a web page which was visited individually by each student of the experimental group (107 students) during 20 "meetings". One of the main emphases was to oblige students rethink their cognitive processes while trying to solve the problems and encounter difficulties in order to monitor their performance.

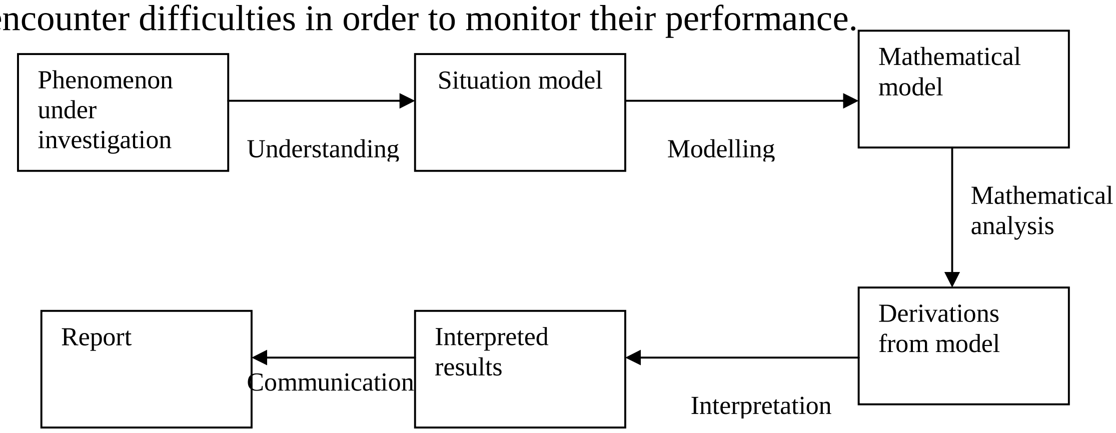

Figure 1: The mathematical model proposed by Verschaffel et al. (2000)

We had organized twenty "individual meetings" of the students with the webpage in order to work with the model (almost 20 minutes each meeting). Using the model used the first four "meetings" for the familiarization with the environment of the computer and for understanding the whole idea of the webpage for the problem solving procedure. The ten following "meetings" concentrated on different stages of the proposed mathematical model. For example at the stage of "understanding the problem" students had to solve problems with not enough data, or with more than the necessary data, they had to answer specific questions about the data of the problem, they had to explain in their own words the problem, to summarize it etc. At the stage of "modeling" they had to work on the classification of mathematical problems by explaining the criteria they used in order to classify the problems. There were problems with the same situational characteristics or the same context in order to oblige students to be concentrated on the structural mathematical characteristics. At the last six "meetings" students should solve mathematical problems by using all the stages of the mathematical model. In each stage the "cartoon" that was the hero of the web page asked questions such as "How did you get that? This isn't a better solution? (for a proposed solution). Do you have any better solution?", in order to force students to self-regulate their cognitive performance. We wanted to have a reflection at all the stages of their work. The students' responses were recorded automatically at a database with details such as when they had worked on the specific task and for how long. The whole procedure is presented at Figure 2.

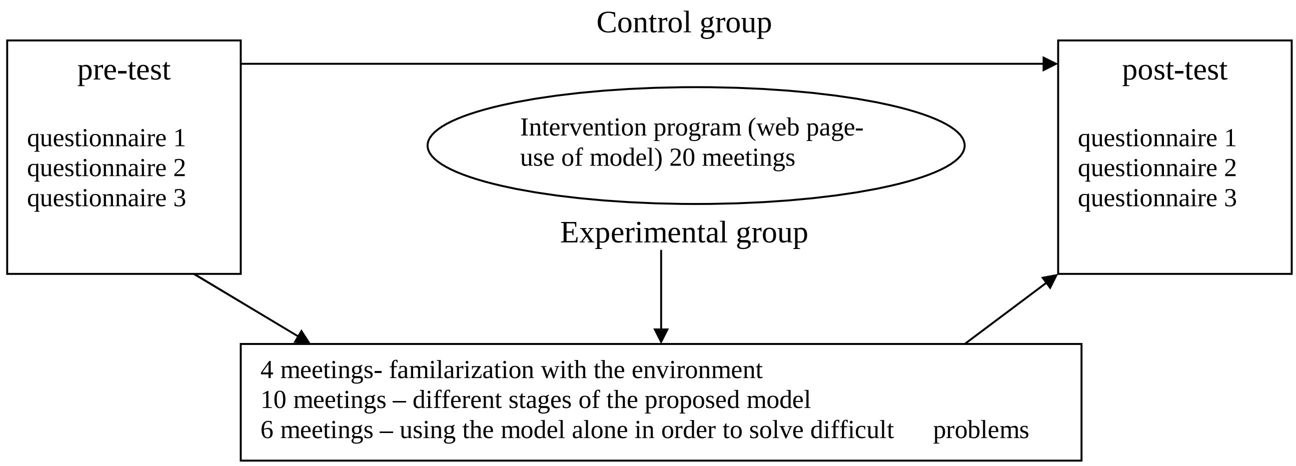

Figure 2: The development of the intervention program

RESULTS

The data about self-representation (1st questionnaire) were first subjected to exploratory factor analysis in order to examine whether the presupposed factors that guided the construction of the items of the first questionnaire were presented in the participants' responses. This analysis resulted in 6 factors with eigenvalues greater than 1, explaining 65.56% of the total variance. After the content analysis, according to the results of the exploratory factor analysis items were classified in the following factors: F1: general self-image about mathematics, F2: self-representation about problem solving abilities, F3: self-representation about the strategies used in order to self-regulate the cognitive performance, F4: self-representation about students' spatial abilities in mathematics, F5: self-representation about the degree of concentration on problem solving procedure, F6: the preference for different types of representations

We concentrated on the three factors which were related with self representation in respect to problem solving and self-regulation (F1, F2 and F3). The comparison of the means of the three factors between the pre and post tests for the experimental and the control group were statistically significant in all cases ($p<0.001$). Nevertheless the improvement was highest for the experimental group in the case of the second and the third factors (Table 1). It is obvious the increase of the control group as well as a consequence of the age development and the impact of teaching and learning (those were factors that could not be controlled). However the improvement was in all cases higher in the case of the experimental group.

	pre - test		post - test	
	experimental	control	experimental	control
F1	3.92	4.00	4.00	4.07
F2	3.22	3.25	3.69	3.57
F3	2.76	2.78	3.35	3.20

Table 1: The means of the experimental and the control group for the three factors at the pre and post test.

At the same time for the experimental group the improvement was highest in the case of the general mathematical performance ($\overline{X}_{1exp}$=0.27, $\overline{X}_{2exp}$=0.63, $\overline{X}_{1control}$=0.27, $\overline{X}_{2control}$=0.52) and the problem solving performance ($\overline{X}_{1exp}$=0.20, $\overline{X}_{2exp}$=0.47, $\overline{X}_{1control}$=0.20, $\overline{X}_{2control}$=0.39). Specifically the highest differences were found in the domain of geometry ($\overline{X}_{1exp}$=0.28, $\overline{X}_{2exp}$=0.47, $\overline{X}_{1control}$=0.29, $\overline{X}_{2control}$=0.44) and statistics ($\overline{X}_{1exp}$=0.38, $\overline{X}_{2exp}$=0.69, $\overline{X}_{1control}$=0.38, $\overline{X}_{2control}$=0.64). This result reveals the positive impact of the use of the specific mathematical model on the mathematical performance.

The most important in the case of self-representation is the accuracy of this feature in relation to the real mathematical performance. We have clustered, depended on cluster analysis, the participants in respect to their general self-image about their mathematical performance into three groups. The first group was consisted of 42 students with low self-image ($\overline{X}$=2.55), the second one of 82 students with medium self-image ($\overline{X}$=3.26) and the third one of 99 students with high self image ($\overline{X}$=3.94). There were statistically significant differences between the first and the third group at the initial phase (pre – test) in respect to their real mathematical performance (F=4.716, df=2, p=0.01, $\overline{X}_1$=0.466, $\overline{X}_2$=0.543, $\overline{X}_3$=0.605). After the program the difference of the groups regarding their general self-image in relation to their mathematical performance (post test) was significant only in the case of the experimental group (F=4.447, df=2, p=0.01, $\overline{X}_1$=0.557, $\overline{X}_2$=0.6059, $\overline{X}_3$=0.699). Those results indicated that most students had accurate self-image in respect to their real mathematical performance and they did not seem to overestimate their abilities. At the same time students' means at the classification of similar mathematical problems according to the mathematical structure of the problems were highest at the post test. The development was statistically higher in the case of the experimental group ($\overline{X}_1$=0.29, $\overline{X}_2$=0.49, t=12.79. p<0.001) than the control group ($\overline{X}_1$=0.29, $\overline{X}_2$=0.41, t=11.69, p<0.001). The difference between the two groups was statistically significant (t=3.32, df=228, p<0.01).

A part of the couples of sentences at the third questionnaire were about the self-regulatory strategies they use in order to encounter difficulties and cognitive obstacles at the problem solving procedure. For the self-regulatory strategies the difference of the means between the two measurements was statistically significant (t=2.93, df=98, $p<0.01$, $\overline{X}_1$=0.65, $\overline{X}_2$=0.69) only in the case of the experimental group. That means that students tended to develop more self-regulatory strategies or tended to believe that they have to develop those strategies. Even the second learning situation is an important step for the change of cognitive and metacognitive behavior, as well.

Students of the experimental group were clustered according to their self-representation about problem solving ability and their general mathematical ability into three groups (low self-representation: 24 students, medium: 36 students, and high self-representation: 34 students). Analysis of variance (ANOVA) indicated that there was a statistically significant difference concerning their self-representation about the use of self-regulatory strategies in mathematics ($F_{2,93}$ =6.094, p=0.003). As it was expected the mean of the group with the high self-representation was higher (0.80) than the other two groups (medium: 0.63 and low: 0.58). The most interesting result was that the students' with medium and low mathematical performance was increased after the program (low: $\overline{X}_1$=0.83, $\overline{X}_2$=0.87, medium: $\overline{X}_1$=0.90, $\overline{X}_2$=0.94, high: $\overline{X}_1$=0.94, $\overline{X}_2$=0.94). In the case of the improvement on the self-representation about the use of self-regulatory strategies for the three groups the changes were similar (low self-representation: $\overline{X}_1$= 0.50, $\overline{X}_2$= 0.53, medium self-representation: $\overline{X}_1$= 0.64, $\overline{X}_2$= 0.67, high self-representation: $\overline{X}_1$= 0.80, $\overline{X}_2$= 0.84). This stability or low increase may indicate that students realized their difficulties and limitations and did not tend to overestimate their abilities in using strategies.

DISCUSSION

Results confirmed that providing students with the opportunity to self-monitor their learning behavior in the case of encountering obstacles in problem solving through the use of modeling is one possible way to enhance students' self-representation about the self-regulatory strategies they use in mathematics and consequently their mathematical performance. It seems that the program with the use of the model created a powerful learning environment in which students were inspired in their own experiences. Nevertheless it is obvious that students with high self-representation about their mathematical abilities in the initial phase were at the same time students with the most self-regulatory strategies after the impact of the intervention program, as well. That means that although the program improved the metacognitive performance and the mathematical performance of the experimental group, further research is needed in order to find ways to change the initial differences among students.

For the development of a more accurate self-representation about mathematical performance and self-regulation in problem solving teachers must create a powerful learning environment, in which children are allowed and inspired to, their own learning experiences. According to the self-regulated learning approach students are self-regulating when they are aware of their capabilities of the strategies and resources required for effectively performing a task (Paris & Paris, 2001). Learners, who decide to ask a more competent person for assistance when faced with a task, indicate that they realize their difficulties and try to find out ways to overcome them. The accurate self-representation about the strengths and limitations is a presupposition for the development of self-regulation. Instruction should mainly lead students to self-questioning as a systematic strategy in helping them control their own learning and organize by themselves the different occasions they may encounter. In the area of mathematics, a number of important questions about metacognition remain unanswered. Much more research is needed to study the different aspects of metacognition in a more systematic and detailed way. We suggest specifically that further research could focus on interactive computer programs that may be designed to provide feedback and hints to assist students in becoming more aware of their cognitive and metacognitive processes. It would be optimistic and naïve to claim that such types of intervention programs would develop the self-regulatory strategies of all students. Possibly different models and programs are suitable for different groups of students.

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Acknowledgements

This paper draws from the Research Project ALPAS, which was funded by the Cyprus Research Foundation with the participation of the authors and the contribution of Erik De Corte and Wim Van Dooren (University of Leuven).