STUDENTS' BELIEFS ABOUT THE USE OF REPRSENTATIONS IN THE LEARNING OF FRACTIONS

Athanasios Gagatsis, Areti Panaoura*, Eleni Deliyianni**and Iliada Elia****

** Frederick University, ** University of Cyprus*

Cognitive development of any mathematical concept is related with affective development. The present study investigates students' beliefs about the use of different types of representations in understanding the concept of fractions and their self-efficacy beliefs about their ability to transfer information between different types of representations. The interest is concentrated on differences among students at primary and secondary education. Results indicated that students at secondary education have less positive beliefs for the use of representations at the learning of mathematics than at primary education. As a consequence they have less positive self-efficacy beliefs about their abilities to use them. Unexpected was their lower performance at solving tasks on fractions for which the information is represented in different forms.

Keywords: representations, beliefs, self-efficacy, fractions

Mathematics is a specialized language with its own contexts, metaphors, symbol systems and purposes (Pimm, 1995). From an epistemological point of view there is a basic difference between mathematics and other domains of scientific knowledge as the only way to access mathematical objects and deal with them is by using signs and semiotic representations (Duval, 2006). Cognitive development is related with metacognitive and affective development. One's behavior and choices, when confronted with a task, are determined by her/his beliefs and personal theories, rather than her/his knowledge of the specifics of the task. Thus, students' academic performance somehow depends on what they have come to believe about their capability, rather than on what they can actually accomplish.

The relationship between cognition and affect has the last decades attracted increased interest on the part of mathematics educators, particularly in the search for causal relationship between affect and achievement in mathematics (Young, 1997). This is due to the fact that the mathematical activity is marked out by a strong interaction between cognitive and emotional aspect. The affective domain is a complex structural system consisting of four main dimensions or components: emotions, attitudes, values and beliefs (Goldin, 2001). At the present study we focus on students' beliefs and mainly their self-efficacy beliefs in using different types of representations in mathematics learning and understanding. We concentrated our attention on the notion of fractions.

Fractions are among the most essential (Harrison & Greer, 1993), but complex mathematical concepts that children meet in school mathematics (Charalambous & Pitta-Pantazi, 2007). An important factor that may contribute to students' difficulties in learning fractions is the transition from primary in secondary school with all the changes that this encompasses in mathematical teaching and learning.

THEORETICAL BACKGROUND

Self-efficacy beliefs

Beliefs is a multifaceted construct, which can be described as one's subjective "understandings, premises, or propositions about the world" (Philipp, 2007, p. 259). According to Pehkonen and Pietila (2003) there are several difficulties in defining concepts related to beliefs. Some researchers consider beliefs to be part of knowledge (e.g. Pajares, 1992), some think beliefs are part of attitudes (e.g. Grigutsch, 1998), and some consider they are part of conceptions (e.g. Thompson, 1992).

The construct of self-efficacy beliefs constitutes a key component in Bandura's social cognitive theory; it signifies a person's perceived ability or capability to successfully perform a given task or behavior. Bandura (1997) defines self-efficacy as one's perceived ability to plan and execute tasks to achieve specific goals. He characterized self-efficacy as being both a product of students' interactions with the world and an influence on the nature and quality of those interactions. Self-efficacy beliefs have received increasing attention in educational research, primarily in studies for academic motivation and self-regulation (Pintrich & Schunk, 1995). It was found that self-efficacy is a major determinant of the choices that individuals make, the effort they expend, the perseverance they exert in the face of difficulties, and the thought patterns and emotional reactions they experience (Bandura, 1986). Furthermore, self-efficacy beliefs play an essential role in achievement motivation, interact with self-regulated learning processes, and mediate academic achievement (Pintrich, 1999).

Multiple representations in mathematics teaching and learning

The representational systems are fundamental for conceptual learning and determine, to a significant extent, what is learnt (Cheng, 2000). Learning involves information that is represented in different forms such as text, diagrams, practical demonstrations, abstract mathematical models, simulations etc (Schuyter & Dekeyser, 2007). Recognizing the same concept in multiple systems of representations, the ability to manipulate the concept within these representations as well as the ability to convert flexibly the concept from one system of representation to another are necessary for the acquisition of the concept (Lesh, Post, & Behr, 1987) and allow students to see rich relationships (Even, 1998). Recently the different types of external representations in teaching and learning mathematics seem to become widely acknowledge by the mathematics education community (NCTM, 2000). The necessity of using a variety of representations or models in supporting and assessing students' constructions of fractions is stressed by a number of studies (Lamon, 2001). The geometric shapes used for introducing the continuous model of fractions are distinguished into two types: the circular model which is based on the use of circles and the linear model which is based on a rectangle divided into a number of equal part (Boulet, 1998).

An issue that has received major attention from the education community over the last years refers to the students' difficulties when moving from elementary to secondary school and to the discontinuities in the curriculum requirements, the use of teaching approaches, aids and methods. According to Schumacher (1998) the transition to secondary school is accompanied by intellectual, moral, social, emotional and physical changes. Pajares and Graham (1999) investigated the extent to which mathematics self-beliefs change during the first year of middle school. By the end of the academic year, students described mathematics as less valuable, and they reported decreased effort and persistence in mathematics. The findings of the Deliyianni, Elia, Panaoura and Gagatsis's (2007) study suggest that there is a noteworthy difference between elementary and secondary education in Cyprus concerning the representations used in mathematics textbooks on fractions. There are also differences in the functions the various representations in the school textbooks fulfil.

The present study investigated Grade 5 to Grade 8 students' beliefs about the use of different representations for the learning of the fractions and their self-efficacy beliefs about the use of those types of representations. That means that it explores the differences of students' beliefs at primary and secondary education concerning the use of different types of representations.

METHOD

The study was conducted among 1701 students of 10 to 14 year of age who were randomly selected from urban and rural schools in Cyprus. Specifically, students belonging to 83 classrooms of primary (Grade 5 and 6) and secondary (Grade 7 and 8) schools (414 in Grade 5, 415 in Grade 6, 406 in Grade 7, 466 in Grade 8) were tested.

A questionnaire was developed for measuring students' beliefs about the use of different types of representations for understanding the concept of fractions. The questionnaire comprised of 27 Likert type items of five points (1=strongly disagree, 5=strongly agree). The reliability of the whole questionnaire was very high (Cronbach's alpha was 0.88). The items of the questionnaire are presented at Table 1.

At the same time a test was developed for measuring students' ability on multiple representation flexibility as far as fraction addition is concerned. The test included 22 fraction addition tasks that examine multiple-representation flexibility and problem-solving ability. There were treatment, recognition, conversion, diagrammatic problem-solving and verbal problem-solving tasks (further details for the tasks can be found at the paper of Deliyianni et al. (2007). Indicative examples of the items are presented at Appendix. Cronbach's alpha for the test was 0.87.

The test and the questionnaire were administered to the students by their teachers at the end of the school year in usual classroom conditions. Right and wrong or no answers were scored as 1 and 0, respectively. Solutions in treatment, recognition and translation tasks were assessed as correct if the appropriate answer, diagram, equation or shading were given respectively, while a solution in the problems was assessed as correct if the right answer was given.

RESULTS

The analysis of students' responses to the items of the questionnaire resulted in six factors (KMO=0.933, $p<0.001$) with eigenvalues greater than 1 (Table 1). The first factor corresponded to students' self-efficacy beliefs about conversion from one type of representation to another. The second factor was associated with their general self-efficacy beliefs in mathematics. The third factor represented their beliefs about the use of the number line, while the forth factor represented their beliefs about the use of models, materials or representations. The fifth factor corresponded to students' beliefs about the use of diagrams in problem solving and the sixth factor to their self-efficacy beliefs about the use of verbal representations.

Item	**F1**	**F2**	**F3**	**F4**	**F5**	**F6**
I can easily find the diagram that corresponds to an equation of fractions.	.53					
I can easily solve tasks than ask toconverse the part of a diagram into an equation.	.62					
I can easily find the diagram that corresponds to an equation of decimals.	.67					
I can easily find the equation of fraction addition that corresponds to a part of a surface of a rectangle.	.63					
I can easily find the equation of fraction addition which is presented with arrows in number line.	.58					
I am very good in solving tasks with decimals.		.70				
I am very good in problem solving fractions.		.78				
I can easily solve tasks with fractions.		.79				
I can easily solve equations of fraction addition.		.70				
I can easily solve equation of decimal addition.		.56				
Number line helps me in problem solving with fractions.			.68			
Number line helps me in solving equations with fractions.			.68			
My teacher usually uses number line in order to explain us the operations of fractions.			.64			
Number line helps me in solving equations with decimals.			.64			
A good student in mathematics can present the solution of a problem by many different ways.				.55		
For the problem solving the use of equation is necessary.				.65		
In mathematics the use of materials (fraction circles, dienes cubes etc) is						

useful mainly for students at primary education				.59		
The diagrams (number line, rectangle etc) are useful for executing operations.				.42		
If I have to explain how I have solved a problem with decimals, I prefer to use an equation.				.57		
If I have to explain how I have solved a problem with fractions, I prefer to use a diagram.					.65	
When I solve a problem with fractions, I use the number line for executing the operations.					.44	
When I solve a problem with fractions by using a diagram, I then try to solve it by using an equation, as well.					.49	
When I solve a problem with decimals I use a diagram.					.67	
I can easily explain how I have solved a problem with decimals by using a diagram.					.47	
I prefer solve problems with decimals which present the data verbally.						.79
I can easily explain verbally how I have solved a problem with decimals.						.69
Eigenvalues	7.87	2.48	1.92	1.58	1.25	1.17
Percentage of variance explained	24.6	9.76	6.77	5.01	4.20	3.34
Cumulative percentage of explained variance	24.6	34.3	41.1	46.1	50.3	53.6

Table 1: Factor loading of the six factors against the items associated with participants' beliefs

Analysis of variance (ANOVA) indicated that there were statistically significant differences in respect to grade for the factors F1, F2, F5 and F6. Specifically in the case of F1 there were differences at the means ($F_{3,1547}$=9.09, p<0.001) between students' self-efficacy beliefs to converse flexibly the concept of fraction addition from one representation to any other who were attending the Grade 8 with the students of the Grades 5, 6 and 7. In the case of the F2 the statistically significant differences ($F_{3,1574}$=31.615, p<0.001) were between the Grade 5 with Grades 7 and 8, the Grade 6 with the Grade 7 and 8. Students at the Grade 8 seemed to have less positive beliefs for the significance of using different types of representations (F5). There were statistically significant differences between Grade 5 and Grade 8, Grade 6 and Grade 8. In the case of their preference for using verbal explanations the differences were between Grade 5 with Grade 7 and 8 and Grade 6 with Grade 7 and 8. Therefore, most of the differences revealed were between the students at primary education and the students at secondary education. All the means are presented at Table 2.

	F	p	$\overline{X}_5$	$\overline{X}_6$	$\overline{X}_7$	$\overline{X}_8$
F1	9.09	<0.001	3.63	3.58	3.53	3.37
F2	31.615	<0.001	4.08	3.93	3.70	3.56
F5	6.209	<0.001	3.29	3.24	3.17	3.09
F6	21.036	<0.001	3.46	3.36	3.15	2.96

Table 2: The means for the factors F1,F2, F5 and F6 with the statistically significant differences in respect to grade.

Very impressive and unexpected were the descriptive results of the students' mathematical performance at the test. As it is obvious in Figure 1 students at the Grade 7 have lower performance than the students at the Grade 6.

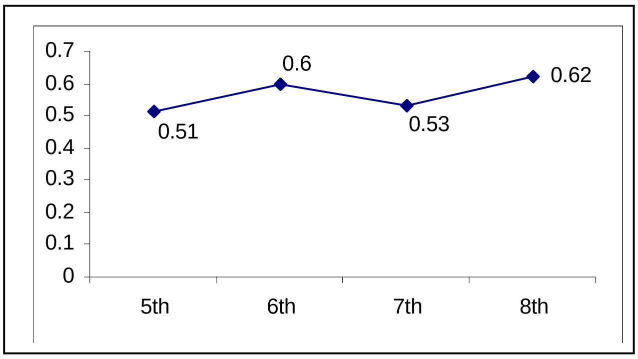


Figure 1: Students' of different grades performance on the mathematical test.

Students were cluster, by using cluster analysis, according to their performance at the test into three groups (Group1: 426 students with low performance, Group2: 788 students with medium performance, Group3: 487 students with high performance) Analysis of variance (ANOVA) with independent variable the three groups and dependent variables the six factors, which were comprised from the abovementioned factor analysis, indicated statistically significant differences in respect to F1 ($F_{2,1547}$=51.819), F2 ($F_{2,1474}$=74.903), F4 ($F_{2,1609}$=12.057) and F6 ($F_{2,1671}$=8.844). In all cases the first group had the most negative beliefs and self-efficacy beliefs and the third group had the most positive beliefs. That means that students with high mathematical performance had at the same time positive beliefs for the use of representations and high self-efficacy beliefs.

Finally students were clustered into two groups according to their general self-efficacy beliefs in mathematics (F2), by using cluster analysis. The group with higher self-efficacy beliefs consisted of 1047 students ($\overline{X}$ =4.31) and the second group consisted of 528 students ($\overline{X}$ =2.82).T-test analysis between the two groups in respect to the other five factors indicated that there were in all cases statistically significant (p<0.01) differences (Table 3). Students with higher general self-efficacy beliefs in mathematics had at the same time more positive beliefs for the use of different forms of representations and more positive self-efficacy beliefs for the use of those representations and their ability to transfer their knowledge.

	t	df	$\overline{X}_1$	$\overline{X}_2$
F1	22.82	1463	3.81	2.97
F3	6.508	1527	3.30	2.99
F4	13.897	1507	4.09	3.57
F5	9.151	1499	3.32	2.96
F6	14.616	1565	3.48	2.73

Table 3: Students' with high and low self-efficacy beliefs differences in respect to their beliefs about the use of representations

DISCUSSION

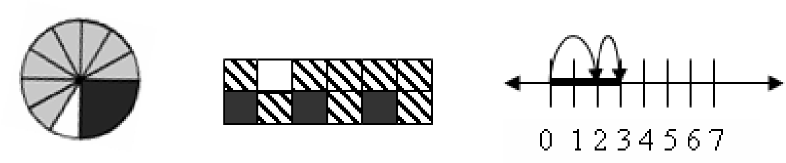
The main emphasis of the present study was on investigating students' self-efficacy beliefs for mathematics in relation to their beliefs about the use of representations for understanding the concept of fraction. The analysis of the data confirms earlier findings that young students have high self-efficacy beliefs (Bandura, 1986) and that they tend to overestimate their abilities. However those beliefs decreased at the secondary education. It seems that students' sense of efficacy diminishes somehow when they compare their abilities with classmates and even more in relation to their mathematical performance as it is revealed by their final grades at mathematics. The influence of those active experiences is too strong and with immediate results. Accepting that the most important step is getting individuals to become aware of their own processes, strengths and limitations in order to have an accurate self-representation, it seems that the specific result is important for the learning of the concept of fractions. Nevertheless it is not positive generally, because there are too many other concepts at the teaching of mathematics at secondary education for which students have to use flexibly different types of representations. For example the concept of function admits a variety of representations, each of which offers information about particular aspects of the concept without being able to describe it completely (Elia et al., 2008).

Interesting and unexpected was the differences between students' performance in the use of different forms of representations at primary and secondary education and mainly the lower performance at secondary education. A possible explanation for the lack of improvement regarding their mathematical performance observed are the differences regarding the representations and their functions in mathematics textbooks used in primary and secondary education in Cyprus (Deliyianni et al., 2007). Furthermore, the secondary school students may had not created referential connections between corresponding elements and related structures in a way that promotes understanding of this concept during their primary schooling. Their difficulties increased in secondary education since no emphasis is placed on learning with multiple representations.

Results confirmed that students with low performance in mathematics have at the same time negative beliefs for the use of different forms of representations because they cannot use them fluently and flexibly as a tool to overcome obstacles while solving tasks and handling the whole situation. It seems that there is a need for further investigation into the subject with the inclusion of a more extended qualitative and quantitative analysis. Most mathematics textbooks today make use of a variety of representations more extensively than every before in order to promote understanding (Elia, Gagatsis & Demetriou, 2007). Much more research is needed for the students' beliefs about the role of those representations regarding different mathematical concepts in relation to their self-efficacy beliefs for using them as a tool for the better understanding of the concepts.

Appendix

1. Circle the diagram or the diagrams whose shaded part corresponds to the equation 2/3 + 1/4.



(*recognition*)

2. Solve the following equation

1/6 + 2/5 = ….. (*treatment*)

3. Write the fraction equation that corresponds to the shaded part of the following diagram:

Equation: .... (*conversion*)

References

Bandura, A.:1986, *Social foundations of thought and action: A social cognitive theory*. Englewood Cliffs, NJ: Prentice-Hall.

Bandura, A.:1997, *Self-efficacy: The exercise of control*. New York: Freeman.

Boulet, G.:1998, Didactical implications of children's difficulties in learning the fraction concept, *Focus on Learning Problems in Mathematics, 20* (4), 19–34.

Charalambous, C.,& Pitta-Pantazi, D.:2007, Drawing on a theoretical model to study students' understandings of fractions. *Educational Studies in Mathematics, 64*, 293- 316.

Cheng, P.C.H.:2000, Unlocking conceptual learning in mathematics and science with effective representational systems. *Computers and Education, 33*, 109-130.

Deliyianni, E., Elia, I, Panaoura, A., & Gagatsis, A.:2007, The functioning of representations in Cyprus mathematics textbooks. In E. P. Avgerinos & A. Gagatsis (Eds.), *Current Trends in Mathematics Education* (pp. 155- 167). Rhodes: Cyprus Mathematics Society & University of Aegean.

Duval, R.:2006, A cognitive analysis of problems of comprehension in learning of mathematics. *Educational Studies in Mathematics, 61*, 103- 131.

Elia, I., Panaoura A., Gagatis, A., Gravvani, K. & Spyrou, P.:2008, Exploring different aspects of the understanding of function: Toward a four-facet model. *Canadian Journal of Science, Mathematic and Technology Education, 8* (1), 49-69.

Elia, I., Gagatsis, A. & Demetriou, A.:2007, The effects of different modes of representation on the solution of one-step additive problems, *Learning and Instruction, 17* (6), 658-672.

Even, R.:1998, Factors involved in linking representations of functions. *The Journal of Mathematical Behavior*, *17*(1), 105-121.

Goldin, G.:2001, Systems of representations and the development of mathematical concepts. In Cuoco, A. A. & Curcio, F. R. (Ed.): *The roles of representation in school mathematics*. Yearbook. Reston, VA: National council of teachers of mathematics, 1-23.

Grigutsch, S.: 1998, On pupils' views of mathematics and self-concept: developments, structures and factors of influence. In E. Pehkonen & G. Törner (Eds.) *The state-ofart in mathematics-related belief research. Results of the MAVI activities (pp.169-197)*. University of Helsinki. Department of Teacher Education. Research report 195.

Harrison, J., & Greer, B.:1993, Children's understanding of fractions in Hong Kong and Northern Ireland. In I. Hirabayashi and N. Nohda (Eds.), *Proceedings of the 17th Conference for the Psychology of Mathematics Education, 3* (pp. 146-153). Tsukuba: University of Tsukuba.

Lamon, S. L.:2001, Presenting and representing: From fractions to rational numbers. In A. Cuoco & F. Curcio (Eds.), *The roles of representations in school mathematics-2001 yearbook* (pp. 146-165). Reston, VA: NCTM.

Lesh, R., Post, T., & Behr, M.:1987, Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics,* (pp. 33-40). Hillsdale, N.J.: Lawrence Erlbaum Associates.

National Council of Teachers of Mathematics:2000, *Principles and standards for school mathematics.* Reston, Va: NCTM.

Pajares, M. F.:1992, Teacher's beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, 62, 307-322.

Pajares, F. & Graham, L.: 1999, Self-efficacy, motivation constructs, and mathematics performance of entering middle school students, *Contemporary Educational Psychology,* 24, 124-139.

Pehkonen, E. & Pietilä, A.:2003, *On Relationships between Beliefs and Knowledge in Mathematics Education*. In: Proceedings of the CERME-3 (Bellaria) meeting. http://www.dm.unipi.it/~didattica/CERME3/draft/proceedings_draft/TG2_draft/

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 257-315). Charlotte: NCTM.

Pimm, D.:1995, *Symbols and meanings in school mathematics*. Routledge.

Pintrich, P.R.:1999, The role of motivation in promoting and sustaining self-regulated learning. *International Journal of Educational Research, 31*, 459-470.

Pintrich, P. R., & Schunk D. H.:1996, *Motivation in education: Theory, research, and applications*. Englewood Cliffs, NJ: Merrill/Prentice Hall.

Schumacher, D.:1998, *The transition to middle school*. ERIC Digest (On-line). ED433119. Abstract from: ERIC Clearinghouse on Elementary and Early Childhood Education. Champaign, IL. Available at: http://www.ed.gov/databases/ERIC_Digests/ed422119.html.

Schuyter, G. & Dekeyser, H.: 2007, Preference for textual information and acting on support devices in multiple representations in computer based learning environment for statistics. *Computers in Human Behavior, 23 (5),* 2285-2301.

Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws (Ed.), *Handbook of research in mathematics teaching and learning.* (p. 127 - 146) New York: MacMillan.

Young, D.J.: 1997, *A multi-level analysis of science and mathematics achievement.* Paper presented at the annual a meeting of the American Research Association in Chicago. Illinois.

Acknowledgements

This paper draws from the medium research project MED19 that investigated the role of multiple representations in mathematics learning during the transition within and between primary and secondary school, which is supported by the University of Cyprus.