"MATHS AND ME": SOFTWARE ANALYSIS OF NARRATIVE DATA ABOUT ATTITUDE TOWARDS MATH

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Some years ago we undertook a research study aimed to obtain a 'grounded' characterization of attitude toward maths through the use of a narrative tool: we assigned to a large sample of Italian students the essay "Maths and me", collecting more than 1600 texts. In this contribution we present some preliminary results, obtained using a piece of software for text analysis, regarding the way students of different grades describe their relationship with mathematics. In particular, we discuss the results from a comparative analysis between students of different school levels in order to find analogies and differences in the description of their own relationship with maths.

INTRODUCTION

Many research studies carried out in the last two decades in mathematics education highlight the relevance of affective factors to analyze and interpret students' maths-related difficulties, and a specific field of research developed in recent years (for an overview see Zan R., Brown L., Evans J., Hannula M. 2006).

Among the affective factors, attitude toward mathematics is one of the most quoted constructs (by researchers in the field, teachers and educational institutions), but this "object" does not seem to have a well-defined and shared meaning. Among studies that explicitly give a definition, we can recognize three main different characterizations of attitude towards mathematics:

a) a "*simple*" definition, that describes attitude as the positive or negative degree of affect associated with mathematics (Haladyna, Shaughnessy J. & Shaughnessy M., 1983; McLeod, 1992);

b) a "*tridimensional*" definition, that recognizes three components in attitude: the degree of affect associated with mathematics, the beliefs regarding mathematics and the behaviour related to mathematics (Hart, 1989);

c) a "*bidimensional*" definition, that includes only emotions and beliefs and does not consider behaviour (Daskalogianni & Simpson, 2000).

Some critical issues are linked to the choice of a definition for attitude (Di Martino & Zan, 2001), in particular: the consistency between the chosen definition of attitude and the instruments to observe/measure it, the definition of *positive/negative* attitude in the case of multidimensional characterizations. To characterize students' attitude toward mathematics *from the bottom*, we carried out a narrative study investigating which dimensions students use to describe their relationship with mathematics. After

the characterization with the same data we could compare attitude of students belonging in different school levels.

In the field of mathematics education, narratives are more and more often used, especially in research about teachers' beliefs and teachers' practice (f.e. Da Ponte, 2001). Outside the field of teacher education, less numerous studies about *affect* make use of narratives: some have adults as their object (Karsenty & Vinner, 2000), others used narrative to report their own research (Hannula, 2003), others have students as their object (Ruffell et al.,1998). In this last case the studies are often carried out to criticize traditional instruments used to observe attitude rather than to carachterize from the bottom the construct itself.

We used students' narratives (autobiographic essay), confident that in this way students could have the possibility to talk about the aspects *they* considered relevant in their own experience with mathematics. The chosen instrument is consistent with an interpretive approach and allows many typologies of data analysis.

From a qualitative analysis of students' description of their relationship with mathematics (Di Martino & Zan, submitted), a multidimensional model for attitude toward mathematics emerges, characterized by three strictly interconnected dimensions: the emotional disposition toward mathematics, the view of mathematics, the perceived competence in mathematics. That suggests the need to overcome the dichotomy between positive/negative attitude, and move to the identification of different profiles of negative attitude.

In this contribution, we present a quantitative analysis of the same data carried out with the help of T-Lab [1], a powerful software for text analysis, giving some preliminary interpretations of these results: in particular comparing the attitude of students from different educational levels.

METHODOLOGY

We proposed the essay "Me and mathematics: my relationship with maths up to now" to students from different school levels. For the administration of the essays we gave the following guidelines: essays had to be anonymous, assigned and collected in the class not by the mathematics teacher. At the end, we collected 1662 essays [2] ranging from grade 1 to grade 13: 874 from 51 classes of 14 primary schools (grade 1-5); 368 from 24 classes of 8 middle schools (grade 6-8); 420 from 29 classes of 10 high schools (grade 9- 13).

In order to perform the statistical analysis with T-Lab we typed all data in a unique Corpus, respecting some specific guidelines, and we classified all essays with three control variables: identification number, grade and school level.

After this phase of data coding, we started to set the customized settings: selection of the lexical units to be included in the analysis, management of the lemmatization's phase, that is the reduction of the Corpus to their respective headwords called *lemmas* (for example general rules of lemmatization are: verbs' forms are taken back to the

infinite tense, nouns to the singular form, and so on).

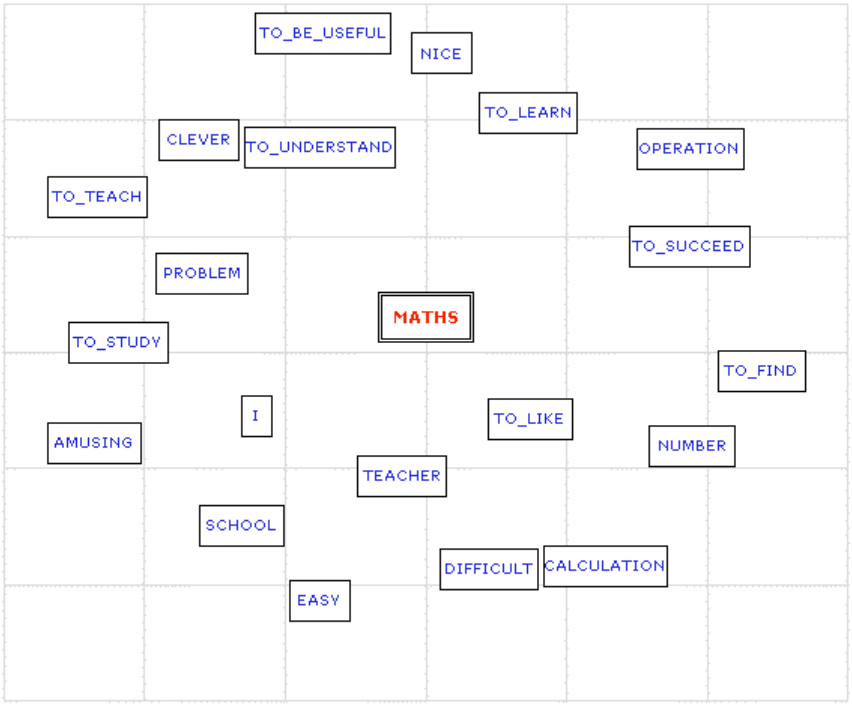
RESULTS AND DISCUSSION

Our attention will be focused on two typologies of analysis: co-occurrence and comparative analysis. The first one is finalized to find lexical units that more frequently are in co-occurrence [3] with some specific lemma, the latter is finalized to identify differences between texts from different subsets of the Corpus identified by some variables (in our case we selected the variable *school level*).

Co-occurrence analysis

Starting from the choice of the key-term '*maths*', the software calculates, in the whole Corpus, the lemmas with more co-occurrence with it through the association index of cosine [4]. This is a way to have a preliminary idea about the lexical units that students, in their autobiographical essays, more frequently associated with maths. Graph 1 is one of the outputs of the analysis: the nearness of each lemma to the central lemma '*maths*' is proportional to its degree of association.

Graph 1: Lemmas associated with *maths*

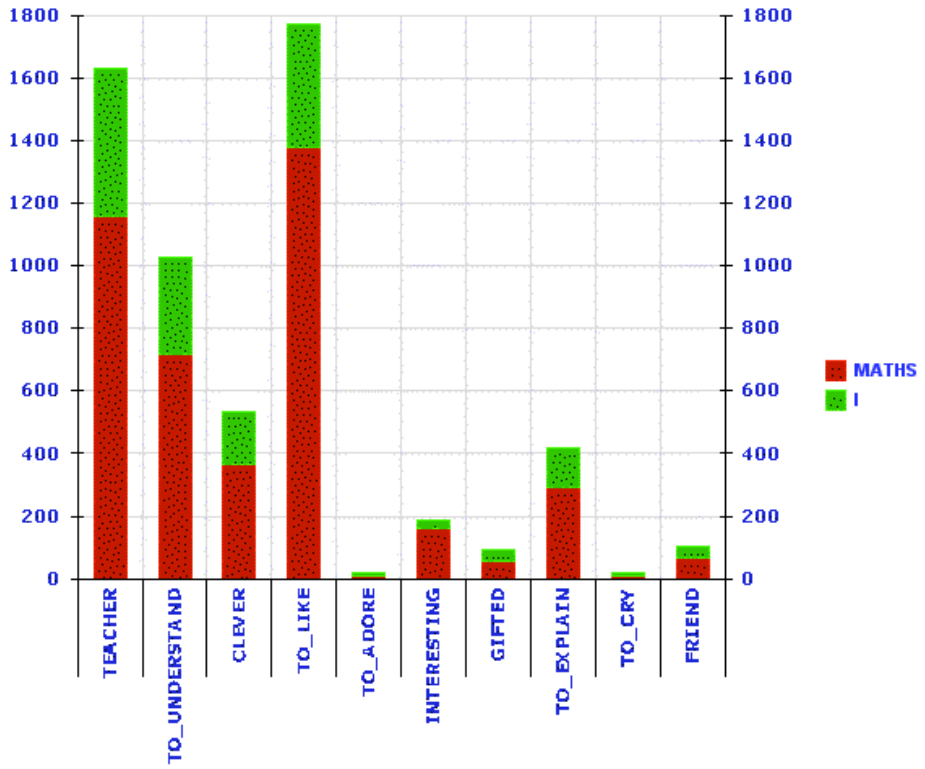


This representation strikingly shows that the emotional disposition (concisely expressed by "*I like/do not like maths*") is very often in co-occurrence with maths: this is an indication that students tend to express their emotional disposition toward mathematics when they tell their relationship with mathematics itself. Moreover, the nearness of '*teacher*' can be interpreted in light of the fact that students recognize the teacher as a protagonist of their story with maths. For what concerns '*I*', it is obvious that, in an autobiographical essay regarding the writer's relationship with maths, the

lemmas *I* and *maths* are in co-occurrence.

Another analysis enables us to find the lemmas that are more correlated to both terms: *I* and *maths*. In graph 2 the co-occurrence with the two terms is shown in decreasing order with respect to the chi square test [5].

Graph 2: co-occurrences with *I* and *maths*



The relevance of the *teacher* in students' building of their own relationship with maths seems to be confirmed. But other two dimensions emerge heavily: an affective one (linked to lemmas as *to_like*, *to_adore*, *to_cry* and also *friend*) and one correlated with the idea of success in maths (associated to lemmas as *to_understand*, *clever*, *gifted*).

Comparative analysis

As we said earlier, with this typology of analysis we try to underline the differences between the three groups of students, as identified by the variable '*school level*'.

The first analysis regards the specificities of each group: T-Lab compares the subset A of the Corpus with the rest of the Corpus, individualizing which lexical units are typical (by the Chi-square test) or exclusive of the subset A. In table 1, for each group (Primary, Middle, High) the ten lemmas with the biggest chi-square value are reported.

Table 1: Specificities of three school levels

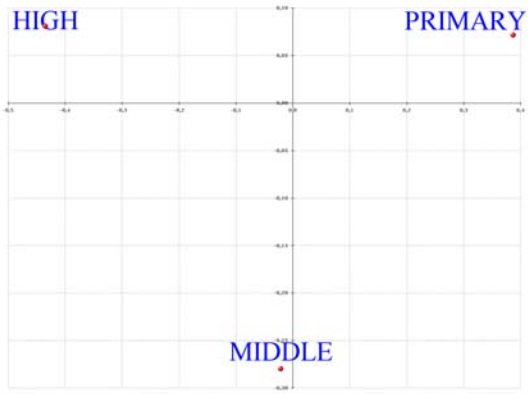
Primary school	Middle school	High School

WORD	CHI²	SUB	TOT	WORD	CHI²	SUB	TOT	WORD	CHI²	SUB	TOT
to_like	459,7	1560	2488	expression	63,7	114	282	to_succeed	254,5	645	1078
operation	222,2	397	539	rule	38,3	50	111	school	185,8	399	638
to_learn	166,0	535	837	to_study	31,2	211	714	to_study	87,9	382	714
nice	149,8	302	423	algebra	27,2	48	118	exam	72,2	66	80
amusing	141,9	284	397	complex	25,9	39	91	time	69,1	156	252
examination	135,4	179	222	arithmetic	21,6	56	154	task	65,9	201	349
number	111,3	410	658	complicated	20,1	56	157	method	60,3	64	82
geometry	89,0	376	619	Easy	17,9	119	401	teacher	52,8	955	2168
calculation	67,6	250	401	important	17,3	84	267	to_apply	52,2	56	72
error	63,5	151	220	maths	14,0	1288	5603	insufficient	49,9	49	61

Sub = number of word's occurrences in subset, Tot = number of word's occurrences in Corpus

One interesting remark is about the strong characterization of the two extreme groups (Primary and High), testified by very high chi-square values. Moreover, looking at the first lemmas for each group, we can observe a shift from a *mastery-oriented* view (*to_learn*) of the relationship to a *performance-oriented* view (*to_succeed*, *exam*) and it is also interesting that the two first lemmas of the Middle group are related to an instrumental view of mathematics. According to our qualitative findings (Di Martino & Zan, submitted), this instrumental view is often combined with negative emotions towards mathematics and low perceived competence. This can be a possible explanation of the fact that in Italy the relationship with mathematics often becomes problematic just at middle school level. A factorial analysis allows us to characterize more precisely the specificity of the three groups: we can visualize their position in the factorial plane.

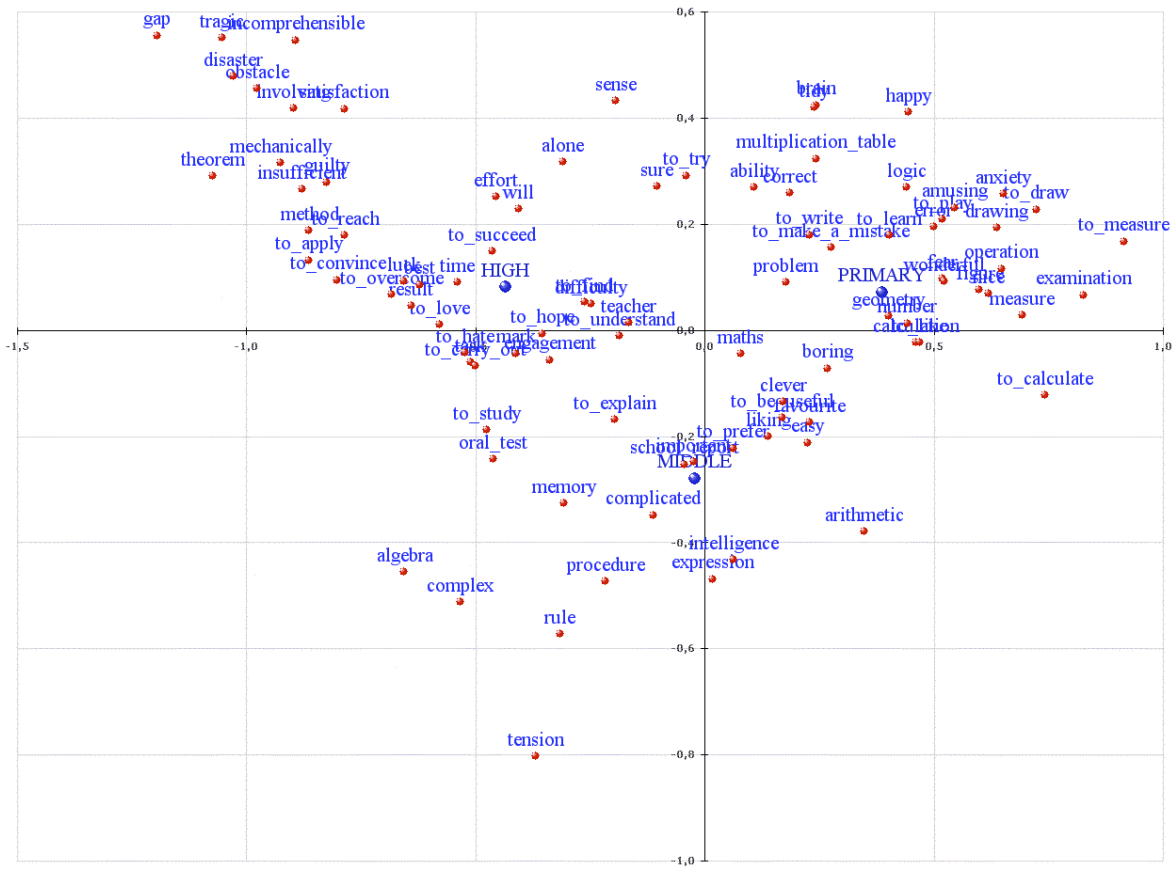
Graph 3: variables' position in factorial plane



So we can observe that Primary and High groups are opposite poles in the X axis, while Middle group is characterized by its negative Y-component. In graph 4 all

lemmas that define the factorial plane are reported: this allows us to interpret the meaning of the distance between the three groups:

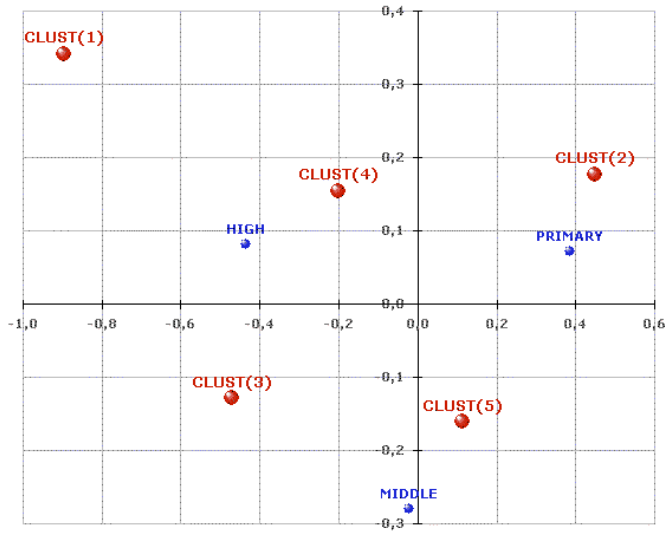
Graph 4: lemmas in factorial plane



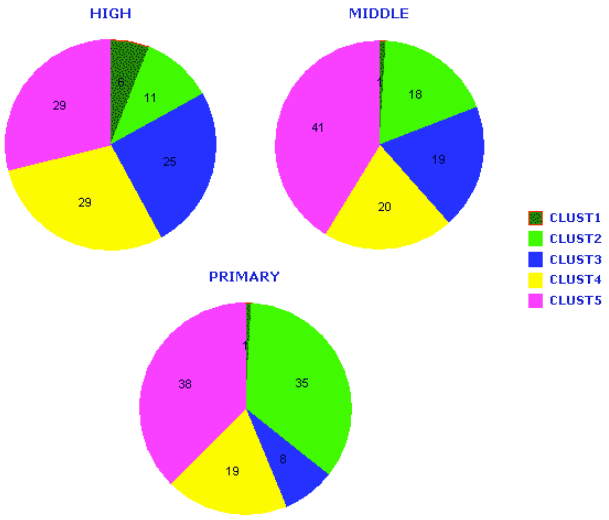
This analysis confirms the interpretations following the analysis of specificities. In particular, the Primary group seems to be characterized by descriptive – illustrative lemmas regarding mathematics (*geometry*, *number*, *calculation*, *measure*, *problem*) and by an often positive judgement of one's mathematical experience (*wonderful*, *nice*). The Middle group, strongly positioned at the negative pole of the Y-axis with respect to the other two groups, has many lemmas referring to an instrumental view of mathematics (*procedure*, *memory*, *rule*). Moreover, this group is in the 0 of the X-axis that is also characterized by emotional responses. Finally, the High group is characterized by very strong emotions (*to_love*, *to_hate*) and also by a particular attention to succeed (*to_succeed*). To summarize these results, it seems that at the beginning of the school experience with mathematics, curiosity prevails over other aspects and *novelty* is often appreciated. Besides, there is little stress related to assessment. After the move to the middle school level, students' attention seems to shift toward some procedural aspects of mathematics, so an instrumental view of mathematics emerges. This view rarely arouses a strong passion (negative or positive). In High school we find opposite lemmas for what concerns emotions (*love*, *hate*) but also perception of success; perhaps, this means that the relationship toward mathematics of these students becomes more radical than the relationship reported by

their youngest colleagues. These interpretations are also reinforced by the cluster analysis that we performed with a partitioning method. We fixed to 5 the cluster numbers because with a smaller one we hadn't a clear distinction between groups identified by variables. We briefly report a table with the lemmas characterizing each cluster and the relationship between clusters and variables.

Graph 5: clusters and variables



Graph 6: Percentage groups subdivision in clusters



The percentages of cluster 1 are very small but it is present for any subdivision in clusters more than two. From an evolutionary point of view, we can observe that cluster 2 becomes less representative passing from 35% at Primary level to 11% at High level and cluster 5 is more or less stable from Primary to Middle level but becomes less representative at High level. While clusters 3 and 4 increase the number of their representatives. So it is very interesting to give a look to lemmas that characterize these four clusters in the following table (lemmas are in decreasing order

of relevance):

Table 2: description of clusters 2, 3, 4 and 5

Cluster 2

to_like, to_learn, number, geometry, operation, nice, calculation, amusing, error, examination, multiplication_table, to_write, to_make_a_mistake, fear, figure, logic, to_calculate, measure, drawing, correct, to_discover, wonderful, get_angry, question, to_play, to_draw, ability, exercise_book, brain, to_read, happy, to_worry, to_measure, anxiety, to_reproach, tidy, heart, to_sweat_blood, to_cry, gaiety, punishment, to_bore, mysterious, angry, test

Cluster 3

to_study, school, to_explain, mark, task, engagement, time, to_hate, to_hope, to_improve, to_carry_out, algebra, to_comprehend, rule, explanation, complex, oral_test, course_book, luck, best, future, to_love, worsening, resolution, cause, gifted, sincere, memory, reasoning, patience, to_overcome, positive, passion, to_forget, fundamental, serious, set_theory, possible, negative, genius, unpleasant, to_attract, to_fascinate, to_repeat_year, competition, to_give_up, theory, able, procedure, nightmare, frightened, torment, unlucky, serene, unbearable, tension, surprise, to_persecute, suffering

Cluster 4

teacher, to_understand, to_succeed, to_find, to_think, difficulty, interesting, to_know, to_believe, to_talk, formula, to_try, attention, will, ugly, to_memorize, immediately, friend, truth, effort, blackboard, sure, alone, strange, to_appreciate, idea, quiet, pleasant, clear, to_reflect, confuse, to_upset, experience, impossible, to_imagine, sense, thought, reality, stupid, to_resign, terrible, dream, terror, to_make_curious, hateful, slow, pride, success, disgusting, sadness, horrible, shame

Cluster 5

maths, I, problem, difficult, clever, to_teach, easy, boring, exercise, to_be_useful, certainty, expression, important, to_solve, simple, liking, arithmetic, useful, complicated, to_reason, game, quickly, severe, exciting, happiness, school_report, mathematician, to_implement, fascinating, tiring, to_support, challenging, to_listen, intelligence, shout, dubious, to_confuse, tremble

Cluster 2 is centred on the description of the *objects* of mathematics as well as on related activities (*to_learn*, *number*, *geometry*, *operation*, *calculation*, *multiplication_table*, *to_write*, *figure*, *logic*, *to_calculate*, *measure*, *drawing*, *to_discover*, *to_play*, *to_draw*, *exercise_book*, *to_read*, *to_measure*). Cluster 3 centres on theories of success (*to_study*, *engagement*, *time*, *to_comprehend*, *rule*, *cause*, *gifted*, *memory*, *reasoning*, *patience*,...) like cluster 4 (*to_understand*, *to_succeed*, *to_find*, *to_think*, *to_know*, *to_believe*, *formula*, *to_try*, *attention*, *will*,

effort,...), but whereas cluster 3 seems to be projected ahead (*to_hope, to_improve, future, to_overcome*), cluster 4 seems to be more *static* and centred on a definitive evaluation of what happened (*impossible, to_resign*,...), cluster 5 seems to be the cluster of balance between difficulties (*difficult, simple, complicated*,…) and usefulness (*to_be_useful, important*,…). Finally, all four clusters have some emotional components: surely clusters 3 and 4 are characterized by lemmas that evoke stronger emotions (*to_hate*, *to_love*, *nightmare*, *frightened*, *torment*, *tension*, *to_persecute*, *suffering* for cluster 3 and *terrible, terror*, *disgusting, hateful*, *pride*, *horrible*, *shame*) than cluster 2, which seems to be the one with the highest number of lemmas linked to *positive* emotions, and cluster 5.

CONCLUSIONS

An important aspect of the described research study is the combination of quantitative analysis with an interpretive approach. All the results we got led us to interpretive hypotheses, that become stronger if compared to, and interconnected with, the qualitative analysis performed on the same material (and partially described in Di Martino & Zan, submitted). We point out that if on the one hand, the obtained results offer extremely interesting stimuli, on the other hand they cannot provide certainties, due to the type of material we analyzed (*open* texts). In this case, we really ought to be cautious: the analysis of open texts based on lexical units only, without an analysis of the contexts within which these lexical units are used, might be problematic. To exemplify, the lemma *to_like* is not always referred to mathematics; the word *problem* might stand for a mathematical problem but also for a real life problem. Therefore, it was really important to compare results of this analysis with those of the qualitative one (described in Di Martino & Zan, ibidem): in particular, the results about the three dimensions characterizing attitude towards mathematics are confirmed.

The ‘evolutionary’ results that emerge from cluster analysis seem to be particularly interesting. A general deterioration of students’ relationship with mathematics can be clearly detected but, most of all, as the school level increases, the lemmas used to describe one’s relationship with mathematics suggest that the latter becomes more and more radical. Moreover, there seems to be a move from a phase of interest in the novelty of mathematics -the pleasure of discovery- to a phase in which succeeding prevails over the subject matter itself. One final remark: the fact that in this phase emotional aspects become more radical provides material for further reflection.

NOTES

1. The bibliography related to T-lab is available on-line: http://www.tlab.it/en/presentazione.asp

2. The collected essays constitute a convenient sample, obtained through a collaboration with teachers and heads of schools who accepted our requests. The schools are situated in six different area of Italy: from North to South.

3. Co-occurrence is when two or more lemmas are present together in the same text.

4. To calculate the cosine index between lemma X and lemma Y we have to consider a = # of essays with lemma X and Y, b = # of essays with lemma X and without lemma Y, c = # of essays with lemma Y and without lemma X. Cosine (lemma X, lemma Y) = a / square root of (a + b) x (a + c).

5. The Chi-square test is a well-known test used to check if the frequency values obtained by a survey are significantly different from the theoretical ones. T-Lab applies this test to 2x2 tables then the threshold values is 3.84 (df=1, p=0.05) or 6.64 (df=1, p=0.01).

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