Basic Characteristics of Algebraic Thinking: ›Signs as Descriptors‹ vs. ›Signs as Creators‹

A reaction to the Plenary talk by Luis Radford: Signs, Gestures, Meanings: Algebraic Thinking from a Cultural Semiotic Perspective

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"To talk about algebraic thinking is a bit risky" (Radford). But perhaps also (for the reactor): "To talk about *a talk about algebraic thinking* might be a bit risky."

To give a first impression I must confess that the plenary paper by Luis Radford offers a deep and broadened new view on: "Algebraic Thinking from a *Cultural Semiotic Perspective*".

The author puts a basic focus on the questions: What is *thinking?* – and – what especially is *algebraic* thinking? A first characterization of algebraic thinking is presented: "… algebraic thinking is about dealing with indeterminacy in analytic ways, … [and]: … There are other semiotic systems than the alphanumeric one to signify indeterminacy – natural language and diagrams, gestures, actions, and rhythm, …" (Radford).

Why this conceptual and theoretical complexity? Is it a help or a hindrance for understanding algebraic mathematical thinking? Wouldn't it be much easier to simply state: Algebraic thinking and school algebra is essentially linked to the correct use of "letters"? (perhaps because such a view directly emphasizes school algebra and learning in everyday classrooms?)

This multilayered and complex conceptual understanding of algebra and algebraic thinking can help:

• to better understand and to reconstruct the broad spectrum of factors involved in students' ways of learning and understanding elementary algebra

• to be aware of hidden difficulties that might depend on some elements of this spectrum of semiotic means (i.e. ›perception‹, as described in the plenary paper)

The development and characterization of algebraic thinking in the broader "zone of emergence of algebraic thinking" is then related to three forms of generalizations: (Factual, Contextual, Icon Formulas – Symbolic Formulas)

Factual

- "In factual generalizations, a "formula" should better be understood as an embodied predicate …"
- "… this generalization occurs within an elementary layer of generality – one in which the universe of discourse does not go beyond particular figures …"
- "… the semiotic means refer to some given objects of reference …" (Radford).

Contextual ("*contextual* generalization"):
- "this generalization applies to non particular figures"
- "… an explicit *contextual* description of the figure, supplemented sometimes by the actions that have to be carried out to find the total of circles in this unspecified figure…" (Radford).

Icon Formulas – Symbolic Formulas
- "…endow […] formulas with new abstract meanings"
- "… transform the iconic meaning of formulas into something that no longer designates concrete objects"
- "… the terms of the formula have to be considered in relation to the signs that they contain …"
- "Resemblances and differences … must no longer be based on spatial and embodied considerations but in *morphological* ones." (Radford).

These three "stages" carefully describe a challenging path of developing algebraic thinking as a body-sign-tool mediated cognitive historical *praxis* using different sorts of objects (concrete, non particular, general) to which body gestures, tools and signs refer to for developing algebraic thinking as a "body-sign-tool mediated cognitive historical *praxis*" (Radford).

Fig. 1 Dot pattern and arithmetical formula (taken from Radford).

This developmental process is accompanied – what was mentioned in the plenary – by an increase of *interaction / communication* between students and also their teacher.

I would like to make two major comments on this developmental way to algebra and algebraic thinking:

First Comment: The spectrum of semiotic means – body, gestures, actions, rhythm, artefacts, signs, symbols – is this meant as:

- Tools for *thinking* and / or tools for *communication*?

  " … thinking is a complex form of reflection mediated by the senses, the body, signs and artifacts." (Radford)

" … ways of thinking result not only from the engagement of the student with mathematical problems but also from the interaction between the students and teachers." (Radford).

It would be most interesting to understand in more concrete details the role of interaction in the development of thinking:

- Could students' algebraic thinking be supported when they become aware of the body-sign-tool use of other students?

- Apprehending gestures, signs and artefacts explicitly also as *means of communication*, allows for investigating the development of students‘ thinking while s/he is trying to take over the perspective of another student – and that is important for students to make changes in the epistemological status of algebraic signs.

Second Comment: School algebra and “letters”: body, gestures, actions, and rhythm, artefacts, signs, symbols: Are they tools of *describing* and / or tools of *creating*?

- “…alphanumeric symbolism is not the only way to designate and express indeterminacy.”
- “… my idea is not to challenge the power of symbolic algebra.”
- “… there are many semiotic ways (other than, and along with, the symbolic one) in which to express the algebraic idea of unknown,…” (Radford).

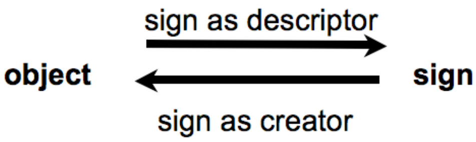

Fig 2. How are signs and objects related to eachother?

“… the simple picture of an independent reality of objects providing a pre-existing field of referents for signs conceived after them, in a naming, pointing, ostending, or referring relation to them, cannot be sustained. … The result is a reversal of the original movement from object to sign. The signs of the system become creative and autonomous.” (Rotman 1987, 27/8).

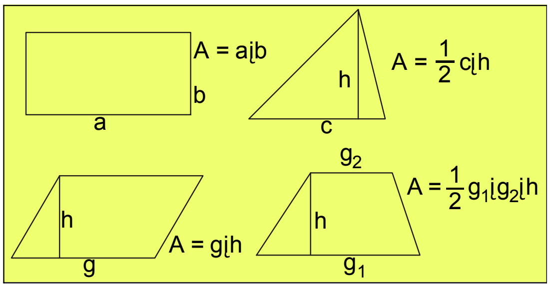

Fig. 3 Different area formulas for different geometric shapes – signs *describing* objects.

Let me illustrate this idea by using an elementary example from school geometry. Students are often used to understand the area formulas for different elementary geometric area shapes as separated and with out any conceptual connections (see Fig. 3). The ordinary understanding can be summarized as: Four different area formulas with signs *describing* “objects”!

One Formula for all four areas (rectangle, triangle, parallelogram, trapezium) with signs *creating* mathematical objects defined by new relations – namely the height (h) and the midline (m) (Fig. 4).

$$\mathbf{A = m \cdot h}$$

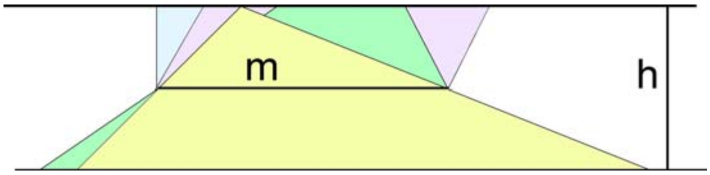

Fig. 4 One area formula for different geometric are shapes – signs *creating* objects.

The new – theoretical formula – uses signs to create new mathematical objects by the defining relations and conditions hidden in the concepts of height and of midline.

I certainly can agree with the statement: "… letters have never been either a necessary or a sufficient condition for thinking algebraically." (Radford), but when signs as *descriptors* change to signs as *creators* then they become indispensable – they do no longer refer to objects – but they now create objects.

- (School-) Algebra and algebraic thinking is not dependent on letters – when simply used as *names for objects*.
- Generic algebraic thinking and also developed school algebra needs letter-like semiotic inscriptions that exist and live autonomously within the operational mathematical structure.

So one could critically comment on the important role signs play in mathematical constructions: "… what … *teachers and students* think they are doing – using *algebraic symbols* as a transparent medium for describing a world of presemiotic *geometric pattern sequences* – is semiotically alienated from what they are … doing – namely, creating that reality *of geometric pattern sequences* through the very language which claims to "describe" it." (Rotman 200, 36/7). [A quotation taken from Rotman (2000) and slightly modified to the development of algebraic thinking in schools.]

The early development of algebraic thinking – described and elaborated by Radford – as a *body-sign-tool mediated cognitive praxis*: for the learning student it is firstly a "movement from object to sign" – with *signs as descriptors.*

Here some thought provoking and research requiring questions could be posed for stimulating further careful scientific investigations in this interesting mathematics education research area:

- How a "reversal" – a movement from sign to object – could be realized?
- How is it possible that "the signs of the system become creative and autonomous" for the learning students later?
- What could be an adequate *body-sign-tool mediated cognitive praxis* for algebra with *signs as creators*?
- "The narrative has to collapse…" (Radford) for *signs as descriptors*. But what is the new narrative about for *signs as creators*?

- How is it possible to induct students into a *creative and autonomous world of elementary algebra* after they have gone their path from “objects” to “signs”?

In this regard I completely agree with Luis Radford's statement: *“Unfortunately, I do not have a cure for this problem – and I do not think that there is a royal road to … algebra.”*

REFERENCES

Rotman, B. (1987). *Signifying Nothing. The semiotics of Zero*. Stanford University Press: Stanford, California.

Rotman, B. (2000). *Mathematics as Sign. Writing, Imaging, Counting*. Stanford University Press: Stanford, California.