PEOPLE AND THEORIES

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People are an essential consideration in networking theories. The dialectical relationship between people and theories is dynamic with regard to development. It is important to consider why people want to develop theories, their motive(s), and how they apply theories, their interpretation(s). Ascriptions of agency to theories, as ways of producing understandings or actions, need to be tempered by considerations of agency on the part of researchers applying theories. The appropriation of a theory by a person starts (and may end) with constructs from the theory.

I saw my role in this CERME plenary as that of reactor to Angelika's contribution. My first reaction is *I think what Angelika is doing is very interesting*. Indeed, "interesting" may be too weak a word as networking local theories is something that I expect to rise to prominence in mathematics education research in the near future. Angelika, Tommy, Ferdinando and I agreed at the outset that the CERME plenary should generate debate. With "debate" in mind I wanted a theme to my reaction that was honest (I did not want to generate debate by simply saying the converse of what Angelika was saying) but addressed issues that Angelika did not address and I focused on people because people network theories.

This paper follows my talk very closely and is in three parts. In the first part I argue that theories cannot be separated from the people theorising. In the second part I look at researchers' motives for adopting/creating theories and their interpretations of data. In the third part I argue that in practice researchers often appropriate parts of theories. I preface these three parts with some preliminary remarks.

PRELIMINARY REMARKS

I am not sure, in general terms, what a theory is. I am aware of discussions in mathematics education and in the social sciences of discussions of this issue. Prediger, Bikner-Ahsbahs & Arzarello (2008) consider the variety of theories in mathematics education research and conclude that 'We can distinguish theories according to the structure of their concepts and relationships' (ibid., p.168). A recent consideration of this issue in the wider social sciences is Ostrom (2005) who considers the difference between frameworks, theories and models with regard to her research interest, institutional analysis. Acknowledging that these terms 'are all used almost interchangeably by diverse social scientists' (ibid., p.27) she goes on to differentiate them according to their function in analysis: frameworks help to identify elements; theories help to specify relevant components for specific questions; models clarify assumptions regarding variables. These authors provide cogent considerations but I am still not sure what a theory is; but I know one when it is presented to me, e.g.

the Theory of Didactical Situations in Mathematics (TDS; Brousseau, 1997). So I will speak of theories with regard to the theories I am aware of that are referred to in mathematics education research.

Mathematics education researchers employ what might be called “out-of-mathematics-education theories” as well as those created within mathematics education. There are, I feel, problems for mathematics education researchers in both of these kinds of theories. The majority of us in mathematics education research are not experts in out-of-mathematics-education theories; most of us do not have a critical insight into all of their ramifications due to a lack of immersion in the academic literatures of philosophy, psychology, sociology etc. Out-of-mathematics-education theories can also miss fine mathematics detail (people interacting with mathematical relationships) that we are so very interested in. Mathematics education theories, on the other hand, can miss the big picture; the sites (classroom, workplace) of most mathematics education research are but a part of the lives of the participants.

THEORIES CANNOT BE SEPARATED FROM THE PEOPLE THEORISING

I present seven statements under this theme.

1 Theories do not exist without people

A theory without someone to interpret the theory is only words (and maybe symbols). A theory accordingly can be considered as a pair, (theory, person). For any given theory and *n* people there will be *n* such pairs. Some pairs will be almost identical, some will differ greatly; any given pair will depend on the interpretation of the theory by the person in the pair.

2 Theories develop and people develop them

(theory, person) pairs are dynamic, they change/develop. It is a bit sad if this does not happen! People develop in their understanding of a theory and through scholarships and research they develop theories. It can also be the case that a person appropriates particular development in the history of a theory, e.g. I am influenced by Davydov’s (1990/72) mid 20th century use of activity theory but activity theory has developed in numerous ways since his time.

3 People hold implicit and explicit theories

I have heard it said that people can only see via a theory and that people adopt theories. I think both claims, without further explanation, are rubbish. We “see” via the artefacts (including implicit and explicit theories) available to us in our phylogenic and ontogenic development (Wartofsky, 1973). The word “adopt” is too passive. I think there is, to draw a close analogy with Guin & Trouche’s (1999) ‘instrumental genesis’, a theoretical genesis in which people with initial ideas (I_I) interact with a theory (T), the person with I_I and T reviews experiences and, if T is convincing for that person, then (T′, P) develops. NB This account is certainly too simple but suffices, for my purposes, as an initial hypothesis.

4 Many people subscribe to more than one theory

Theory_1 informs us on … and Theory_2 informs us on … With regard to person-theory pairs we do not just have (T1, P) and (T2, P) but (some combination of T1 and T2, P). Maybe this is where networking theories becomes really important.

5 A continuum with regard to theory expertise

At one extreme there are leading theorists; at the other extreme there are those who do not appear to understand a theory; and there are many intermediate positions. In France there is a maximal element in the pair (TDS, Brousseau) but I believe that it is intellectually dangerous to grant absolute authority to leading theorists.

6 Mathematics education researchers network and partially absorb others' ideas

We (mathematics education researchers) read but we also talk – to people. I was introduced to the anthropological theory of didactics (ATD; Chevallard, 1999) by talking to J-b Lagrange. I did eventually read the paper but my understanding of the theory was through my conversations with J-b Lagrange and his research.

1-7 Theories arise in communities and cultures

As academic we may aspire to objectivity but we cannot escape cultural and community influences in our work. The plenary debate took place in France and I have alluded to ATD and TDS above in homage to mathematics education theories from France. I referred to J-b Lagrange introducing me to ATD above but our relationships with this theory will be distinct simply because J-b Lagrange is a French mathematics educator and his, and not my, identity is partially shaped in relation to this French theory.

A different example is provided by Nkhoma (2002), a black South African mathematics educator. This paper comments on attempts to import learner-centred instruction from the USA into Black SA classrooms:

> It is not beneficial to stereotype classrooms practices into, simply, teacher-centred therefore bad, and learner-centred therefore good ... rich experiences can be provided in practices that appear teacher-centred. (p.112)

In reading Nkhoma's paper it is difficult not to feel his anger at the importation of a "foreign" theory.

MOTIVES AND INTERPRETATIONS

I now look deeper into people and theories and examine researchers' motives for adopting/creating theories and their interpretations of data within theoretical frameworks.

To examine motives I consider a paper by Kieran & Drijvers (2006) and a response to this paper by Monaghan & Ozmantar (2007). Kieran & Drijvers worked in a form of ATD with they call "task-technique-theory" (TTT). It is a long and interesting paper on the interplay between computer algebra systems (CAS) techniques and by-hand techniques. The students were working on factorisations of x^n-1, and the CAS required specific values for n and did not give the classic factorisation every time. They state:

> According to the TTT … a student's mathematical theorizing is deemed to be intertwined with the techniques … tasks … we distinguish the following three theoretical elements.
>
> 1. Patterns in the factors of $x^n - 1$: Seeing a general form and expressing it symbolically
>
> 2. Complete factorization: Developing awareness of the role played by the exponent in $x^n - 1$ …
>
> 3. Proving: Theorizing more deeply on the factorization of $x^n - 1$ (pp.242-243)

We viewed this with regard to Hershkowitz, Schwarz & Dreyfus's (2001) abstraction in context (AiC) recognising and building-with actions. Students' prior work had involved factorising binomial expressions with regard to the difference of squares and sums and differences of cubes. They *recognise* that expressions of the form x^5-1 can be factored and *build-with* this knowledge artefact to produce factorisations $x^5 - 1 = (x-1)(x^4 + x^3 + x^2 + x + 1)$.

We also viewed it via Davydov's ascent to the concrete (an inspiration for AiC) whereby an abstraction progresses from an initial entity to a consistent final form. This progression depends on the

> disclosure of *contradictions* between the aspects of a relationship that is established in an initial abstraction ... It is of theoretical importance to find and designate these contradictions. (p.291)

One student says, with regard to $x^{135} - 1$, 'how are we supposed to know if it's valid or not?' – the initial abstraction is fragile and limited to specific whole number exponents. The teacher introduced $x^n - 1$ and this required a *vertical* reorganisation of their knowledge. Student work shortly after includes *recognising* and *building-with* but on a higher vertical level than when the exponents were specific whole numbers.

Later in the Kieran & Drijvers paper we see students grappling with contradictions created from attempts to reconcile paper-and-pencil and CAS techniques and how this attempt at reconciliation led to synthesis and further insights.

> The difference-of-squares 'proof', for example, with its accompanying treatment of the case $x^{n/2}$ +1, for odd values of $n/2$, helped to extend even further the thinking of students in the class. The $x^n - 1$ conjecture, which had issued from the earlier work of some students with the factoring of $x^{10} - 1$, helped others to integrate their ideas about odd, even, and prime exponents - theoretical ideas that had been generated in interaction with various CAS and paper-and-pencil techniques ... (p.253)

We also viewed Noss & Hoyles' (1996) situated abstraction and webbing as closely related to Davydov's ascent to the concrete

> learning as the construction of a *web* of connections – between classes of problems, mathematical objects and relationships, 'real' entities and personal-specific experiences. (p. 105)

We are certainly networking theories (though in a different sense to how Angelika networks theories) but we also attend to differences arising from our ascribed personal motives of the theorist to theorise. Minimal ascriptions of motive are:

- Kieran & Drijvers – to understand the interplay of machine and by-hand techniques;
- Davydov – to develop theory to aid instructional design;
- Noss & Hoyles – to account for mathematical meaning making and the structuring of mathematical activities;
- Hershkowitz et al. - dissatisfaction with empirical theories of abstraction re students' actual development.

I now briefly consider interpretation. Angelika talks about a case where two theories are successfully networked. In CERME 6 Working Group 9 "Different theoretical perspectives and approaches in research: Strategies and difficulties when connecting theories" some papers focused on difficulties in networking theories. This is not new, six years ago Even & Schwarz (2003, p.283) commented 'We exemplify how analyses of a lesson by using two different theoretical perspectives lead to different interpretations …' I have no problem with this but question whether different interpretations are only the result of different theoretical perspectives. Research is often a team effort. Have you ever disagreed with a colleague during data analysis? I have and the outcome is usually compromise or an impasse. I think this tends not to get reported in papers. This is a further refinement to my point 1 **'**any given pair will depend on the interpretation of the theory by the person in the pair' but with regard to the interpretation of data via the theoretical perspective. Is at least one interpretation wrong?

ISSUES, CONSTRUCTS AND CONSISTENCY

In this final section I consider the extent to which theories lead research, theories and constructs and consistency issues. This section expands on my "motive" considerations above in that people often turn to/develop theories in order to address issues that they regard as important. But often it is parts of theories that they appropriate and this can lead to potential consistency problems.

Radford (2008, p.320) states that 'a theory can be seen as a way of producing understandings and ways of actions based on' a system of basic principles, a methodology and a set of paradigmatic research questions. I take partial issue with this, as a generality. I consider issues and specific research projects.

In my experience there are fundamental issues that mathematics education researchers return many times over their working lives. In my case one of these is the link between school mathematics and out-of-school mathematics. I have grappled with this over many decades. Research questions, methodologies and principles have come and gone but the issue remains. The construct "transfer" often arises in discussions of this issue for something akin to transfer is central in linking school to out-of-school maths. Personally I hate the term and largely agree with the old Lave (1988) critique but the issue haunts me and I am prepared to consider any theory that will further my understanding of this issue.

I now consider research projects with regard to theories. These generally have a shorter time scale than "issues". I, like most CERME delegates, write formal proposals with a theoretical framework, research questions and methodology. Almost every time, however, I develop in the process. I encounter unexpected phenomena (and revise the research questions) or experience problems in data analysis (and revise the methodology) or develop the theoretical framework. My point is that sometimes theories lead research, sometimes they do not; and, whatever the case, researcher development is in the dialectic mix. I now consider constructs.

A construct may be regarded as a proper part of a theory, e.g. *didactical contract* in TDS. I think people often appropriate a construct of a theory without appropriating the whole theory. I further think that if a person appropriates a theory, then they appropriate constructs of that theory prior to appropriating the theory. I included 'I think' in the previous two sentences because I base these remarks on my reflections of my own development; with regard to my point on "theoretical genesis" in (3) above I am not aware of research that traces the genesis of theory acquisition amongst academics but such research would be relevant to my reflections.

As an instance of construct appropriation in my own development I return to my comments above that J-b Lagrange introduced me to ATD. This is true, he did introduce me to ATD, but what I initially appropriated was the 'task-technique' part of ATD (and this focus as the only part of ATD I made sense of lasted several years). This focus was, I am sure, due to my prior experience. I had long experience of working with students and with teachers on using ICT-mathematics tools and the term "technique" in my country's everyday mathematics-education-speech refers to value-free manipulation. To view, as Lagrange's exposition of ATD does, techniques as not only being not value-free but techniques having both epistemic and pragmatic values and being viewed with respect to tasks was, quite frankly, a huge revelation and very relevant to my ICT work. Monaghan (2000) provides published evidence of this narrow focus. Perhaps it was due to the big impact this construct had on my thinking that appropriating other aspects of ATD took me a longer period of time.

I do not think the above (ATD, me) is an isolated example. I think the theory-person development is similar to that which I outline in (3) above: a person with a theoretical approach (T, P) interact with a construct C, the person with theoretical approach and construct reviews experiences and, if C is convincing for that person, then (T+C, P) develops. As with my comments in (3) this is almost certainly simplistic.

As I prepared for the CERME plenary I kept returning to T and C, in (T+C, P), with the thought that T and C must, in some sense, be consistent. I tried to formulate consistency criteria but failed, my attempts to frame consistency criteria ended with grand but empty phrases. This failure may be a personal failure but it may be that there is not a suitable meta-language in which to couch consistency criteria for non-specific theories and if this is the case, then perhaps we just need to resolve consistency tensions in our own research in case and theory specific ways.

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