NETWORKING OF THEORIES: WHY AND HOW?

Angelika Bikner-Ahsbahs

Universität Bremen

This contribution presents a short overview of the current discussion about a meta-theoretical standpoint of working with theories: the networking of theories as a practice of research. It explains some principles on which this kind of research practice is based. Based on a methodological frame, an example is worked out showing how the networking of theories can lead to deepening insight into a problem and to methodologically reflecting the process of connecting theories.

During the last four years a new kind of research practice has been investigated: the networking of theories (Bikner-Ahsbahs & Prediger, 2006; Prediger, Arzarello, Bosch & Lenfant, 2008; Prediger, Bikner-Ahsbahs & Arzarello, 2008). What does this mean? Networking of theories is regarded as a systematic way of linking theories (Bikner-Ahsbahs & Prediger, 2009). Linking theories is not a new idea. Within conceptual frameworks (Eisenhart, 1991) different theoretical approaches are used to build a consistent frame for research. In the case of design research, Cobb (2007) argues for connecting theories as a kind of “bricolage” in order to capitalize on different views. In addition, triangulation has developed as a kind of evaluation criterion for qualitative research (Schoenfeld, 2002; Denzin, 1989).

A lot of scholars in the community of mathematics educators have already triangulated different theoretical perspectives in their research projects to enhance insight. However, the networking of theories means more than that, it means going beyond triangulation and developing methodological tools for *systematically* connecting theories, theoretical approaches and theory use. To be a bit more precise, I will describe the networking of theories as a process of

- analyzing the same phenomenon in mathematics education from different theoretical perspectives or within different theories,
- reflecting the use of these different theories,
- respecting the identity of each theory,
- exhausting the possibilities for linking them, and
- linking them

Meanwhile some research has been executed which has led to the development of strategies, methods and techniques for the networking of theories and to some insights about the benefit that can be reached this way (Prediger et al., 2008). An interesting example is shown by Kidron (2008). Based on data she explains in detail why more than one theory is needed to understand limit concepts. She networks three theories analyzing the discrete continuous interplay of limits and shows how these three theories - the concept of procept, the instrumentation approach, and the theory of abstraction in context - provide complementary insights and, hence, deepens understanding of limit concepts like the definition of the derivative. This way, Kidron is also able to show strengths, weaknesses and the limitations of the three theories.

On a product level, the networking of theories might lead to types of networked theories. However, since only first steps have been made in this direction, e.g. at CERME 4, 5, and 6 and elsewhere (ZDM 40 (2) for an overview), it is not yet clear, how these products might look. As Radford (2008) stated, the kinds of products will depend on the aims of networking, for instance, developing the identity of theories, experiencing the limits of linking theories, developing new methodological tools and new kinds of questions etc. One current result of this effort is a landscape of networking strategies that was worked out on the base of the contributions to the theory working group of CERME 5 (Prediger, Bikner-Ahsbahs & Arzarello, 2008).

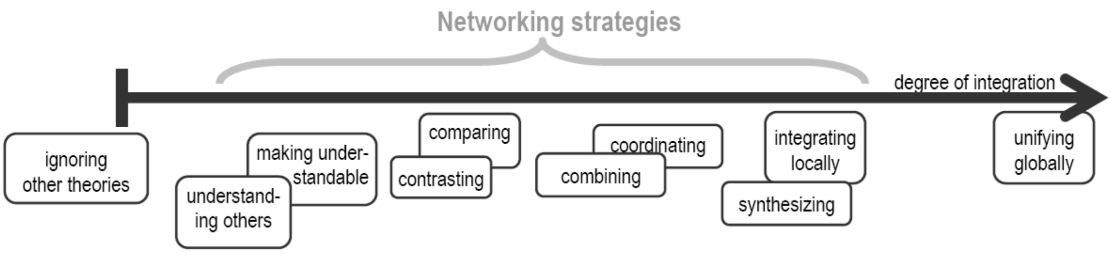

Figure 1: Networking strategies (Prediger et al., 2008)

This landscape represents a continuum of strategies for relating theories and theoretical approaches to each other including the extreme poles of non-relation between theories on the one hand and unifying them globally on the other. The term connecting theories means all kinds of building theory relations whereas networking strategies exclude the extreme poles. This landscape is ordered in complementary pairs of strategies according to their potential for integration. An example below will illuminate some of these strategies.

The idea of the networking of theories is based on some principles, the principle of

1. regarding the diversity of theories as a form of scientific richness,
2. acknowledging the specificity of theories,
3. looking for the connectivity of theories and research results,
4. developing theory and theory use to inform practice.

The first two principles acknowledge the diversity of theories in the field of mathematics education and accept diversity as a resource for scientific progress (Bikner-Ahsbahs & Prediger 2009). The third principle assumes that research in mathematics education produces much more connectivity than is visible at first sight. Related to different viewpoints, the networking of theories provides the opportunity to make these implicit aspects more explicit. The different ways of connecting theories presented at the theory Working Group 9 at CERME 6 illustrate the value and variability of the third principle. The fourth principle does not necessarily need to be shared by all the researchers in our field; however, it helps to keep research about the networking of theories grounded in practical problems producing concepts with an empirical load that is not empty (Jungwirth, 2009).

We are all busy doing research within and about mathematics education. If research demands the use of different theories we should use them being aware that this has to be justified somehow. But why is it necessary to engage in a meta-theoretical discourse about theory use? Why do we need to reflect about linking theories?

1. WHY DO WE NEED THE NETWORKING OF THEORIES?

In order to inform practice, theories facing specific practical problems are needed. Therefore a variety of theories of middle range scope, so-called foreground theories (Mason & Waywood, 1996), have been developed, for instance different theories about abstraction (Mitchelmore & White, 2007). Furthermore, the objects of mathematics education research can be viewed from different theoretical perspectives, e.g. cognitive, semiotic, social, …. Thus, a variety of research perspectives and various theories have been used leading to theory development in different directions. Researchers normally know what their theory is about but often the theories' limitations remain implicit. Limitations of theories can be experienced through the failure to apply them. A systematic way to provoke these experiences is critique. It can lead to a change of view (Steinbring, 2008) but also to the development of theories in that concepts and their limitations become more precise, additional concepts are constructed or the theories' parts become interconnected more deeply. Therefore, the diversity of theories can be regarded as a resource for and a consequence of critique (see also Lerman, 2006) and is scientifically necessary.

However, the diversity of theories has also caused problems (Prediger, Bikner-Ahsbahs & Arzarello, 2008), for instance a language problem and a connectivity problem. The first problem arises whenever researchers from different theoretical traditions try to talk to each other, since different theories might use the same words in different ways (e.g. social interaction in different tradition, see for example Kidron et al., 2006) or different theories use different words for the same or very similar phenomena (for example *interest-dense situation* and *a-didactic situation*, see Kidron et al., 2006). The connectivity problem is related to the question of how research results from different theoretical traditions can be connected to understand and solve practical problems.

So we need scientific ways of dealing with the diversity of theories that encounter these problems. The idea of the networking of theories might be a promising concept for this task which has the potential to induce the development of a common language among different research traditions and to investigate the ways in which theories and research results can be linked.

I will now present an example that shows how these goals can partly be achieved.

2. HOW CAN THEORIES BE NETWORKED?

In order to connect theories, a framework is needed that allows building relations among them. Radford assumes a semiosphere that comprises the collection of the semiotic parts of the different theoretical cultures within mathematics education (Radford, 2008). He explains that a semiosphere is

> "an uneven multi-cultural space of meaning-making processes and understandings generated by individuals as they come to know and interact with each other." (Radford, 2008, p. 318)

Theories within this semiosphere can be described as triplets (P, M, Q) that establish languages and allow the building of relationships between them. In these triplets, P represents the system of principles, M is a sign for a system of methodologies that can be connected to these principles in an appropriate way, and Q represents a set of paradigmatic questions related to P and M. A connection between two theories establishes a specific relation that depends on the theories' structures and the goal of this connection.

Using this frame, I will present an example of the networking of two theories illuminating the benefit of critique for developing insight into a problem. Methodological reflections will uncover five steps through which the process of networking has passed. This example refers to a data set that was used by Arzarello and Sabena (Arzarello, Bikner-Ahsbahs & Sabena, 2009). I will use it to explicitly show benefits and limits of networking practices.

An episode about the growth of the exponential function

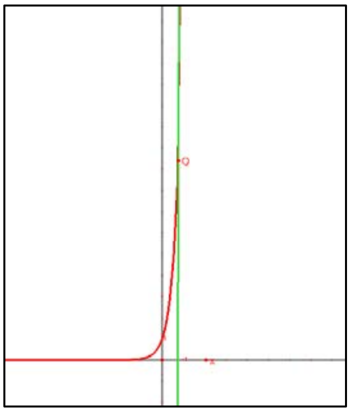

Figure 2

Two students of grade 10 are working in a pair on an exploratory activity on the exponential function and its growth. They use Cabri Geomètre to explore the graph's tangents. In this situation the teacher asks the students: What happens to the exponential function for very big x. The transcript shows the dialogue among the students G, C and the teacher.

Now I would like to invite the reader to participate in a short exercise using just a few pictures.

Figure 2 shows the computer screen the students observe.

Figure 3 presents two pairs of pictures. The left pair shows the student's gestures accompanying his utterances: his left hand goes up. The right pair illustrates the teacher's gestures accompanying his utterances: he crosses two fingers going to the right.

Figure 3: The student's gestures (left pair of pictures) and the teacher's gestures (right pair of pictures)

Please imagine for a moment what the teacher and the student are talking about. How does the student answer the question about the growth of the exponential function for very big x and how does the teacher react? – The student describes his perception of the screen meaning that the graph seems to approximate a vertical straight line. The teacher wants to show that this is wrong because every vertical straight line would be passed by the graph.

We now consider the beginning of the discussion.

1 G: but always for a very big this straight line (pointing at the screen), when they meet each others, there it is again…that is it approximates the, the function very well, because…

2 T: what straight line, sorry?

3 G: this … (pointing at the screen) this, for x very, very big

With broken language the student tells something about the growth of the exponential function for big x. This broken language is an indicator for thinking aloud. Saying "sorry" the teacher interrupts the student's train of thought indicating that this question is important. However, the student does not answer the question. Instead, he defends the choice of the term "vertical straight line". The student reacts to the so-called illocutionary level (telling something through saying something) of the teacher's question. Illocutionarily, the teacher's disruption is an indicator that there is something wrong with the vertical straight line while on the locutionary level (what is said) the teacher wants to know what vertical straight line G refers to.

During the following dialogue the student and the teacher talk about the function's growth, but, illocutionarily they negotiate about whose train of thought will be followed. The student begins to become involved repeatedly but is disrupted every time. In the end the teacher wins.

We now have a look at the last utterances.

14 T: eh, this is what seems to you by looking at; but you have here x = 100 billion, is this barrier overcome sooner or later, or not?

15 G: yes

16 T: in the moment it (the vertical straight line) is overcome, this x 100 billion, how many x do you have at your disposal, after 100 billion?

17 G: infinite

18 T: infinite… and how much can you go ahead after 100 billion?

19 G: infinite (points)

We see: The teacher is involved in arguing and the student's involvement is reduced to one (or two) word sentences (for a more detailed analysis of this episode see Arzarello, Bikner-Ahsbahs & Sabena, 2009).

A case of networking

Two theories were used to understand the episode above (for a short introduction: Arzarello et al., 2009b); a theory about the emergence of interest-dense situations and a theoretical approach about how a semiotic game between the teacher and the students shape the transition of mathematical knowledge.

The perspective of interest-dense situations

The first analysis is done from the view of the theory of the emergence of interest-dense situations. This theory – regarded as a triplet – is based on the following principles, methodology and questions:

- P1: Mathematical knowledge is socially constructed through interpretations of the others' utterances (see as well: Kidron et al., 2008).
- P2: The object of research is "meaning-making" within the process of social interaction.
- P3: In an interest-dense situation successful learning takes place as learners are deeply involved in the activity of social interactions constructing mathematical meanings in a deepening way. In these situations learning with interest is supported.
- P4: If the teacher focuses on the students' train of thought the emergence of an interest-dense situation is supported, if the teacher pushes the student to follow the teacher's train of thought the emergence of an interest-dense situation is hindered.
- M: Main part of the methodology is speech analysis on three levels. On the locutionary level an interlocutor says something; on the illocutionary level he tells something by saying something; on the perlocutionary level the intentions and the impact are taken into account.

The analysis is executed according to three questions:

- Q1: Did an interest-dense situation emerge?
- Q2: What conditions fostered or hindered it?
- Q3: How was mathematical knowledge constructed?

From the perspective of the emergence of an interest-dense situation the dialogues do not lead to increasing student involvement. Locutionarily (what is said) the student and the teacher negotiated the growth of the exponential function for very big x.

Illocutionarily (telling something through what was said) the student and the teacher struggle whose train of thought is followed. In some instances the teacher starts to focus on the student's thinking process but changes his argumentation immediately according to his own train of thought, namely to work out a "proof of contradiction": Given a vertical straight line –seen as a asymptote- this line would be passed by the graph of the exponential function. The degree of the student's involvement decreases while the teacher follows his own ideas, although the teacher tries to connect them with the student's utterances. Several times, an interest-dense situation is about to emerge, but this process is interrupted by the teacher's behaviour forcing the student to follow the teacher's train of thought. The construction of mathematical knowledge is carried out by the teacher; the contribution of the student is very low.

The semiotic bundle approach (Arzarello, 2006; Arzarello et al., 2009a)

- P1: Mathematics is transferred through a semiotic game with the help of the teacher.
- P2: The object of research is the semiotic game and its semiotic bundle.
- P3: Successful learning is interiorisation of mathematics by the help of the semiotic game.
- M: Analysis of the semiotic game according to the use of the semiotic bundle meaning the interplay of speech, gesture, representations and the transition of sign use.
- Q1: How was the mathematical content transferred through the semiotic game?
- Q2: Did the teacher tune speech and gestures with the student's ones?

From the semiotic bundle approach the semiotic game seems to be successful: The teacher takes over the student's words, using more precise explanations or following the students' ideas for a while. He points to the computer screen showing what is wrong in the way of the student's perception. He underpins his explanation and the proof of contradiction using gestures and tunes his words with those from the student. As far as the teacher is concerned, the semiotic game seems to be fruitful. From the perspective of the teacher's options to engage in the semiotic game he has done a lot of things to successfully transfer the mathematical content to the student. The student seems to be convinced, since, in the end, he correctly answers the teacher's questions.

The networking of the theories

At first glance, these results seem to be contradictory. Each theory serves as a resource for criticizing the other. After the networking process we found that the results are complementary since we could add an aspect that provided the integration of the different results: The teacher tries to tune his words with those from the student; but the gestures show that the epistemological views of the teacher and the student are different and they do not converge. The student uses his perception and extrapolates the growth of the graph of the exponential function for very big x: the function seems to grow like a vertical straight line. The teacher's view is theoretical requiring potential infinity. Neither the teacher nor the student is able to bridge this gap.

Some methodological reflections

The contradictory results were a reason for us to meet and refresh our analysis. During this process five steps emerged:

1. *Re-analysis*: Analysing the data together again from both perspectives made our theories mutually more understandable.
2. *Comparing and contrasting*: As we contrasted and compared our theories we began to juxtapose some principles and methodologies. For example: our views on theory require different uses of the data.
3. *Establishing a common ground*: From the perspective of interest-dense-situations I could explain how the emergence of an interest-dense situation was hindered, but I could not explain why hindrance occurred. We agreed that the semiotic game was not successful as shown from the other theoretical perspective. The question was: why?
4. *Complementary analysis*: A hypothesis occurred as we looked at the semiotic game, the gestures and the speech complementarily: The student's epistemological resource was his perception of the computer screen: he extrapolated the growth of the exponential function for very big x. The teacher's epistemological resource was theoretical. This caused a gap that could not be bridged.
5. *Establishing an inclusive methodology*: We used the three levels of speech in a complementary way for the analysis of gestures and utterances and re-analysed the data carefully. Again we reconstructed the gap between the epistemological resources that could not be bridged through the semiotic game as it was executed.

Conclusions

Did we move forward? Well – yes, we did. The starting point was the contradiction of our results that served as a resource for critique and a challenge for the networking of our theoretical backgrounds. We developed a common methodology including gesture analysis and the levels of speech into one analysis. We have gained a methodological overlap but we do not know yet whether our views will converge. If we do not dig too deep we can say we followed the same question: How is mathematical knowledge gained? However, this question is still understood a bit differently because our principles and paradigmatic questions remained the same. In the end, we deepened our insights and widened our theoretical perspectives. This was possible because the grain sizes of analysis were similar and the theories' principles were close enough to include the epistemological resource as a matter for explanation.

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